Journal of Optics

PAPER

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J. Opt. **15** (2013) 035502 (5pp) doi:10.1088/2040-8978/15/3/035502

Analysis of a channel-drop filter based on dispersive waveguides and two resonant cavities

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Received 30 October 2012, accepted for publication 15 January 2013
Published 29 January 2013
Online at stacks.iop.org/JOpt/15/035502

Abstract
By use of coupled-mode theory we analyze a channel add–drop filter based on two dispersive waveguides symmetrically coupled with two resonant optical cavities. We show new solutions for the channel-drop filter processes compared to the solutions found by Manolatou *et al* (1999 *IEEE J. Quantum Electron.* **35** 1322). For a special choice of the dispersion of the waveguides, we reveal a frequency region with sufficient total reflection.

Keywords: channel dropping, dispersive waveguides, optical micro-cavities

(Some figures may appear in colour only in the online journal)

1. Introduction

Micro-cavities or resonators formed by point defects and waveguides formed by line defects in photonic crystals (PhCs) have been the subjects of a great deal of research because of their capability to confine photons within a small volume, and they are expected to be key building blocks for miniature photonic functional devices and photonic-integrated circuits. Among various PhC-based devices, ultra-compact channel-drop filters (CDFs) based on resonant coupling between cavity modes of point defects and waveguide modes of line defects have drawn primary interest.

Formally, the CDFs can be split into two basic structures: four- and three-port systems, although the resonant transmission between continua through resonant micro-cavities is a key mechanism in both structures. A single standing wave cavity side coupled to a signal or bus waveguide can only pick up half of the signal power. In order to couple out all of the signal power a second side coupled resonator–reflector is required [2]. Two identical coupled micro-cavities give rise to, at least, two resonances of different symmetry. Then a degeneracy of these resonances provides 100% drop efficiency at the resonant frequency [1, 3]. In order to improve passband characteristics of the CDF a more sophisticated design of micro-resonators [3–5] or micro-rings [6–10] is explored.

The mass of CDFs based on two resonant micro-cavities was studied theoretically and practically: four- [1, 5, 9, 11–18] and three-port systems [15, 18–22]. The four-port system is typically two parallel waveguides with two optical micro-resonators in between, where each resonator is presented by a single monopole eigenmode. Therefore, these two micro-resonators can be substituted by a single micro-cavity with higher order eigenmodes [23–27]. A clear channel-drop operation was successfully demonstrated by employing an ultrahigh quality factor single micro-resonator and a suitably designed waveguide bend [28]. Very high efficiency of channel dropping was demonstrated in the PhC system of two parallel waveguides and three cavities [17]. Also we refer to the theoretical study of surface-emitting CDFs using channel-drop tunneling processes in two-dimensional photonic crystal slabs [29].

The most striking result of the CMT consideration by Manolatou *et al* [1] is that there is no reflection irrespective of the input signal's frequency, which is extremely important for integrated circuits. However, that result becomes only approximate if referring to real PhC structures. One of the reasons is the difficulty of satisfying Manolatou *et al*'s condition for the ideal reflection of the defects' discrete positions, which play the role of resonators.

Moreover, these studies did not demonstrate a practical design capable of realizing in-plane PC devices with finite

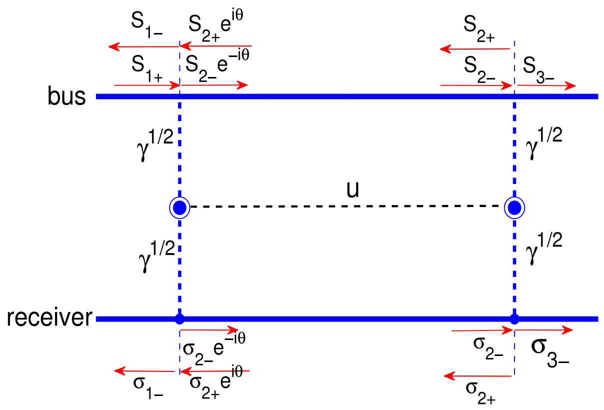

Figure 1. A symmetric add/drop filter based on two identical coupled single-mode cavities.

thicknesses because of light leakage outside of the cavity [28]. Also there is another hurdle which was disregard previously in CMT considerations. In the PhC waveguides, the optical length $k(\omega)L$ depends on the frequency ω. This gives rise to the fact that the reflection equals zero only at the selected frequency value. However, we find new solutions for the dispersive waveguides with two optical resonant cavities which can serve as the CDF.

