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On Low-frequency Oscillations of a Bloch-point in a Nanodisk

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The paper is devoted to the problem of low frequency oscillations of a magnetic vortex of small amplitude in isolated cylindrical ferromagnetic of submicronic size. The problem is solved analytically in the context of the rigid vortex model. This model assumes a certain constant magnetization distribution near the Bloch point when the vortex core is displaced from the equilibrium position. Numerical results are presented in the form of diagrams. These diagrams can be used to predict how frequencies of modes depend on geometrical sizes of a nanocylinder.

Keywords: nanodot, magnetic vortex, Bloch point, magnetic reversal.

Introduction

Within the last years there is an interest in the synthesis and study of nanodots structures. Nanodots are the objects of submicronic size that can take various shapes, such as disks, spheroids, parallelepipeds and so on. In spite of small size magnetic nanodots are not superparamagnetic particles. Nanodots have stable and controllable distribution of magnetization.

Let us take a nanodot in the shape of a disk (the thickness of the disk is much less than its radius). The disk is made of soft ferromagnetic. It is well known that in the main state some vortical structure of magnetization may exist, and Bloch point of this structure lies on the disk axis (see Fig. 1).

Vortical distribution of magnetization is stable when external magnetic field is absent, the disk radius is less than 10-6 m and thickness of the disk is about 10-8 m [1,2]. For disks of larger size vortical distribution of magnetization becomes metastable.

The study of nanodot structures might have practical applications. For example, such structures can be used in super dense data recording devices. The main challenges now are to understand the mechanisms of magnetization reversal and to find the stable and energetically favorable way to control magnetization. The search of the most energetically efficient way of magnetization reversal is the very important issue in study of nanodots. It is now obvious that application of variable or pulse fields shows considerable promise. Under the action of such fields vortex core behaves like an oscillating mechanical system with the distinct resonant response.

Often vortex core is treated as a quasi-particle subjected to an effective force. Gyroscopic force, similar to Lorentz force, acts on the vortex core in the presence of external magnetic

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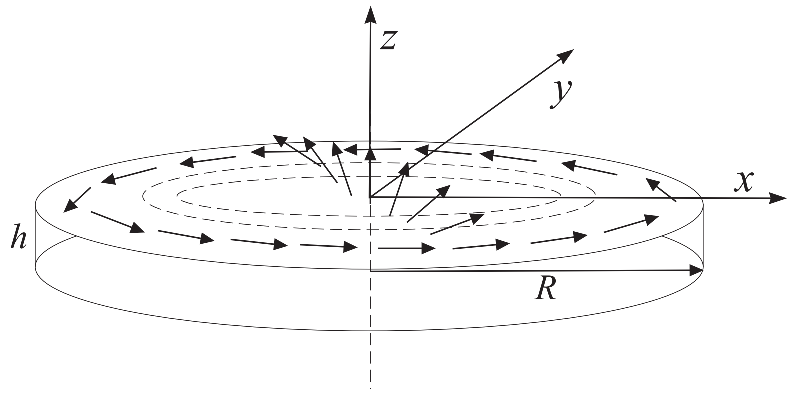

Fig. 1. Example of vortical structure of nanodisk magnetization

field [3,4]. The vortex movement is periodic in character. Usually two limiting cases are considered: low and high frequency magnon modes [5]. In these two cases different approaches with appropriate approximations are needed to obtain results. Very often the computer simulation is used to study the process of magnetization reversal (see, for example, [6–8] and references there).

In the present study we propose a simple method to calculate the natural frequency of oscillation of magnetic vortex in the low frequency case. This method provides a way of determining the dependence of natural frequency on nanodisk geometry.

1. Model of the Main State of Nanodisk Magnetization

Let us consider a disk of radius R and thickness h (see Fig. 1). The thickness of the disk is much less than the correlation radius of magnetization in the material domain wall: $h \ll \delta_W$, where $\delta_W = \sqrt{A/K_{eff}}$, A and K_{eff} are exchange constant and effective anisotropy constant, respectively.

The crystallographic anisotropy constant is included into the effective anisotropy constant. The induced form anisotropy gives the main contribution to the effective anisotropy constant and provides preferable position of magnetization in the disk plane [9].

