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SUPERCONDUCTIVITY

Electromagnetic Waves with a Negative Group Velocity in a Randomly Inhomogeneous Josephson Junction

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Abstract—Electromagnetic waves in a randomly inhomogeneous Josephson junction have been investigated by the averaged Green's function method for a nonmonotonic decay of the correlations of inhomogeneities. Modifications of the spectrum and the decay of these excitations caused by spatial fluctuations of the critical current of the Josephson junction have been studied. The regions of the values of the frequency, the wave number, and the stochastic parameters of the medium, at which the waves have a negative group velocity, have been determined.

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1. INTRODUCTION

In recent time, propagation of light pulses with a negative group velocity in various optical media has been intensively investigated experimentally [1–3]. This phenomenon, in addition to the fundamental importance [4–6], can be of practical importance for the development of means for signal control based on it [2]. Achievements in optics stimulated the search of materials in which their inherent excitations possess a negative group velocity. Continuous media, in which the propagation with a negative group velocity of acoustic [7–9] and spin [10, 11] waves as well as excitons and polarons [12] is possible, are considered. The existence of solitons with a negative group velocity in one-dimensional arrays of "small" Josephson junctions was predicted in [13].

In addition to solitons, collective electromagnetic plasma-like excitations occur in the Josephson junctions (the Josephson plasma waves), the investigation of which attracts great attention [14]. The frequency of these excitations in some superconducting materials and structures varies from a few hundred gigahertz to tens terahertz. This frequency range of the electromagnetic radiation, which is important in many respects and attracts the attention of specialists in the field of physics of the solid state, the high energy physics, biology, and medicine, is intermediate between the microwave and infrared spectral regions. Therefore, it is inaccessible for conventional devices of signal generation and reception, which makes the Josephson plasma waves promising for adoption of this frequency range [15]. The thickness of the Josephson junctions usually does not exceed several nanometers; therefore, the influence of various inhomogeneities, mainly of random character, especially strongly affects them. Such inhomogeneities can be caused, for example, by the spatial variation in the thickness and composition of the dielectric layer, by inhomogeneity of contact banks, etc. The influence of random inhomogeneities on solitons (fluxons) in the Josephson junction was investigated in [16], where the model was suggested, according to which, inhomogeneities of geometric and physical parameters of the junction manifest themselves in spatial fluctuations of its critical current. The same model was used in [17] to investigate the Josephson plasma waves in the junction with one-dimensional random inhomogeneities during the exponential and monotonic decay of their correlations.

This study is devoted to the investigation of electromagnetic waves in a randomly inhomogeneous Josephson junction during the nonmonotonic decay of inhomogeneity correlations. It is shown that such correlation properties of spatial fluctuations of the junction lead to a minimum in the wave spectrum with a nonzero value of the wave number and possibility of excitation propagation with a negative group velocity.

2. MODEL AND WAVE EQUATION

Let us consider two identical superconductors separated by a thin dielectric layer with thickness w, which is located in the xy coordinate plane. The origin of count along axis z perpendicular to the contact plane of superconductors is selected in the layer center. With the coherence length of the superconductor much larger than w, the Josephson electric current $j_z = j_c \sin\varphi$, where j_c is the critical current of the Josephson junction and φ is the phase difference of wave functions of superconducting electrons between junction edges, flows across the contact. It is known [18, 19] that the phase difference for the homogeneous Josephson junction in the absence of losses in it is described by the equation

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}-\frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial t^2} = \frac{\sin\varphi}{\lambda_J^2}. \quad (1)$$

Here, t is time, $c_0 = c\sqrt{w/\varepsilon d}$ is the propagation velocity of electromagnetic waves in the junction (the Swihart velocity); c is the velocity of light in vacuum; ε is the dielectric constant of the junction; $d = w + 2\lambda$, where λ is the penetration depth of the magnetic field into the superconductor; and λ_J is the Josephson penetration depth. We will further consider the "large" Josephson junction: $L_x, L_y \gg \lambda_J$, where L_x, L_y, and L_z are the sample sizes in directions of corresponding coordinate axes; the sample volume is $V = L_xL_yL_z$.

