# Multirelaxation Generalized Refractive Mixing Dielectric Model of Moist Soils

V. L. Mironov, P. P. Bobrov, and S. V. Fomin

***Abstract*—In this letter, a multirelaxation generalized refractive mixing dielectric model (GRMDM) for moist soil is proposed and substantiated in the frequency range from 0.04 to 26.5 GHz. This model is based on the methodology of a single-relaxation GRMDM which accounts only for the dipole relaxation of water molecules in the gigahertz frequency range. The proposed multirelaxation GRMDM takes into account both the dipole (Debye) and ionic (Maxwell–Wagner) relaxations of soil water molecules. For this purpose, it uses a two-frequency Debye relaxation equation for the dielectric spectra of bound water. The spectroscopic parameters of the multirelaxation GRMDM were derived by fitting the spectra calculated by this model to the respective measured ones. The main advantage of this model is that it predicts the complex dielectric constant of moist soils throughout the megahertz and gigahertz frequency ranges with the same error as the single-relaxation GRMDM does only in the gigahertz range.**

***Index Terms*—Bound soil water, dielectric model, dielectric relaxations, dielectric spectra, free soil water, moist soil.**

## I. Introduction

AT PRESENT, the generalized refractive mixing dielectric model (GRMDM) proposed in [1] is an effective tool for predicting the dielectric spectra of moist soils in the microwave band. This model takes into account only the dipole relaxation of water molecules in the gigahertz frequency range and can be identified as a single-relaxation GRMDM. With the single-relaxation GRMDM as a basic element, the dependence of moist soil dielectric spectra on moisture, temperature, and soil texture appeared to be adequately taken into account [2], [3]. The error in dielectric predictions using the models in [2] and [3] proved to be significantly smaller than that of the semiempirical dielectric model proposed in [4]. It is worth noting that the model in [3] has been thoroughly tested [5] and implemented (instead of the model in [4]) in the data processing algorithm of the Soil Moisture and Ocean Salinity mission run by the European Space Agency [6].

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Meanwhile, the dielectric models in [3] and [4] are limited to the gigahertz frequency range and do not account for a substantial increase in the measured dielectric constant of moist soils as the frequency decreases in the megahertz frequency range [7]–[11]. This increase occurs due to the interfacial (Maxwell–Wagner) relaxation (see [12]) of water in the soil. It takes place when charge carriers are trapped at interfaces of dielectric layers existing at the interface of water films and solids, as well as between the films of bound water and free water. Periodic recharge of layers under the influence of an alternating electromagnetic field results in an increase in the dielectric constant and loss factor of moist soil. So far, no spectroscopic dielectric model has been proposed to account for the interfacial dielectric relaxation in moist soils in the megahertz frequency range. However, there is a great need for such a model in order to develop radars, radiometers, time domain reflectometers, and capacitance sensors used for soil moisture sensing in the megahertz frequency range.

In our recent work [13], a two-relaxation Debye equation was proposed to adequately account for the increase of the complex dielectric constant of bound water in bentonitic clay in the megahertz frequency range. We applied this finding to develop a multirelaxation GRMDM for moist soils, which covers both the megahertz and gigahertz frequency ranges. For this purpose, we made use of the complex permittivity spectra of moist soils measured at the temperature of 20 °C in [7] and [8] in the ranges of frequency from 0.04 to 26.5 GHz and 0.1 to 10 GHz, respectively. In the case of one silty sand and two clay soils, the parameters of multirelaxation GRMDM were obtained. The error of the multirelaxation GRMDM was estimated in terms of the Pearson coefficient and standard deviation.