2. Four-port system with a pair of identical single-mode cavities

Let us consider one of the most studied four-port systems shown in figure 1 consisting of two linear waveguides and two optical cavities. Following [1], we write the CMT equations for the amplitudes A_1, A_2 of two nonlinear optical cavities

$$\begin{aligned}(\omega-\omega_0-2i\gamma)A_1+(u-2i\gamma e^{i\theta})A_2&=-i\sqrt{\gamma}S_{1+},\\ (\omega-\omega_0-2i\gamma)A_2+(u-2i\gamma e^{i\theta})A_1&=-i\sqrt{\gamma}S_{1+}e^{i\theta}.\end{aligned}\qquad(1)$$

$\sqrt{\gamma}$ is the coupling constant between the cavities and the waveguides, u is a direct coupling between the cavities, θ represents the phase shift incurred as the waveguide mode travels from the first cavity to the second, and ω_0 is the resonant frequency of the cavities. Equations for transmission amplitudes have the following form:

$$\begin{aligned}S_{3-}&=S_{1+}e^{i\theta}-\sqrt{\gamma}A_1e^{i\theta}-\sqrt{\gamma}A_2\\ S_{1-}&=-\sqrt{\gamma}A_1-\sqrt{\gamma}A_2e^{i\theta}\\ \sigma_{3-}&=-\sqrt{\gamma}A_1e^{i\theta}-\sqrt{\gamma}A_2\\ \sigma_{1-}&=-\sqrt{\gamma}A_1-\sqrt{\gamma}A_2e^{i\theta}.\end{aligned}\qquad(2)$$

Since the transmission and reflection amplitudes are given by the ratios of outgoing amplitudes (2) and the ingoing amplitude S_{1+}, we put below $S_{1+}=1$. Equations (1) and (2) constitute the stationary CMT equations based on preservation of light flows [30] and were applied to consider CDF circuits in many works [1, 5, 14, 17, 18, 20].

3. Analysis of the channel-drop filtering

We start with the condition that the total reflection equals zero. Then from equation (2) we have

$$A_1+A_2e^{i\theta}=0.\qquad(3)$$

Next, we write the solution of equation (1)

$$\begin{aligned}A_1&=-\frac{i\sqrt{\gamma}}{D}[\omega-\omega_0-2i\gamma+(2i\gamma e^{i\theta}-u)e^{i\theta}],\\ A_2&=-\frac{i\sqrt{\gamma}}{D}[(\omega-\omega_0-2i\gamma)e^{i\theta}+2i\gamma e^{i\theta}-u],\end{aligned}\qquad(4)$$

with the determinant

$$D=(\omega-\omega_0-2i\gamma)^2-(2i\gamma e^{i\theta}-u)^2.\qquad(5)$$

Using equations (3)–(5), we obtain the equation which defines the phase θ at which we have no reflection:

$$e^{i\theta}=\frac{u\pm\sqrt{u^2-(\omega-\omega_0)^2-4\gamma^2}}{\omega-\omega_0+2i\gamma}.\qquad(6)$$

Moreover, we imply that the light has to fully drop into the lower waveguide $|\sigma_{2-}|^2=\gamma|A_1e^{i\theta}+A_2|^2=1$. Combining this condition with equation (3) we obtain

$$4\gamma|A_1|^2\sin^2\theta=1.\qquad(7)$$

On the other hand, substituting (3) into equation (1) we have

$$\gamma=|A_1|^2[(\omega-\omega_0)^2-2u(\omega-\omega_0)\cos\theta+u^2].\qquad(8)$$

Finally, combination of equations (7) and (8) gives us the following equation:

$$\omega-\omega_0=u\cos\theta\pm\sin\theta\sqrt{4\gamma^2-u^2}.\qquad(9)$$

Intersection of the lines, given by equations (6) and (9), gives us points where the system operates as an ideal CDF. It is important to notice that equation (9) has a solution only for $u^2\leq 4\gamma^2$.

One can see from equation (6) that there is the solution $u=-2\gamma, \omega=\omega_0, \theta=\pi/2+2\pi n$ and $u=2\gamma, \omega=\omega_0, \theta=3\pi/2+2\pi n$ obtained by Manolatou *et al* [1]. One of these solutions is marked in figure 2 by a star. Moreover, there are extra solutions shown by open circles in figure 2 with the CDF. However, these solutions are singular because the determinant (5) equals zero. They correspond to the bound states in continuum (BSC) [31, 32], where the collapse of the Fano resonance occurs [33]. A value of the channel-drop transmission depends on the way to approach the BSC points marked by open circles in figure 2(b). Because of the analytical behavior of the channel-drop transmission near the BSC solution, it cannot serve as the CDF.