The effective constant can be estimated as follows: $K_{eff} \approx 2\pi\mu_0 M_S^2$, here M_S is magnetization of saturation. For descriptions of magnetization distribution we choose cylindrical coordinate system, where z axis is directed along the disk axis and azimuthal angle β and radius vector ρ are in the disk plane.

Because of cylindrical geometry it is convenient to express the components of magnetizations in terms of the polar angle ϑ and azimuthal angle ϕ in cylindrical coordinate system. In this coordinate system components of magnetization are:

$$\begin{aligned} m_\rho &= \frac{M_\rho}{M_S} = \sin(\vartheta)\cos(\beta-\phi), \\ m_\beta &= \frac{M_\beta}{M_S} = \sin(\vartheta)\sin(\beta-\phi), \\ m_z &= \frac{M_z}{M_S} = \cos(\vartheta). \end{aligned} \tag{1}$$

Density of the exchange energy is:

$$w_{ex} = A\left[(\nabla m_\rho)^2 + (\nabla m_\beta)^2 + (\nabla m_z)^2\right]. \tag{2}$$

As we look for the solution with cylindrical symmetry, it is reasonable to assume that components of magnetization do not depend on coordinate β. Because of small thickness of the disk one can assume that magnetization do not depend on z coordinate. In this case we have:

$$\begin{aligned}
\nabla m_\rho &= \cos(\vartheta)\cos(\beta-\phi)\frac{\partial\vartheta}{\partial\rho}\mathbf{n}_\rho + \sin(\vartheta)\sin(\beta-\phi)\frac{\partial\phi}{\partial\rho}\mathbf{n}_\rho - \frac{\sin(\vartheta)\sin(\beta-\phi)}{\rho}\mathbf{n}_\beta,\\
\nabla m_\beta &= \cos(\vartheta)\sin(\beta-\phi)\frac{\partial\vartheta}{\partial\rho}\mathbf{n}_\rho - \sin(\vartheta)\cos(\beta-\phi)\frac{\partial\phi}{\partial\rho}\mathbf{n}_\rho + \frac{\sin(\vartheta)\cos(\beta-\phi)}{\rho}\mathbf{n}_\beta,\\
\nabla m_z &= -\sin(\vartheta)\frac{\partial\vartheta}{\partial\rho}\mathbf{n}_\rho.
\end{aligned} \tag{3}$$

Here $\mathbf{n}_\rho$, $\mathbf{n}_\beta$, $\mathbf{n}_z$ are unit vectors of coordinate system.

Taking into account equation (3) expression (2) becomes:

$$w_{ex} = A\left[\left(\frac{\partial\vartheta}{\partial\rho}\right)^2 + \sin^2(\vartheta)\left(\frac{\partial\phi}{\partial\rho}\right)^2 + \frac{1}{\rho^2}\sin^2(\vartheta)\right]. \tag{4}$$

Energy of anisotropy is represented in the following form:

$$w_{an} = K_{eff}\cos^2(\vartheta). \tag{5}$$

Taking into account expressions (4) and (5) total magnetic energy of the disk is written as

$$W = 2\pi h\int_0^R (w_{ex} + w_{an})\,\rho d\rho. \tag{6}$$

To obtain the equilibrium distribution of magnetization which corresponds to the minimum value of energy defined in (6), we use Euler's equations:

$$\frac{\partial}{\partial\rho}\frac{\partial w}{\partial\vartheta'_\rho} = \frac{\partial w}{\partial\vartheta}, \qquad \frac{\partial}{\partial\rho}\frac{\partial w}{\partial\phi'_\rho} = \frac{\partial w}{\partial\phi}. \tag{7}$$

Then we get the system of equations:

$$\frac{\partial^2\vartheta}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\vartheta}{\partial\rho} - \frac{1}{2}\sin(2\vartheta)\left[\left(\frac{\partial\phi}{\partial\rho}\right)^2 + \frac{1}{\rho^2} - \frac{1}{\delta_W^2}\right] = 0, \tag{8}$$

$$\tan(\vartheta)\left[\frac{\partial^2\phi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\phi}{\partial\rho}\right] + 2\frac{\partial\vartheta}{\partial\rho}\frac{\partial\phi}{\partial\rho} = 0. \tag{9}$$