In the case of a randomly inhomogeneous Josephson junction, physical quantities in Eq. (1) are random functions of coordinates. In order to simplify the model, we will assume following [16] that velocity c_0 is uniform, while the Josephson penetration depth fluctuates

$$\lambda_J^{-2}(\mathbf{x}) = \lambda_J^{-2}[1+\gamma\rho(\mathbf{x})], \quad (2)$$

where $\rho(\mathbf{x})$ is the statically uniform random function, which is centered ($\langle\rho\rangle = 0$) and normalized ($\langle\rho^2\rangle = 1$). Angle brackets denote averaging over the ensemble of realizations of random function $\rho(\mathbf{x})$; $\mathbf{x} = \{x, y, z\}$; and γ is the relative root-mean-square fluctuation of the critical current, $0 \le \gamma < 1$. Using formula (2) in Eq. (1), assuming $\varphi \ll 1$, and performing the Fourier transform over time, we derive

$$\frac{\partial^2\varphi(\omega,\mathbf{x})}{\partial x^2}+\frac{\partial^2\varphi(\omega,\mathbf{x})}{\partial y^2}+[\nu-\eta\rho(\mathbf{x})]\varphi(\omega,\mathbf{x}) = 0, \quad (3)$$

where $\nu = (\omega^2 - \omega_J^2)/c_0^2$; ω is the wave frequency, $\omega_J = c_0/\lambda_J$ is the Josephson plasma frequency; $\eta = \gamma/\lambda_J^2$; $\varphi \sim \exp[i(\mathbf{kx} - \omega t)]$; and $\mathbf{k} = \{k_x, k_y\}$. For the homogeneous junction ($\gamma = 0$), it follows from expression (3) that

$$\nu(k) = k^2; \quad (4)$$

from here, we derive the formula for the wave spectrum

$$\omega = \sqrt{\omega_J^2 + c_0^2k^2}, \quad (5)$$

according to which the group velocity $v_{g0} = d\omega/dk$ has the form

$$v_{g0} = \frac{kc_0^2}{\omega}. \quad (6)$$

In order to investigate the Josephson plasma waves in the randomly inhomogeneous junction ($\gamma \neq 0$), let us use the Kraichnan approximation [20], which makes it possible to take into account the multiple wave scattering at inhomogeneities and is also known as the self-consistent approximation [21]. A simple formulation of this approximation is given in [22]. According to the approach stated in these publications, the Fourier image of the averaged Green's function, which corresponds to Eq. (3), has the form

$$\bar{G}(\mathbf{k},\nu) = \frac{(2\pi)^{-3}}{\nu - k^2 - \Sigma(\mathbf{k},\nu)}, \quad (7)$$

where mass operator $\Sigma(\mathbf{k}, \nu)$ follows the integral equation

$$\Sigma(\mathbf{k},\nu) = \eta^2\int\frac{S(\mathbf{k}-\mathbf{k}_1)d\mathbf{k}_1}{\nu - k_1^2 - \Sigma(\mathbf{k}_1,\nu)}. \quad (8)$$

Here, $S(\mathbf{k})$ is the spectral density related to the correlation function of inhomogeneities $K_\rho(\mathbf{r}) = \langle\rho(\mathbf{x})\rho(\mathbf{x}+\mathbf{r})\rangle$; $k_1 = |\mathbf{k}_1|$ by the Fourier transform.