The question arises as to whether the CDF could be used for $u\neq\pm2\gamma$. After combination of equations (6) and (9), we obtain the following equation:

$$(\omega-\omega_0)^2\left[(\omega-\omega_0)^2-u^2\right]\left[(\omega-\omega_0)^2+4\gamma^2\right]=0,\qquad(10)$$

which gives us the following roots: $\omega=\omega_0+u$, $\omega=\omega_0-u$ and $\omega=\omega_0$. Respectively, the phase $\theta=2\pi n, \theta=\pi+2\pi n$

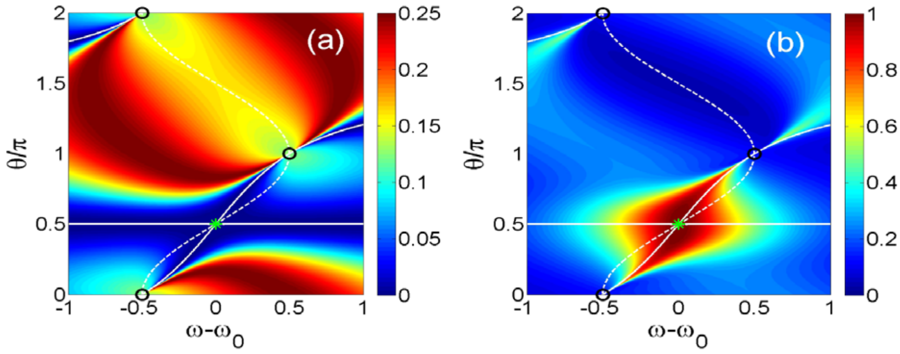

Figure 2. (a) The reflection and (b) channel-drop transmission into the lower waveguide for the case $u^2 = 4\gamma^2$. Solid lines show where the reflection equals zero. Dashed lines are given by equation (9). The parameters are chosen as $\gamma = 0.25$, $u = -2\gamma$.

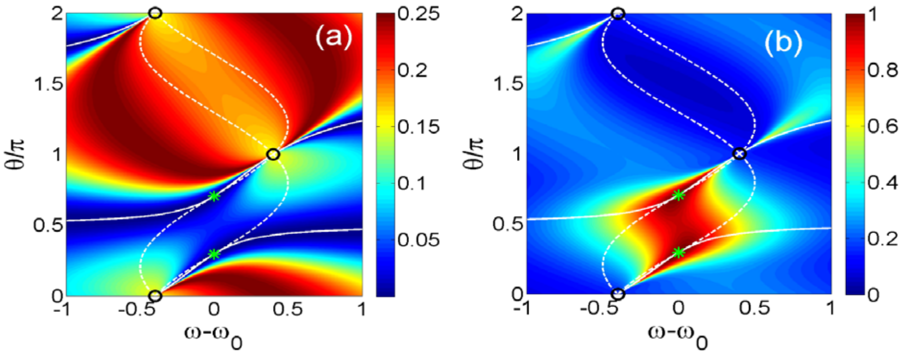

Figure 3. (a) The reflection and (b) channel-drop transmission into the lower waveguide for $u^2 < 4\gamma^2$, $u = -0.4$, $\gamma = 0.25$. Solid lines show where the reflection equals zero. Dashed lines are given by equation (9). The parameters are the same as in figure 2.

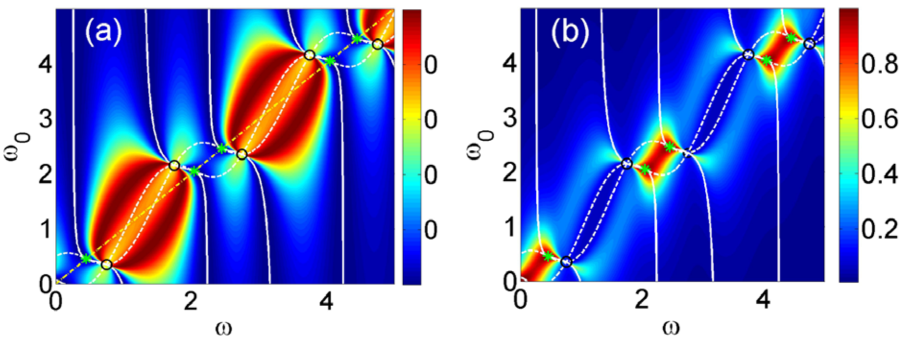

Figure 4. (a) The reflection and (b) channel-drop transmission for the case of dispersive waveguides with $\theta = \pi(1/4 + \omega)$. Solid lines show the solutions of equation (6) at which the reflection equals zero. Dashed lines show ω_0 defined by equation (12). The parameters of the CMT model are the same as in the previous figures.