After the change of variables $f = \partial\phi/\partial\rho$, we obtain the following system of equations:

$$\frac{\partial^2\vartheta}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\vartheta}{\partial\rho} - \frac{1}{2}\sin(2\vartheta)\left[f^2 + \frac{1}{\rho^2} - \frac{1}{\delta_W^2}\right] = 0, \tag{10}$$

$$\tan(\vartheta)\left[\frac{\partial f}{\partial\rho} + \frac{f}{\rho}\right] + 2\frac{\partial\vartheta}{\partial\rho}f = 0. \tag{11}$$

Let us present equation (11) in the form:

$$\frac{\partial f}{\partial\rho} + \eta f, \qquad \eta = \frac{1}{\rho} + \frac{2}{\tan(\vartheta)}\frac{\partial\vartheta}{\partial\rho}. \tag{12}$$

The solution of equation (12) is

$$f = \frac{\partial\phi}{\partial\rho} = \frac{C}{\rho\sin^2(\vartheta)}. \tag{13}$$

The constant C is determined from the following boundary condition. At $\rho = R$ parameter f should be equal to zero (angle ϕ does not depend on ρ), therefore $C = 0$. This is conceivable if the nanodisk radius is comparable or exceeds characteristic linear size of magnetic vortex, i.e. $R \geqslant \delta_W$. This assumptiom is true in our model. According to experimental results the correlation radius of magnetization is in the range from 2 to 20 nanometers in permalloy nanodots [2, 6, 7]. Our analysis is valid for the disk radius is up to several hundred nanometers. In this case equation (10) becomes

$$\frac{\partial^2 \vartheta}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta}{\partial r} - \frac{1}{2}\sin(2\vartheta)\left[\frac{1}{r^2} - 1\right] = 0, \qquad (14)$$

here $r = \rho/\delta_W$ is dimensionless variable.

The solution of equation (14) is expressed in terms of special functions. It is very inconvenient to use these functions in further analysis. Therefore we solve equation (14) numerically and find an acceptable approximation for the numerical data.

Fig. 2 shows the results of numerical solution of equation (14) and approximating function

$$m_z(r) = \cos(\vartheta(r)) = \frac{\exp\left(-0.1r^2\right)}{1 + 0.6r^2}. \qquad (15)$$

Boundary conditions for equation (14)are $\vartheta(0) = 0$, $\vartheta(R/\delta_w) = \pi/2$.

In early works of other authors not much attention has been given to magnetization distribution. The equations similar to equation (14) were solved approximately and some simple approximating function were taken. Often approximating function was taken in the form $m_z \sim \exp(-r^2)$ [2, 8] or material point approach was used [10]. Generally those approaches are justified, because the function form of $m_z(r)$ practically does not influence the dynamic characteristics of magnetic vortex. What is important that the function describing magnetization distribution should be sufficiently localized.

However, the form of approximating function matters for determining other characteristics of magnetic vortex. For example, magnetic characteristics of nanodisk material can be determined from the average value of $\langle m_z \rangle$. In this case the approximate solution of equation (14) should be as close to the exact solution as possible.

2. Potential Magnetic Energy of Magnetic Subsystem

As it was already mentioned that the Bloch point is in the nanodisk center for the equilibrium state of magnetization. When the center of a vortex is displaced from the center of disk, energy of magnetic subsystem is increased.

Let us consider a simplified model of displacement of a rigid magnetic vortex from the equilibrium state. In the case of small displacement such model is acceptable for certain ratios of the core size of the vortex to nanodot radius [1]. In this model we assume that the form of magnetic vortex is not changed when the Bloch point is shifted from the nanodisk center. In this case the magnetostatic charges appear on the lateral surface of the disk and they interact with each other. As a result additional potential energy arises and quasielastic restoring force is exerted on the vortex core. The approach of rigid distribution of magnetization allows simple calculation of surface charges distribution on the lateral surface of the disk.