Let us consider the Josephson junction with random inhomogeneities possessing the nonmonotonic correlation decay. To describe such inhomogeneities, we will use the correlation function and the spectral density in the form

$$K_\rho(\mathbf{r}) = \left(1-\frac{rk_c}{3}\right)e^{-k_cr}, \quad S(\mathbf{k}) = \frac{4k^2k_c}{3\pi^2(k_c^2+k^2)^3}, \quad (9)$$

where k_c is the correlation wave number of inhomogeneities, $r = |\mathbf{r}|$, and $k = |\mathbf{k}|$. Function $\rho(\mathbf{x})$ and the correlation decay are assumed to be rather smooth (correlation radius $r_c = 1/k_c \gg a_0$, where a_0 is the interatomic distance). Expression (9) describes a nonmonotonic correlation decay of inhomogeneities. It assumes the presence of local correlations between the positive and negative fluctuations [23], which lead to equality $\int_{V_0}\rho(\mathbf{x})d\mathbf{x} = 0$, where V_0 is a small local volume. It follows from this condition for the correlation function that $\int_V K(\mathbf{x})dx = 0$, from here, we have $S(0) = 0$. The notion of a nonmonotonic decay is widely used when studying the randomly inhomogeneous materials (e.g., [23–28]). In [28], the possibility of anomalous dispersion of bulk plasma waves in the conductor with spatial fluctuations of the lattice potential, the correlation properties of which are described by functions (9), is shown in the second order of the perturbation theory. Such correlations are inherent to inhomogeneities with an average size proportional to r_c (the spectral density in (9) has a maximum at $k = k_s \equiv k_c/\sqrt{2}$). Particularly, the nonmonotonic correlation decay implies the absence of uniform realizations in the ensemble of random functions. Temporal fluctuations, the spectral density of which $S(\omega)$ turns to zero not only at $\omega \longrightarrow \infty$ but also at $\omega \longrightarrow 0$ are also known; they were called "the green noise". Its influence was investigated in [29], including the Josephson junctions.