## II. Concept of the Multirelaxation GRMDM

According to Mironov *et al.* [1], the dielectric constant $\varepsilon'_m$ and loss factor $\varepsilon''_m$ of moist soils as a function of volumetric moisture $m_v$ can be represented in the form of the refractive mixing dielectric model as

$$\varepsilon'_m = n_m^2 - \kappa_m^2, \varepsilon''_m = 2n_m\kappa_m \tag{1}$$

$$n_m = \begin{cases} n_d + (n_b - 1)m_v, & m_v \le m_{vt} \\ n_d + (n_b - 1)m_{vt} \\ \quad +(n_u - 1)(m_v - m_{vt}), & m_v \ge m_{vt} \end{cases} \tag{2}$$

$$\kappa_m = \begin{cases} \kappa_d + \kappa_b m_v, & m_v \le m_{vt} \\ \kappa_d + \kappa_b m_{vt} + \kappa_u(m_v - m_{vt}), & m_v \ge m_{vt} \end{cases} \tag{3}$$

where $n_m$, $n_d$, $n_b$, $n_u$ and $\kappa_m$, $\kappa_d$, $\kappa_b$, $\kappa_u$ are respectively the values of the refractive index and normalized attenuation coefficient, which is a ratio of the standard attenuation

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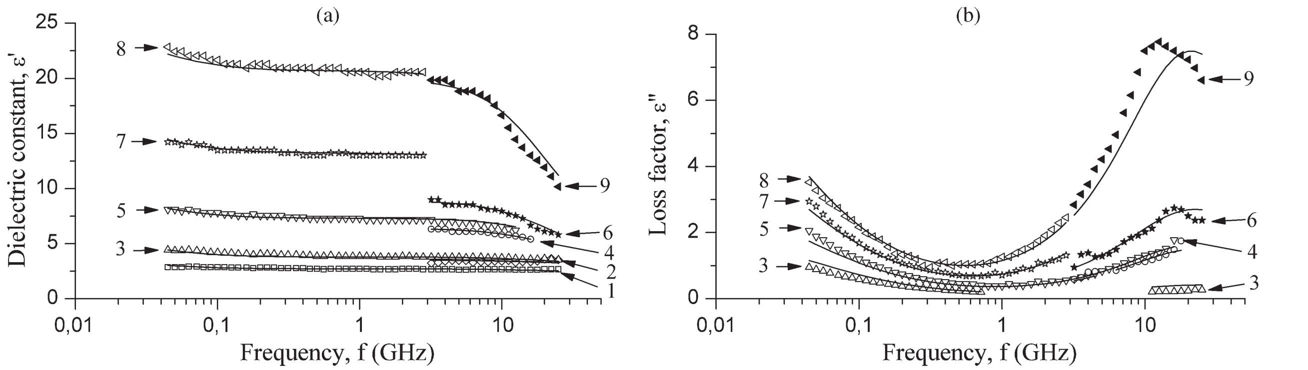


Fig. 1. (Symbols) Measured and (solid lines) predicted spectra of (a) dielectric constant and (b) loss factor. Silty sand soil [7]. Temperature of 20 °C. Moistures, $m_v$ cm$^3$/cm$^3$: 1) 0.018; 2) 0.021; 3) 0.03; 4) 0.1; 5) 0.103; 6) 0.162; 7) 0.229; 8) 0.319; and 9) 0.321.