Formation of magnetostatic charges is schematically shown in Fig. 3. With a rigid shift of magnetization the perpendicular to the surface component of magnetization arises on the lateral surface of the disk. The value of this component is equal to the surface charge density:

$$\sigma(\beta) = M_\rho(R, \beta). \qquad (16)$$

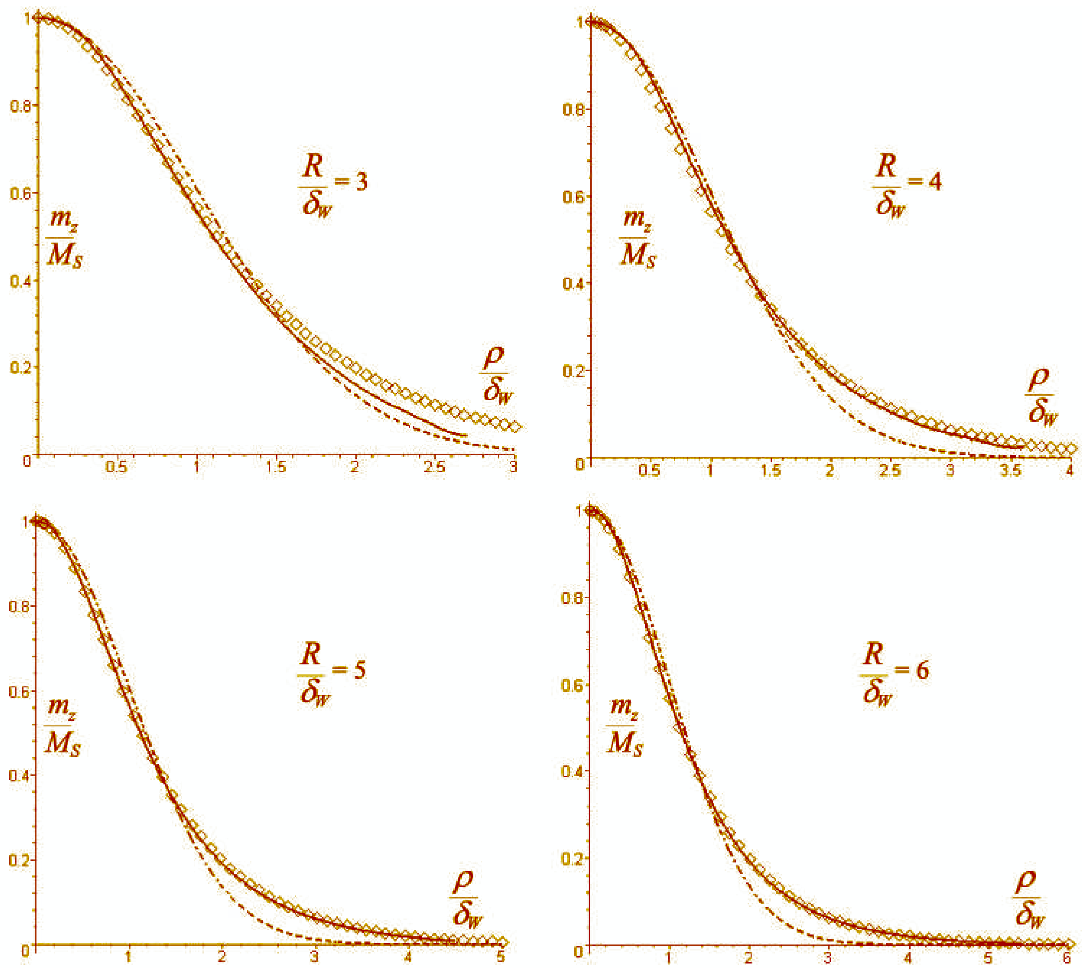

Fig. 2. Comparison of the numerical solution with two approximate solutions. Points — approximating function (15), solid line — numerical solution of equation (14), dotted line — approximating function $\exp(-r^2)$

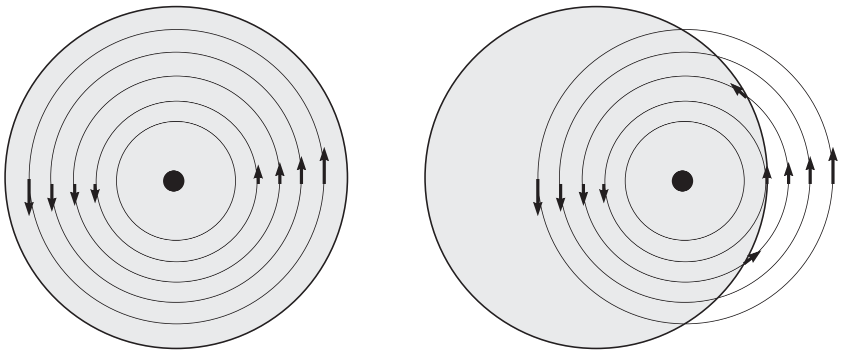

Fig. 3. The mechanism of formation of surface charges when the Bloch point is shifted from the disk center