and $\theta = \pm \arctan u/\sqrt{4\gamma^2 - u^2} + 2\pi n$ where n is an integer. The first two solutions are marked by open circles and the third solution is marked by stars in figure 3. Similar to the previous case shown in figure 2, the first solution cannot serve as the CDF solutions, while the second solution marked by stars can.

However, to practically realize a sufficiently high drop efficiency there are hurdles. In fact, in photonic crystal waveguides, the phase $\theta = k(\omega)L$ depends on frequency ω, where L is the distance between the cavities [34]. Following [35], we approximate that dependence as

$$\theta(\omega) \approx \theta_0 + \theta_1\omega. \tag{11}$$

It instantly implies that the channel dropping occurs only at some discrete set of frequencies ω_n, as follows from equation (9). Substituting equation (11) into equation (6), which defines the condition of total reflection, gives the following equation:

$$\omega_0 = \omega_n - \frac{u + 2\gamma \sin\theta(\omega_n)}{\cos\theta(\omega_n)}. \tag{12}$$

This discrete set of frequencies is marked by stars and open circles in figure 4. Again similar to the former cases, only the solutions marked by stars can serve as the CDF solutions; they are positioned at $\omega_0 = \omega$ as shown in figure 4(a) by the yellow dashed–dotted line.

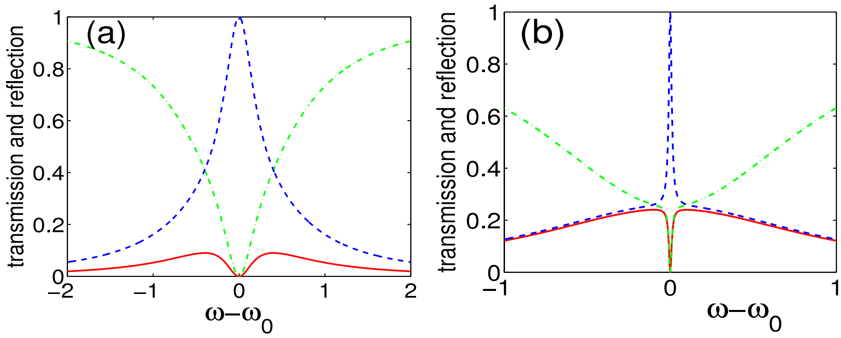

Figure 5. Reflection (red solid), transmission up (dashed–dotted green lines) and the channel-drop transmission (blue dashed line) for the case of constant phase $\theta_0 = \arctan(\frac{u}{\sqrt{4\gamma^2-u^2}})$, $\theta_1 = 0$, $\gamma = 0.25$. (a) $u = -0.4$ and (b) $u = -0.1$.

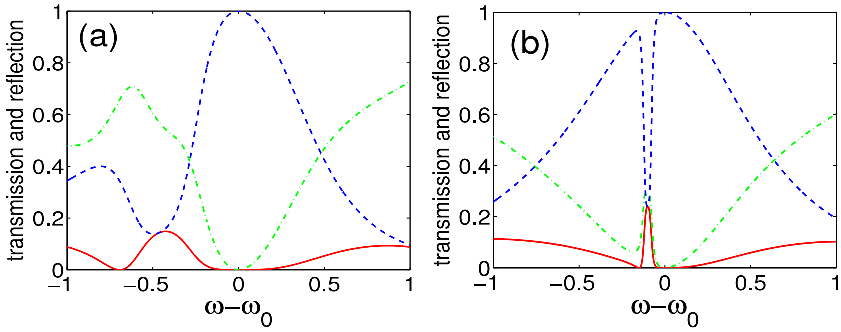

Figure 6. Reflection (red solid), transmission up (dashed–dotted green lines) and the channel-drop transmission (blue dashed line) for the phase dependent on frequency $\theta_0 = \arctan(\frac{u}{\sqrt{4\gamma^2-u^2}})$, $\theta_1 = 1/2\gamma$, $\omega_0 = 0$. (a) $u = -0.4$ and (b) $u = -0.1$.