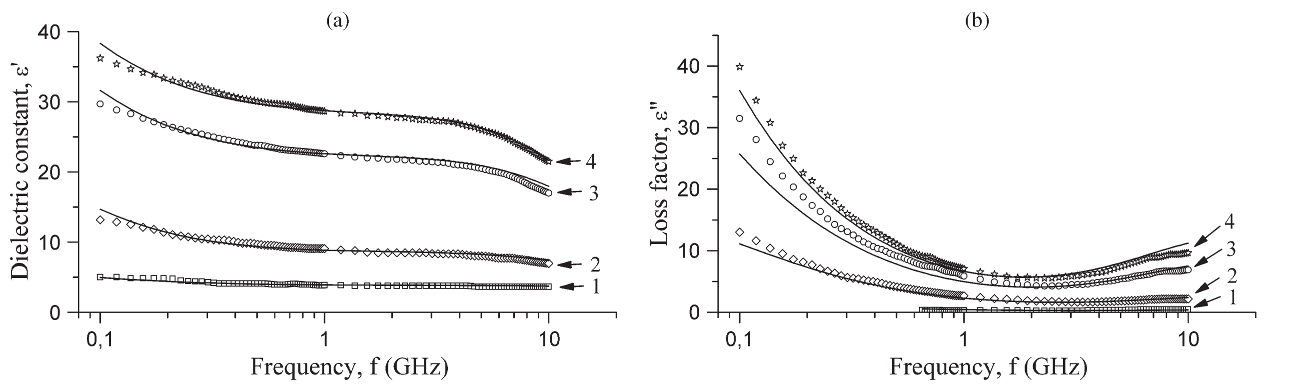


Fig. 2. (Symbols) Measured and (solid lines) predicted spectra of (a) dielectric constant and (b) loss factor. Silty clay loam soil [8]. Temperature of 20 °C. Moistures, $m_v$ cm$^3$/cm$^3$: 1) 0.032; 2) 0.174; 3) 0.344; and 4) 0.462.

coefficient to the free space propagation constant [1]. The subscripts $m$, $d$, $b$, and $u$ in (1)–(3), and in the following, denote the moist soil, dry soil, bound soil water, and unbound or free water in the soil, respectively, and $m_{vt}$ is the maximum bound water fraction in a given type of moist soil. In general, the complex refractive indices of bound and free water in the soil depend on the soil texture $C$, temperature $T$, and wave frequency [3]. Based on observations of the compactness of dry soil changing with moisture [1], we imply in (1) a dependence of the dry soil complex refractive index on soil moisture.

The values of the refractive index and normalized attenuation coefficient can be expressed in terms of their dielectric constants and loss factors by the following formulas:

$$n_p\sqrt{2} = \sqrt{\sqrt{(\varepsilon'_P)^2 + (\varepsilon''_p)^2} + (\varepsilon'_P)}$$
$$\kappa_p\sqrt{2} = \sqrt{\sqrt{(\varepsilon'_P)^2 + (\varepsilon''_p)^2} - (\varepsilon'_P)}. \qquad (4)$$

Here, the subscript $p$ may stand for the moist soil $(p = m)$, dry $(p = d)$ soil, bound $(p = b)$, and free $(p = u)$ water in the soil. We define the dielectric constant and loss factor of soil water by the two-relaxation Debye equations

$$\varepsilon'_p = \frac{\varepsilon_{0pL} - \varepsilon_{0pH}}{1 + (2\pi f \tau_{pL})^2} + \frac{\varepsilon_{0pH} - \varepsilon_{\infty pH}}{1 + (2\pi f \tau_{pH})^2} + \varepsilon_{\infty pH}$$
$$\varepsilon''_p = \frac{\varepsilon_{0pL} - \varepsilon_{0pH}}{1 + (2\pi f \tau_{pL})^2} 2\pi f \tau_{pL} + \frac{\varepsilon_{0pH} - \varepsilon_{\infty pH}}{1 + (2\pi f \tau_{pH})^2} 2\pi f \tau_{pH} + \frac{\sigma_p}{2\pi \varepsilon_r f}. \qquad (5)$$

Here, $\varepsilon_{0pq}$ and $\varepsilon_{\infty pq}$ are the dielectric constants in the low- and high-frequency limits, respectively, in the cases of the ionic $(q = L)$ and dipole $(q = H)$ relaxations, respectively, and $\varepsilon_{\infty pL} = \varepsilon_{0pH}$, while $\tau_{pL}$ and $\tau_{pH}$ denote the relaxation times, also in the cases of the ionic $(q = L)$ and dipole $(q = H)$ relaxations. All these parameters must be referred to the bound $(p = b)$ or free $(p = u)$ water in the soil; $\varepsilon_r = 8854 \cdot 10^{-15}$ F/m—dielectric permittivity of free space. Single-relaxation Debye formulas follow from (5) when $\varepsilon_{0uL} = \varepsilon_{0uH}$.