To determine the energy increase we find the function of distribution of surface charges. The Bloch point is shifted on the distance x scaled by δ_W. Other parameters are shown in Fig. 4. As is seen from Fig. 4 the angle between the projection of magnetization to the plane of the disk and ρ axis is $\gamma = -\alpha + \pi/2$. The application of the sine theorem to expression (16) gives

$$\sigma(\beta) = M_S \sin(\vartheta)\cos(\gamma) = M_S \sin(\vartheta)\sin(\alpha) = M_S \frac{x\sin(\beta)\sin(\vartheta(x,\beta))}{\sqrt{x^2 + R^2 - 2xR\cos(\beta)}}. \tag{17}$$

It should be noted that the polar angle ϑ depends on the distance between the nanodisk center and the Bloch point (center of symmetry of magnetization distribution). Therefore, $\vartheta(r)$ is the function of parameters x and β: $\vartheta(x,\beta) = \vartheta\left(\sqrt{x^2 + R^2 - 2xR\cos(\beta)}\right)$, from here on R is dimensionless.

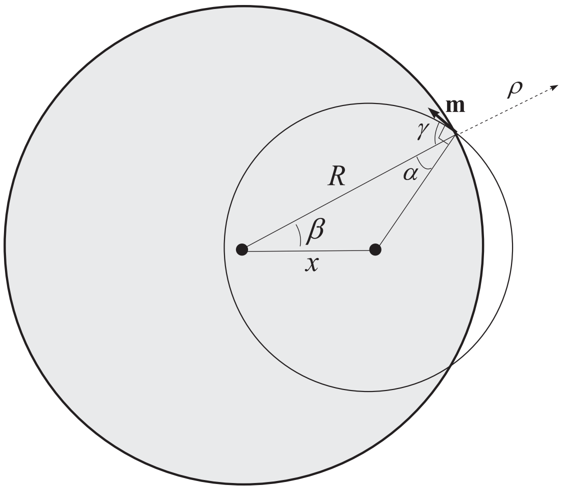

Fig. 4. Set of parameters for the definition of component M_ρ

Let us write the expression for the energy of interaction of surface charges in an explicit form. We represent the lateral surface as a set of small surface elements. Then the energy of pair interaction of two elements is

$$dW_1 = \frac{\mu_0}{4\pi}\frac{\delta_W^3\sigma(\beta)\sigma(\beta')}{\sqrt{2R^2\left(1-\cos(\beta-\beta')\right)}}h^2R^2 d\beta d\beta' = \frac{\mu_0\delta_W^3\sigma(\beta)\sigma(\beta')}{4\pi\sin(\frac{beta-\beta'}{2})}h^2 R d\beta d\beta'. \tag{18}$$

From here on h is dimensionless thickness of the disk. The total energy W_1 is

$$W_1 = \frac{\mu_0 h^2\delta_W^3 R}{4\pi}\int_0^{2\pi}\int_0^{2\pi}\frac{\sigma(\beta)\sigma(\beta')}{\sin(\frac{\beta-\beta'}{2})}d\beta d\beta'. \tag{19}$$

Taking into account (17) we obtain the following expression for total energy (19):

$$2_S\delta_W^3 h^2 R x^2 W_1 = \frac{\mu_0 M}{4\pi}\times$$

$$\times\int_0^{2\pi}\int_0^{2\pi}\frac{\sin(\beta)\sin(\beta')\sin(\vartheta(x,\beta))\sin(\vartheta(x,\beta'))}{\sin(\frac{beta-\beta'}{2})\sqrt{(x^2+R^2-2xR\cos(\beta))\left(x^2+R^2-2xR\cos(\beta')\right)}}d\beta d\beta'. \tag{20}$$

Here the polar angle of magnetization ϑ is defined by equation (15). Expression (20) can be written in another form:

$$W_1 = \frac{\mu_0 M_S^2\delta_W^3 h^2}{4\pi}I(x,R). \tag{21}$$

The dimensionless function $I(x,R)$ depends on the disk size and the distance x.