Also we consider selected cases for the reflection and transmissions presented in figure 5 and 6. First let us consider $\theta_1 = 0$ in equation (11), as considered in [1, 3–5, 11–18, 24]. Figure 5 shows that the reflection and transmission peaks are narrowed with a decrease of the coupling constant u for $\tan\theta_0 = \pm\frac{u}{\sqrt{4\gamma^2-u^2}}$. Further, let us take the case where the phase is dependent on the frequency, however, for the special case $\theta_1 = \frac{d\theta}{d\omega}|_{\omega=\omega_0} = \frac{1}{2\gamma}$ calculated from equation (6). This case is presented in figure 6 for two choices of u. One can see that this case gives rise to a frequency region with sufficient optical isolation.

4. Summary

In this paper we analyzed the system of two identical linear optical cavities each only presented by a single mode with the eigen-frequency ω_0. The cavities are positioned symmetrically between two parallel directional waveguides. We implied from the condition that there is (i) no reflection back from the cavities and (ii) the transmission onto the lower receiver waveguide equals unit, i.e., the full channel-drop transmission takes place. Due to that we derived analytical equations which define a discrete set of values for the frequency of the incoming wave and the phase θ which acquires light between the cavities. Besides the well-known solutions $\theta = \pi/2 + \pi n$ found by Manolatou *et al* [1] for the special case $u = -2\gamma\sin\theta$, we presented new solutions for $u^2 < 4\gamma^2$ and $\theta = \pm\arctan(\frac{u}{\sqrt{4\gamma^2-u^2}})$. Here u is the direct coupling between the cavities, and $\sqrt{\gamma}$ is the coupling constant between the cavities and the waveguides.

Next, we took into account the fact that in the waveguides, the phase depends on the frequency $\theta(\omega) = \theta_0 + \theta_1\omega$, $\theta_1 = \frac{1}{2\gamma}$. There is a discrete set for the frequencies $\omega = \omega_0$ at which the system operates as the CDF, where ω_0 is the eigen-frequency of the cavities. We considered also the special case of the waveguide with $\theta_1 = \frac{d\theta}{d\omega}|_{\omega=\omega_0}$, which demonstrates the channel-drop filtering at a frequency region around the eigen-frequency of the cavities. However, that case also requires defect rods to be specially designed in the PhC structures, similar to the case of Manolatou *et al*.

Acknowledgments

The work is partially supported by RFBR grant 13-07-98018-a and RFBR grant 'Sibir' 13-07-00497.

References

[1] Manolatou C, Khan M J, Fan S, Villeneuve P R, Haus H A and Joannopoulos J D 1999 Coupling of modes analysis of resonant channel add–drop filters *IEEE J. Quantum Electron.* **35** 1322
[2] Haus H A and Lai Y 1992 Theory of cascaded quarter wave shifted distributed feedback resonators *J. Quantum Electron.* **28** 205