As seen from (1)–(5), in the context of the multifrequency relaxation GRMDM, the dielectric spectrum of a certain type of moist soil can be calculated through the following set of spectroscopic parameters: 1) refractive index $n_d$ and normalized attenuation coefficient $\kappa_d$ for dry soil; 2) maximum bound water fraction $m_{vt}$; 3) dielectric constants in the low $\varepsilon_{0pq}$ and the high $\varepsilon_{\infty q}$ frequency limit for the bound $(p = b)$ and free $(p = u)$ water in the soil, concerning both the ionic $(q = L)$ and dipole $(q = H)$ relaxations; 4) relaxation times for ionic $\tau_{pL}$ and dipole $\tau_{pH}$ relaxations of the bound $(p = b)$ and free $(p = u)$ water in the soil; and 5) ohmic conductivities for bound $\sigma_b$ and free $\sigma_u$ water in the soil. In the next section, these parameters will be derived by using (1)–(5) and the dielectric data in [7] and [8].

## III. Methodology of the Multirelaxation GRMDM

To determine the spectroscopic parameters in (1)–(5), we used the proposed methodology in [14] of fitting the calculated dielectric constant and loss factor to the measured ones and the

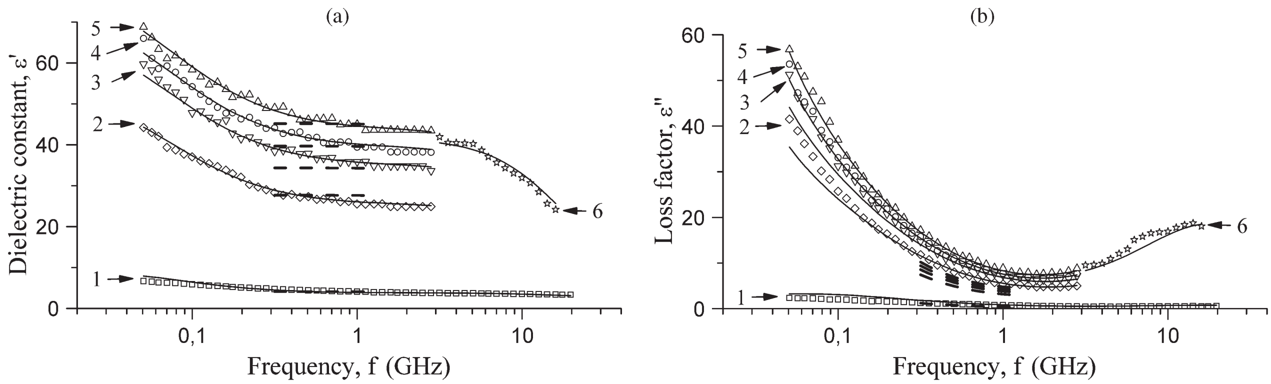


Fig. 3. (Symbols) Measured and (solid lines) predicted spectra of (a) dielectric constant and (b) loss factor. Gray clay soil [7]. Temperature of 20 °C. Moistures, $m_v$ cm$^3$/cm$^3$: 1) 0.07; 2) 0.462; 3) 0.52; 4) 0.571; 5) 0.62; and 6) 0.61. The predictions of Peplinski's model in [16] are also shown in dashed lines.

TABLE I

SPECTROSCOPIC PARAMETERS AND ERRORS OF THE MULTIRELAXATION DIELECTRIC MODEL FOR THE SILTY SAND AND GRAY CLAY MEASURED AT THE TEMPERATURE OF 20 °C BY CURTIS *ET AL.* [7] AND SILTY CLAY LOAM MEASURED BY WAGNER *ET AL.* [8]

software OriginLab [15]. We analyzed the data measured at the temperature of 20 °C for several types of soil, i.e.: 1) silty sand [7]; 2) silty clay loam [8]; and 3) gray clay [7], which are shown in Fig. 1(a) and (b), Fig. 2(a) and (b), and Fig. 3(a) and (b). The textures of these soils are given in Table I. A noticeable increase in the measured dielectric constant is observed in Figs. 1–3, when the frequency is decreasing in the megahertz frequency range, thus indicating the presence of the Maxwell–Wagner relaxation.