The numerical results for $I(x,R)$ and chosen approximation are shown in Fig. 5. With a good accuracy the potential energy (21) can be presented in the form

$$W_1 = \frac{180\mu_0 M_S^2 \delta_W^3 h^2}{4\pi R} x^2. \tag{22}$$

Similar result has been received earlier for the case of large x comparable with R [10].

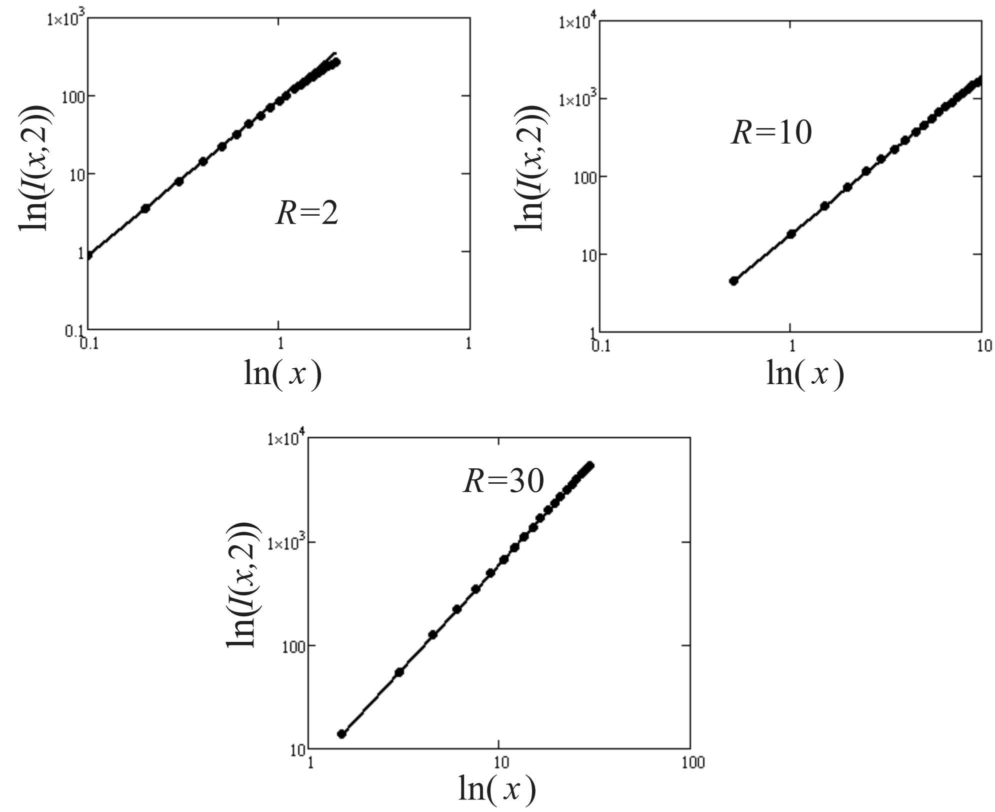

Fig. 5. Points are calculated values of integral $I(x,R)$ from (21). Solid line is approximating function $180x^2/R$

The expression for quasielastic force follows from expression (21):

$$\kappa = \frac{90\mu_0 M_S^2 \delta_W h^2}{\pi R}. \tag{23}$$

3. Kinetic Energy of Magnetic Vortex

It has been known [11,12] that rotation of magnetization behaves like an inert quasiparticle. Then the kinetic energy of that quasiparticle is proportional to the product of the square of quasiparticle velocity by effective weight. The existence of effective weight is due to the magnetic moments precession in dispersion fields created by domain wall.