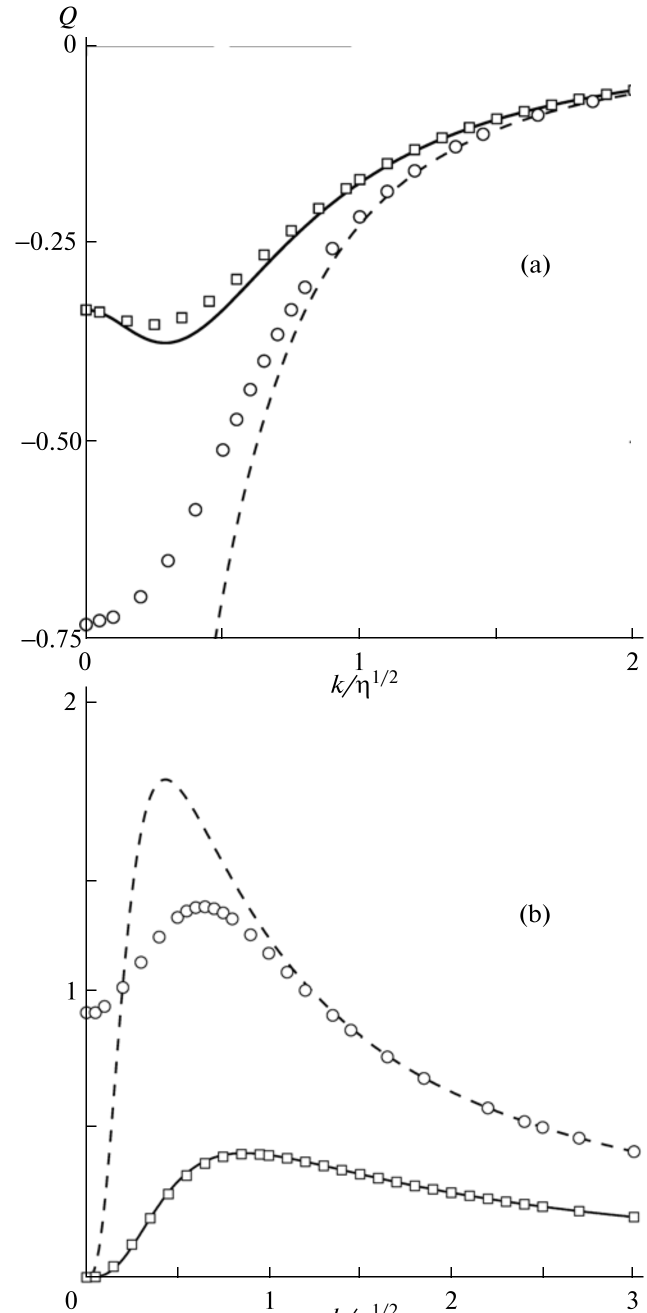

Fig. 1. Spectrum ν' of the Josephson plasma waves and position ν_m of the peak of function $\bar{G}''(k, \nu)$: $Q = (\nu_m - k^2)/\eta$ (points), $Q = (\nu' - k^2)/\eta$ (lines). Attenuation ν'' of the wave and width $\Delta\nu$ of the peak of function $\bar{G}''(k, \nu)$: $R = \Delta\nu/\eta$ (points) and $R = 2\nu''/\eta$ (lines). Squares and circles (they are determined by formulas (7) and (8)), as well as solid and dashed curves (they are specified by expression (14)) are plotted at $k_c/\sqrt{\eta} = 1.0$ and 0.5, respectively.

3. SPECTRUM AND GROUP VELOCITY OF WAVES

Using expression (9) for the spectral density in integral equation (8) and assuming that the mass operator in the integrand of this equation depends only on $|\mathbf{k}_1|$, let us find its numerical solution; substituting it into formula (7), let us determine the dependence of position ν_m and width $\Delta\nu$ of the peak of function $\bar{G}''(k, \nu) = \mathrm{Im}\bar{G}(k, \nu)$ on k. Functions $\nu_m(k)$ and $\Delta\nu(k)$ are shown in Figs. 1a and 1b, respectively. Particularly, Fig. 1a shows the shift of peak $\bar{G}''(k, \nu)$ to the lower-frequency region ($\nu_m - k^2 < 0$) compared with its position in the homogeneous medium, which is reflected by straight line $Q = 0$. The value of $|\nu_m - k^2|$ is largest in region $k \leq \sqrt{\eta}$, where a nonmonotonic dependence of difference $\nu_m - k^2$ on k (squares), which is characteristic of $k_c \gtrsim \sqrt{\eta}$, is found. At $k_c < \sqrt{\eta}$, difference $\nu_m - k^2$ increases monotonically as k increases (circles). Width $\Delta\nu$ of the peak of $\bar{G}''(k, \nu)$, which was calculated at its half-height, also behaves differently depending on k_c. This is illustrated in Fig. 1b by the sequence of points found at various values of k_c. For example, for all $k_c \gtrsim \sqrt{\eta}$, function $\Delta\nu(k)$ tends to zero at $k \longrightarrow 0$ (squares), while at $k_c < \sqrt{\eta}$, it remains finite (circles). The latter is caused by the fact that the nonuniform (fluctuation) broadening, which is determined by a stochastic spread of resonant frequencies, contributes to the peak width calculated in the self-consistent approximation.

We will determine the spectrum of the Josephson plasma waves in the region of values of stochastic junction parameters $k_c \gtrsim \sqrt{\eta}$. At such k_c and η in the denominator of the integrand in the right side of Eq. (8), we can use the approximation $\Sigma(\mathbf{k}_1, \nu) = \Sigma(k, \nu)$ suggested in [22]. This substitution is admissible [30] if inequalities are valid:

$$\left|\frac{d\Sigma(k,\nu)}{dk}\right| \ll 2k, \quad \left|\frac{d^2\Sigma(k,\nu)}{dk^2}\right| \ll 2. \tag{10}$$

Indeed, from the expansion of the integrand denominator of Eq. (8) in the vicinity of point $k_1 = k$ into the power series

$$\begin{aligned}\nu - k_1^2 - \Sigma(k_1, \nu) = g - \left[2k + \frac{d\Sigma(k,\nu)}{dk}\right](k - k_1) \\ - \left[1 + \frac{1}{2}\frac{d^2\Sigma(k,\nu)}{dk^2}\right](k - k_1)^2 - \ldots,\end{aligned} \tag{11}$$

with the conservation of three first terms in it and fulfillment of inequalities (10), it follows that

$$\nu - k_1^2 - \Sigma(k_1, \nu) \approx \nu - k_1^2 - \Sigma(k, \nu). \tag{12}$$