[3] Fan S, Villeneuve P R, Joannopoulos J D and Haus H A 1998 Channel drop filters in photonic crystals *Opt. Express* **3** 4
[4] Khan M J, Manolatou C, Fan Sh, Villeneuve P R, Haus H A and Joannopoulos J D 1999 Mode-coupling analysis of multipole symmetric resonant add/drop filters *IEEE J. Quantum Electron.* **35** 1451
[5] Akahane Y, Asano T, Takano H, Song B-S, Takana Y and Noda S 2005 Two-dimensional photonic-crystal-slab channel-drop filter with flat-top response *Opt. Express* **13** 2512
[6] Little B E, Chu S T, Haus H A, Foresi J and Laine J-P 1997 Microring resonator channel dropping filters *J. Light. Technol.* **15** 998
[7] Barwicz T, Popovic M A, Rakich P T, Watts M R, Haus H A, Ippen E P and Smith H I 2004 Microring-resonator-based add-drop filters in SiN: fabrication and analysis *Opt. Express* **12** 1437
[8] Chak P and Sipe J E 2006 Minimizing finite-size effects in artificial resonance tunneling structures *Opt. Lett.* **31** 2568
[9] Qiang Z, Zhou W and Soref R A 2007 Optical add-drop filters based on photonic crystal ring resonators *Opt. Express* **15** 823
[10] Shang L, Wen A, Li B and Wang T 2011 Coupled spiral-shaped microring resonator-based unidirectional add–drop filters with gapless coupling *J. Opt.* **13** 015503
[11] Song B S, Noda S and Asano T 2003 Photonic devices based on in-plane hetero photonic crystals *Science* **300** 1537
[12] Min B K, Kim J E and Park H Y 2004 Channel drop filters using resonant tunneling processes in two-dimensional triangular lattice photonic crystal slabs *Opt. Commun.* **237** 59
[13] Li Z-Y, Sang H-Y, Lin L-L and Ho K-M 2005 Evanescent-wave-assisted wideband continuous tunability in photonic crystal channel-drop filters *Phys. Rev.* B **72** 035103
[14] Hwang K H and Song G H 2005 Design of a high-Q channel add-drop multiplexer based on the two-dimensional photonic-crystal membrane structure *Opt. Express* **13** 1948
[15] Shinya A, Mitsugi S, Kuramochi E and Notomi M 2005 Ultrasmall multi-channel resonant-tunneling filter using mode gap of width-tuned photonic-crystal waveguide *Opt. Express* **13** 4202
[16] Xu Q, Sandhu S, Povinelli M L, Shakya J, Fan S and Lipson M 2006 Experimental realization of an on-chip all-optical analogue to electromagnetically induced transparency *Phys. Rev. Lett.* **96** 123901
[17] Djavid M, Ghaffari A, Monifi F and Abrishamian M S 2008 Heterostructure photonic crystal channel drop filters using mirror cavities *J. Opt. A: Pure Appl. Opt.* **10** 055203
[18] Fasihi K and Mohammadnejad S 2009 Highly efficient channel-drop filter with a coupled cavity-based wavelength-selective reflection feedback *Opt. Express* **17** 8983
[19] Noda S, Chutinan A and Imada M 2000 Trapping and emission of photons by a single defect in a photonic bandgap structure *Nature* **407** 608
[20] Kim S, Park I, Lim H and Kee C-S 2004 Highly efficient photonic crystal-based multi- channel drop filters of three-port system with reflection feedback *Opt. Express* **12** 5518
[21] Song B-S, Asano T, Akahane Y and Noda S 2005 Role of interfaces in heterophotonic crystals for manipulation of photons *Phys. Rev.* B **71** 195101
[22] Ren H, Jiang C, Hu W, Gao M and Wang J 2006 Photonic crystal channel drop filter with a wavelength-selective reflection micro-cavity *Opt. Express* **14** 2446
[23] Romero-Vivas J, Chigrin D N, Lavrinenko A V and Torres C M S 2005 Resonant add-drop filter based on a photonic quasicrystal *Opt. Express* **13** 826
[24] Zhang Z and Qiu M 2005 Compact in-plane channel drop filter design using a single cavity with two degenerate modes in 2D photonic crystal slabs *Opt. Express* **13** 2596
[25] Ď'Orazio A, De Sario M, Marrocco V, Petruzzelli V and Prudenzano F 2008 Photonic crystal drop filter exploiting resonant cavity configuration *IEEE Trans. Nanotechnol.* **7** 10
[26] Fu J-X, Lian J, Liu R-J, Gan L and Lia Z-Y 2011 Unidirectional channel-drop filter by one-way gyromagnetic photonic crystal waveguides *Appl. Phys. Lett.* **98** 211104
[27] Zhao Y-N, Li K-Z, Wang X-H and Jin C-J 2011 A compact in-plane photonic crystal channel drop filter *Chin. Phys.* B **20** 047210
[28] Takano H, Song B-S, Aasano T and Noda S 2006 Highly effective in-plane channel-drop filters in two-dimensional heterostructure photonic-crystal slab *Japan. J. Appl. Phys.* **45** 6078
[29] Min B-K, Kim J-E and Parka H Y 2005 High-efficiency surface-emitting channel drop filters in two-dimensional photonic crystal slabs *Appl. Phys. Lett.* **86** 011106
[30] Suh W, Wang Z and Fan S 2004 Temporal coupled-mode theory and the presence of non-orthogonal modes in lossless multimode cavities *IEEE J. Quantum Electron.* **40** 1511
[31] Bulgakov E N, Pichugin K N, Sadreev A F and Rotter I 2006 *JETP Lett.* **84** 508
[32] Sadreev A F, Bulgakov E N and Rotter I 2006 *Phys. Rev.* B **73** 235342
[33] Kim C S, Satanin A M, Joe Y S and Cosby R M 1999 *Phys. Rev.* B **60** 10962
[34] Joannopoulos J, Meade R D and Winn J 1995 *Photonic Crystals* (Princeton, NJ: Princeton University Press)
[35] Bulgakov E N and Sadreev A F 2010 Bound states in photonic Fabry–Perot resonator with nonlinear off-channel defects *Phys. Rev.* B **81** 115128