Let us estimate the effective weight of the vortex. The additional induction of demagnetizing field due to the motion of the vortex can be estimate as [12]:

$$B_x = \mu_0 M_x. \tag{24}$$

As spins is subjected to field (24) they precess around x axis with the velocity

$$\omega = \frac{d\zeta}{dt} = \gamma B_x = -\gamma\mu_0 M_x, \tag{25}$$

here γ is the gyromagnetic ratio. We can also write

$$\omega = \frac{d\zeta}{dt} = \frac{d\zeta}{dx}\frac{dx}{dt} = -\frac{d\zeta}{dx}v. \tag{26}$$

Comparing (25) and (26) we have:

$$M_x = \frac{1}{\gamma\mu_0}\frac{d\zeta}{dx}v. \tag{27}$$

The increase of energy of the vortex due to additional dispersion field is

$$W_2 = -\frac{1}{2}\int_V M_x B_x d\mathbf{r}^3. \tag{28}$$

Here the integration is carried out over the volume of the disk. Then in cylindrical coordinate system the kinetic energy is defined as follows:

$$W_2 = \frac{h\delta_W}{2\gamma^2\mu_0}v^2\int_0^{2\pi}\int_0^R\left(\frac{d\zeta}{dx}\right)^2 r dr d\beta. \tag{29}$$

The factor at v^2 is the effective weight:

$$m = \frac{h\delta_W}{2\gamma^2\mu_0}\int_0^{2\pi}\int_0^R\left(\frac{d\zeta}{dx}\right)^2 r dr d\beta. \tag{30}$$

Let us write the integrand of (29) in the form

$$\frac{d\zeta}{dx} = \frac{d\zeta}{dr}\frac{dr}{dx} + \frac{d\zeta}{d\beta}\frac{d\beta}{dx}. \tag{31}$$

Note that ζ is the angle measured from y axis in the yz plane (this plane is perpendicular to the disk surface) and $\phi = -\beta + \pi/2$ in the expression for magnetization distribution. For this angle value we have

$$\tan(\zeta) = \frac{\cos(\vartheta(r))}{\sin(\vartheta(r))\cos(\beta)}. \tag{32}$$

Besides we have an obvious relation $x = r\cos(\beta)$. Then taking into account (32) the left hand side of (31) becomes

$$\begin{cases} \dfrac{d\zeta}{dr} = \dfrac{-1}{\cos^2(\beta) + \tan^{-2}(\vartheta(r))}\dfrac{\cos(\beta)}{\sin^2(\vartheta(r))}\dfrac{d\vartheta(r)}{dr}, \\ \dfrac{d\zeta}{d\beta} = \dfrac{1}{\cos^2(\beta) + \tan^{-2}(\vartheta(r))}\dfrac{\sin(\beta)}{\tan(\vartheta(r))}, \\ \dfrac{dr}{dx} = \dfrac{1}{\frac{dx}{dr}} = \dfrac{1}{\cos(\beta)}, \\ \dfrac{d\beta}{dx} = \dfrac{1}{\frac{dx}{d\beta}} = \dfrac{-1}{r\sin(\beta)}. \end{cases} \tag{33}$$

Considering the last expressions integral (30) takes the form

$$m = \frac{h\delta_W}{2\gamma^2\mu_0}\int_0^{2\pi}\int_0^R \frac{1}{\left(\cos^2(\beta)+\tan^{-2}(\vartheta(r))\right)^2}\left(\frac{1}{\sin^2(\vartheta(r))}\frac{d\vartheta(r)}{dr}+\frac{1}{r\tan^2(\vartheta(r))}\right)^2 r dr d\beta. \tag{34}$$

It is obviously not possible to evaluate this integral analytically, so we resort to numerical integration. The results of numerical calculation of integral (34) and approximating function are shown in Fig. 6.

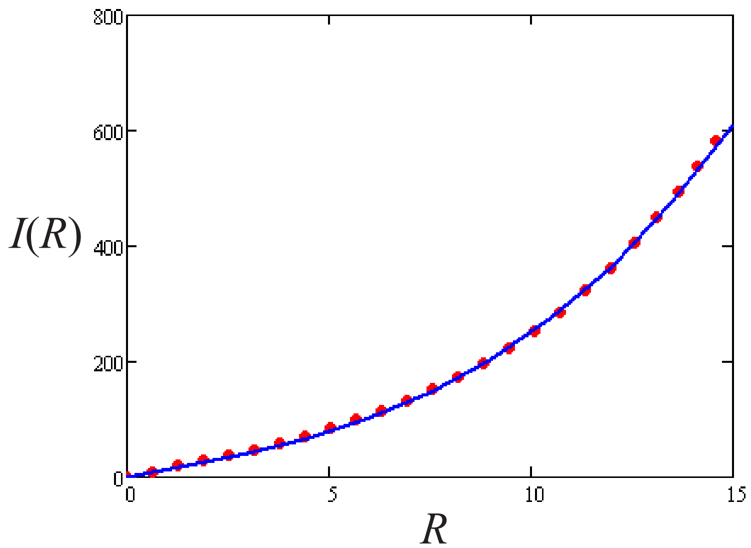

Fig. 6. Dots are results of numerical calculation of integral (34). Solid line is approximating function $I(R) = 12.5R + 0.125R^3$