In expression (11), $g = \nu - k^2 - \Sigma(k, \nu)$ is the denominator of the Green's function (7). Inequalities (10) are fulfilled for the Josephson plasma waves at

$k_c > k_{c1} \approx 0.7\sqrt{\eta}$ and $k < k_c/2$ as well as at $k \gg \sqrt{\eta}$ irrespective of the magnitude of k_c. Thus, integrating in the right side of Eq. (8), we derive

$$\Sigma(k, \nu) = -\eta^2 \frac{F_1 + iF_2}{3[(g + k_c^2)^2 + 4k^2k_c^2]^2}, \quad (13)$$

where $F_1 = k_c^6 + k_c^4(7g + 12k^2) + gk_c^2(3g - 4k^2) - 3g^3$ and $F_2 = 8k_c\sqrt{g + k^2}(g^2 + gk_c^2 + 2k^2k_c^2)$. Expression (13) is reduced to the sixth-order algebraic equation relative to $\sqrt{g + k^2}$. Its numerical solution allows us to find magnitude $\Sigma(k, \nu)$, which coincides with the solution of starting integral equation (8) at $k_c \ge k_{c1}$. This correspondence allows us to apply formula (13) for determining the dispersion law of averaged waves. Using equality $g = 0$ in expression (13), we derive

$$\nu(k) - k^2 = -\eta^2 \frac{k_c^2 + 12k^2}{3(k_c^2 + 4k^2)^2} - \eta^2 \frac{16ik^3}{3k_c(k_c^2 + 4k^2)^2}. \quad (14)$$

The real part of function $\nu(k)$ and the doubled value of its imaginary part are shown in Figs. 1a and 1b, respectively: $\nu' = \mathrm{Re}\,\nu(k)$, $\nu'' = -\mathrm{Im}\,\nu(k)$. Good coincidence of solid lines and square-marked sequences of points is seen. Such a behavior is characteristic of the spectrum and wave attenuation at $k_c \gtrsim \sqrt{\eta}$. The first term in the right side of expression (14) has a minimum at $k = k_\nu \equiv k_c/2\sqrt{3}$, which coincides with the position of maximum k_s of the spectral density by the order of magnitude and indicates the cause of nonmonotonicity of the dispersion curve: the modification of the wave spectrum is especially large at those k which correspond to the vicinity of the maximal value of the spectral density. Particularly, for the monotonic decay of inhomogeneity correlations, when function $S(k)$ is maximal at $k = 0$, the frequency of uniform excitations is subjected to the largest modification.

Inequality $\mathrm{Im}\,\omega^2 \ll \mathrm{Re}\,\omega^2$ follows from formula (14) and definition of ν at $k_c > k_{c1}$. This inequality allows us to simplify the expressions for spectrum $\omega' = \mathrm{Re}\,\omega(k)$ and attenuation $\omega'' = -\mathrm{Im}\,\omega(k)$ of the waves. In such approximation, we have

$$\omega' = \left[\omega_J^2 + c_0^2k^2 - \eta^2 c_0^2 \frac{k_c^2 + 12k^2}{3(k_c^2 + 4k^2)^2}\right]^{1/2}, \quad (15)$$

from here, for the group velocity $v_g = d\omega'/dk$, we derive

$$v_g = \frac{c_0^2 k}{\omega'}\left[1 - \eta^2 \frac{4(k_c^2 - 12k^2)}{3(k_c^2 + 4k^2)^3}\right]. \quad (16)$$