At the first phase of fitting, the values of $n_d$, $\kappa_d$, and $m_{vt}$, as well as the dipole relaxation parameters, $\varepsilon_{0pH}$, $\varepsilon_{\infty pH}$, and $\tau_{pH}$, and ohmic conductivities $\sigma_p$ of the bound $(p = b)$ and free $(p = u)$ soil water, respectively, were calculated. For this purpose, the formulas from [3] and the values of clay contents given in Table I were applied. Then, the parameters of ionic relaxation, i.e., $\varepsilon_{0bL}$, $\tau_{bL}$, and $\sigma_b$, were derived. At that, the values of $n_d$, $\kappa_d$, $m_{vt}$, $\varepsilon_{0pH}$, $\varepsilon_{\infty pH}$, $\tau_{pH}$, and $\sigma_p$ calculated at the first phase were used as initial values to be refined in the second phase of regression analysis.

All the spectroscopic parameters obtained by this method are summarized in Table I. These parameters and (1)–(5) represent the multirelaxation GRMDM. Fig. 1(a) and (b), Fig. 2(a) and (b), and Fig. 3(a) and (b) also show the graphs for the spectra of the dielectric constant and loss factor calculated by using

(1)–(5) and the spectroscopic parameters from Table I. As seen from these graphs, the spectra corresponding to the multirelaxation GRMDM (solid lines) fit the measured values very well over the entire megahertz and gigahertz frequency range. This was made possible by simultaneously taking into account both the dipole and ionic relaxations of bound soil water.

## IV. Error of the Multirelaxation GRMDM

To estimate the deviations of the values predicted by using the multirelaxation GRMDM from the measured ones, we correlated them with each other and calculated the Pearson coefficient $\rho$ and standard deviation $\sigma$ corresponding to a linear regression. In addition, to estimate an error corresponding to a bias of the linear regression from the respective bisectors, the equations of linear regression were obtained. The calculated values of the Pearson coefficient $\rho$ and standard deviation $\sigma$ and the equations of linear regression are shown in Table I. As seen from these estimations, the error of the multirelaxation GRMDM in the megahertz and gigahertz frequency ranges is on the order of that corresponding to dielectric measurements themselves, as was the case for the single-relaxation GRMDM in the gigahertz frequency range [1].

In the literature, a semiempirical dielectric model for a number of soils was proposed [16], [17] in the frequency range from 0.3 to 1.3 GHz, in which the complex dielectric constant is affected to some extent by the interfacial relaxation, as is seen from Figs. 1–3. To compare the errors of the multirelaxation GRMDM and the model in [16] and [17], we obtained the multirelaxation GRMDM dielectric predictions for the soils and moistures measured in [16] and [17] and found the standard deviations of the predictions from the measurements for the dielectric constant to be equal to 0.014. At the same time, this value in the case of the semiempirical model in [16] and [17] was found to be 0.3. However, the major advantage of the multirelaxation GRMDM is that it provides predictions with the same error down to the frequency of 40 MHz, while the model in [16] and [17] fails to do that as it gives frequency dispersionless predictions of the dielectric constant in the entire megahertz frequency range [see Fig. 3(a)].

## V. Conclusion

As a result of the analysis carried out in this research, a multirelaxation GRMDM was developed to provide predictions of the complex dielectric constant for moist soil as a function of moisture and frequency for each soil type. The proposed model uses a two-relaxation spectrum for the complex dielectric constant of the bound water in the soil over the wide range of frequencies from 40 MHz to 26.5 GHz. A procedure for deriving the spectroscopic parameters of the multirelaxation GRMDM was proposed, using the measured dielectric data of moist soils. These studies can be developed further since, using the multirelaxation GRMDM established here, it will be possible to create dielectric models in the range from 40 MHz to 26.5 GHz that are able to take into account the dependence of complex dielectric constant spectra on texture as well as temperature of soil, as is done in [3]–[7] using the single-relaxation GRMDM.

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