Thus the effective weight can be approximated as follows

$$m = 6.25\frac{h\delta_W R}{\gamma^2\mu_0}\left(1+0.01R^2\right). \tag{35}$$

If damping in the system is neglected then we obtain with the use of (23) and (35) the following expression for the eigenfrequency of vortex oscillation:

$$\Omega = \sqrt{\frac{\kappa}{m}} = \left[\frac{14.4\mu_0^2 M_S^2 h\gamma^2}{\pi R^2\left(1+0.01R^2\right)}\right]^{\frac{1}{2}}. \tag{36}$$

Conclusion

Expression (36) allows us to predict the qualitative dependences of oscillation frequency of the magnetic vortex core on nanodisk sizes. The example of such dependences is shown in Fig. 7.

Comparison with known experimental data shows that presented above simple "mechanistic" approach is quite justified for the description of low frequency oscillation of magnetization in nanodots [13].

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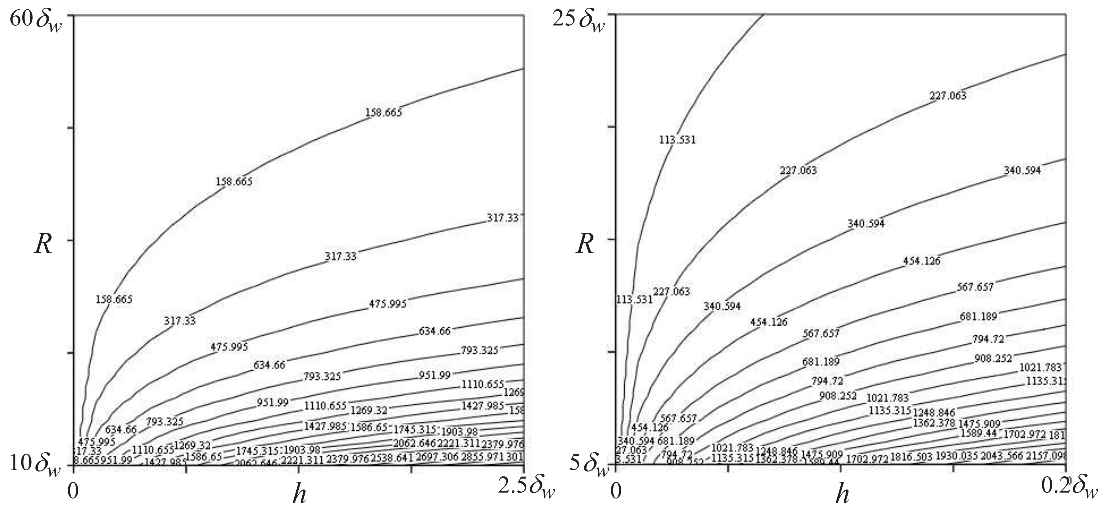

Fig. 7. Contour diagrams of frequency of vortex oscillation as a function of disk thickness and disk radius for $\delta_w = 0.02$ мкм. Frequencies are given in megahertz

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О низкочастотных колебаниях точки Блоха в нанодиске

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Петр Д. Ким

В настоящей работе аналитически решается задача о низкочастотных колебаниях магнитного вихря малой амплитуды в изолированном цилиндрическом ферромагнетике субмикронных размеров. Расчеты проводятся в модели "жесткого" вихря, т.е. распределение намагниченности вблизи точки Блоха считается неизменной при смещении ядра из положения равновесия. Построены диаграммы, позволяющие предсказать частоты мод в зависимости от геометрических размеров наноцилиндра.

Ключевые слова: наноточка, магнитный вихрь, точка Блоха, перемагничивание.