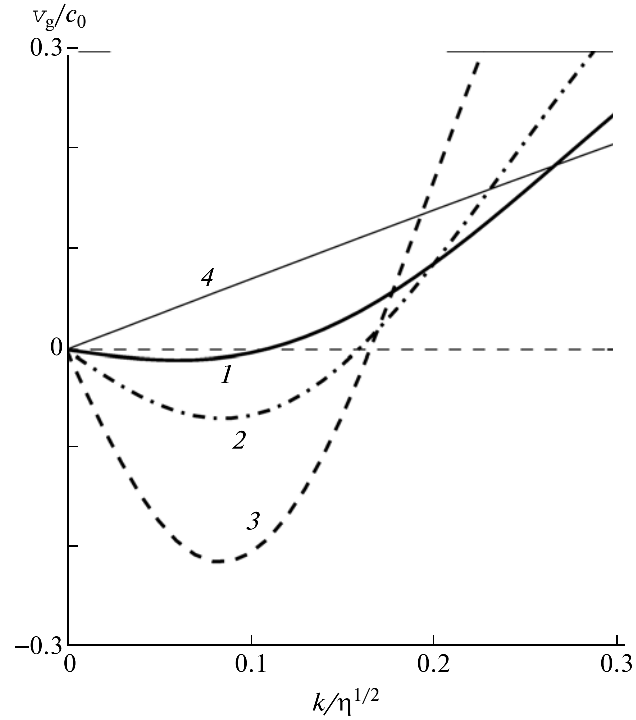

Fig. 2. Wave-number dependence of the group velocity. Curves are plotted according to formula (16) at $k_c/\sqrt{\eta}$ = (*1*) 1, (*2*) 0.85, and (*3*) 0.7. Line *4* corresponds to expression (6).

Figure 2 shows dependences $v_g(k)$ at several values of k_c. It is seen that $v_g < 0$ at $k < k_g$, where k_g can be found from the equality to zero of the bracketed expression in (16),

$$k_g = \sqrt{\eta \frac{2A^2 - 3AK_c^2 - 6}{12A}}. \quad (17)$$

Here, $A = (9K_c^2 + 3\sqrt{3 + 9K_c^4})^{1/3}$, $K_c = k_c/\sqrt{\eta}$. Inequality $k_c < k_{c2} \equiv (4/3)^{1/4}\sqrt{\eta}$ follows from the requirement of positivity of the radicand in formula (17). Function $k_g(k_c)$, which is shown in Fig. 3, reaches the maximum $k_g = \sqrt{\eta}/6$ in the point $k_c = \sqrt{5\eta}/3$ in range $k_{c1} < k_c < k_{c2}$, and tends to zero at $k_c \longrightarrow k_{c2}$ remaining finite at $k_c = k_{c1}$. Thus, inequality $v_g < 0$ is fulfilled inside the region bounded by curve $k_g(k_c)$ and straight line $k_c = k_{c1}$. Within the limits of this region, inequality $k \ll k_c$ is valid, which allows us to use the power series expansion with respect to k in expression (14) and to obtain

$$\omega \approx \omega_0 + \frac{k^2c_0^2}{2\omega_0}\left(1 - \frac{4\eta^2}{3k_c^4}\right) - i\frac{8\eta^2c_0^2k^3}{3\omega_0k_c^5}, \quad (18)$$

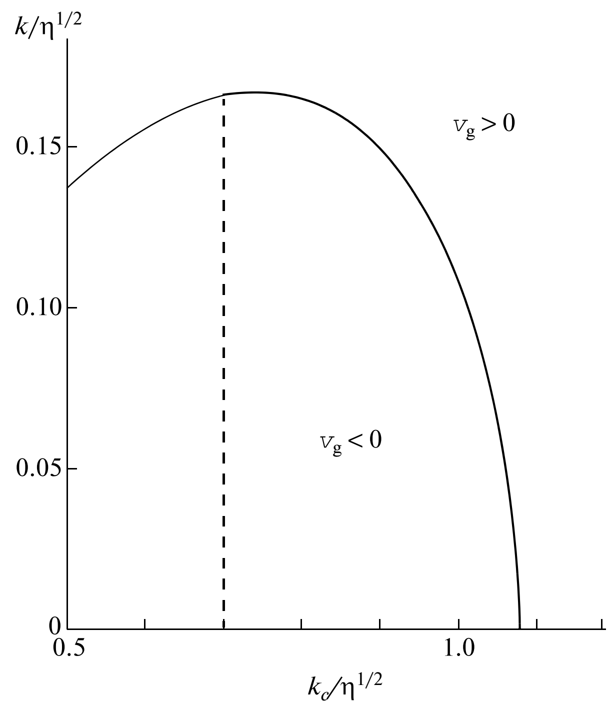

Fig. 3. Regions of group velocities of different signs. The solid line is determined by expression (17), and the dashed line is specified by equality $k_c = k_{c1}$.

where $\omega_0 = \omega_J(1 - \gamma\eta/3k_c^2)^{1/2}$. It follows from here that at $\gamma = 0.5$ and $k_{c1} < k_c < k_{c2}$, the range of varying ω' is determined by inequalities $-0.25 < (\omega' - \omega_J)/\omega_J < -0.08$. In addition, it follows from formula (18) for wave attenuation that $\omega'' \ll |\omega' - \omega_J|$ and $\omega'' \propto k^3$. A similar dependence of attenuation on the wave vector was obtained in study [27] for spin waves in a ferromagnet with the fluctuating magnetic anisotropy with the Gaussian nonmonotonic decay of inhomogeneity correlations.

4. CONCLUSIONS

Using the method of the averaged Green's functions, the modification of the spectrum and the attenuation of electromagnetic waves in a randomly inhomogeneous Josephson junction with the nonmonotonic decay of inhomogeneity correlations have been investigated. Dependences of frequency and attenuation of averaged waves as well as positions ν_m and widths $\Delta\nu$ of the imaginary part of the Fourier image of the averaged Green's function have been determined based on the self-consistent approximation, which makes it possible to take into account the multiple wave scattering at inhomogeneities. The evolution of these dependences have been investigated when measuring the correlation radius and relative root-mean-square fluctuations of inhomogeneities. The region of existence of a negative group velocity of the Josephson plasma waves caused by random inhomogeneities of the junction is determined. This effect is most pronounced at $k_c \sim \sqrt{\eta}$ ($k_{c1} < k_c < k_{c2}$), from where it follows that $v_g \sim -kc_0^2/3\omega'$, i.e., $|v_g| \sim v_{g0}/3$. If $k_c \sim \sqrt{\eta}$, then the correlation radius is $r_c \sim \lambda_J/\sqrt{\gamma}$. At $\lambda_J = 10^{-4}$ cm and $\gamma \sim 0.5$, inhomogeneities with $r_c \sim 1.4 \times 10^{-4}$ cm are effective. The stochastic parameters of the junction r_c and are formed during its fabrication including the targeted formation of random inhomogeneities with desired properties. As a rule, strong effect on the sample is required to vary values of r_c and γ. Therefore, it would be possible to "tune" to the effect of negative group velocity by varying λ_J, for example, varying the junction temperature. An appropriate object for experimental investigations of phenomena considered in this study possibly would be the Josephson junctions from the $YBa_2Cu_3O_{7-x}$ high-temperature superconductor, in which the nonuniform current distribution was mentioned [15].

We note that formula (14), which determines the spectrum and attenuation of the waves, can be obtained in the second order of the perturbation theory [31] if $\gamma \ll 1$ and $k_c \gg \sqrt{\gamma\eta/3}$. The self-consistent approximation that we used substantially extends the applicability region of expression (14) and formulas (15) and (16) obtained based on it for the spectrum of waves and their group velocity.

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REFERENCES

1. L. Zhang, L. Zhan, K. Qian, J. Liu, Q. Shen, X. Hu, and S. Luo, Phys. Rev. Lett. **107**, 093903 (2011).
2. K. Qian, L. Zhan, L. Zhang, Z. Q. Zhu, J. S. Peng, Z. C. Gu, X. Hu, S. Y. Luo, and Y. X. Xia, Opt. Lett. **36**, 2185 (2011).
3. R. T. Glasser, U. Vogl, and P. D. Lett, Opt. Express **20**, 13702 (2012); R. T. Glasser, U. Vogl, and P. D. Lett, Phys. Rev. Lett. **108**, 173902 (2012).
4. V. G. Veselago, Sov. Phys.—Usp. **10** (4), 509 (1967); V. G. Veselago, Phys.—Usp. **54** (11), 1161 (2011).
5. R. A. Silin, Phys.—Usp. **49** (5), 542 (2006).
6. V. P. Makarov and A. A. Rukhadze, Phys.—Usp. **54** (12), 1285 (2011).
7. A. A. Maznev and A. G. Every, Appl. Phys. Lett. **95**, 011903 (2009).
8. V. A. Burov, V. B. Voloshinov, K. V. Dmitriev, and N. V. Polikarpova, Phys.—Usp. **54** (11), 1165 (2011).
9. M. I. Shalaev, S. A. Myslivets, V. V. Slabko, and A. K. Popov, Opt. Lett. **36**, 3861 (2011).
10. A. V. Vashkovskii and E. H. Lock, Phys.—Usp. **49** (4), 389 (2006).

11. A. V. Chumak, P. Dhagat, A. Jander, A. A. Serga, and B. Hillebrands, Phys. Rev. B: Condens. Matter **81**, 140404R (2010).
12. V. M. Agranovich and Yu. N. Gartstein, Phys.—Usp. **49** (10), 1029 (2006).
13. E. Arévalo, Eur. Phys. Lett. **83**, 10004 (2008).
14. S. Savel'ev, V. A. Yampol'skii, A. L. Rakhmanov, and F. Nori, Rep. Prog. Phys. **73**, 026501 (2010).
15. Yu. Ya. Divin, U. Poppe, I. M. Kotelyanskii, V. N. Gubankov, and K. Urban, J. Commun. Technol. Electron. **53** (10), 1137 (2008).
16. M. B. Mineev, M. F. Feigel'man, and V. V. Shmidt, Sov. Phys. JETP **54** (1), 155 (1981).
17. Yu. I. Mankov, Tech. Phys. **56** (8), 1147 (2011).
18. A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982; Mir, Moscow, 1984).
19. K. K. Likharev, *Dynamics of Josephson Junctions and Circuits* (Nauka, Moscow, 1985; Gordon and Breach, Amsterdam, 1986).
20. R. H. Kraichnan, J. Math. Phys. **2**, 124 (1961).
21. G. Brown, V. Celli, M. Haller, A. A. Maradudin, and A. Marvin, Phys. Rev. B: Condens. Matter **31**, 4993 (1985).
22. V. A. Ignatchenko and V. A. Felk, Phys. Rev. B: Condens. Matter **71**, 094417 (2005).
23. M. A. Krivoglaz, in *Problems of Physics of Metals and Metallography* (Academy of Sciences of the Ukrainian SSR, Kiev, 1961), Vol. 13, p. 17 [in Russian].
24. E. Schlömann, Phys. Rev. [Sect.] A **135**, 413 (1964).
25. M. V. Medvedev, Fiz. Met. Metalloved. **55**, 629 (1983).
26. V. A. Ignatchenko and R. S. Iskhakov, Fiz. Met. Metalloved. **65**, 679 (1988); V. A. Ignatchenko and R. S. Iskhakov, Fiz. Met. Metalloved., No. 9, 5 (1990).
27. V. A. Ignatchenko and I. V. Bogomaz, Fiz. Met. Metalloved. **67**, 866 (1989).
28. V. A. Ignatchenko and Yu. I. Mankov, J. Phys.: Condens. Matter **3**, 5837 (1991).
29. S. A. Guz, Yu. G. Krasnikov, and M. V. Sviridov, Dokl. Phys. **44** (3), 127 (1999).
30. Yu. I. Mankov, Phys. Solid State **54** (7), 1323 (2012).
31. V. A. Ignatchenko and R. S. Iskhakov, Sov. Phys. JETP **45** (3), 526 (1977).

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