

Quantum and Classical Correlations in the Solid-State NMR Free Induction Decay

V. E. Zobov · A. A. Lundin

Received: 16 June 2014/Revised: 29 July 2014/Published online: 13 August 2014
© Springer-Verlag Wien 2014

Abstract The free induction decay (FID) of the transverse magnetization in a dipolar-coupled rigid lattice is a fundamental problem in magnetic resonance and in the theory of many-body systems. As it was shown earlier the FID shapes for the systems of classical magnetic moments and for quantum nuclear spin ones coincide if there are many nearly equivalent nearest neighbors n in a solid lattice. In this paper, we reduce a multispin density matrix of above system to a two-spin matrix. Then we obtain analytic expressions for the mutual information and the quantum and classical parts of correlations at the arbitrary spin quantum number S , in the high-temperature approximation. The time dependence of these functions is expressed via the derivative of the FID shape. To extract classical correlations for $S > 1/2$ we provide generalized POVM measurement (positive-operator-valued measure) using the basis of spin coherent states. We show that in every pair of spins the portion of quantum correlations changes from $1/2$ to $1/(S + 1)$ when S is growing up, and quantum properties disappear completely only if $S \rightarrow \infty$.

1 Introduction

Nuclear spin systems observed by nuclear magnetic resonance (NMR) really for a long time and yet now are performing a suitable laboratory for studying the physics of nonequilibrium processes in quantum many-body systems. Some of the most of fundamental lines of that type activities are the emergence and growth of correlations, spin dynamics and so on [1]. Quite recently applications of the NMR

V. E. Zobov (✉)

L. V. Kirensky Institute of Physics, Siberian Branch, RAS, 660036 Krasnoyarsk, Russia
e-mail: rsa@iph.krasn.ru

A. A. Lundin

N. N. Semenov Institute of Chemical Physics, RAS, 117977 Moscow, Russia

spin dynamics to investigate quantum information processing were initiated [2]. It is usually assumed that the quantum correlation existing both at low and at high temperatures influence the performance speed of quantum computer [3]. In this regard, the interest of researchers has shifted from the calculation of the correlation function as a whole to their partitioning into quantum and classical parts (e.g. see the review [3]). On the other hand different time correlation functions determine observed NMR signals in conventional NMR [1]. However, their decomposition in quantum and classical components has not been done yet. In the present article it will be done for one of the most significant NMR time correlation function, namely, for the free induction decay (FID) function.

The FID shape links to the shape of NMR absorption line via Fourier transform [1]. In the many-body spin systems of solids, the calculation of the time correlation functions is a very challenging problem and different approaches to its solutions have been widely discussed. In light of the above, we point to the works [4, 5]. In the article [4], the numerical simulation has been used to derive FID curves for a simple cubic lattice with 216 classical magnetic moments (classical spins) coupled by dipole–dipole interaction. It was found that the calculated FID shape is close to the FID shape of fluorine nuclei (nuclear spin $S = 1/2$) which was experimentally measured in CaF_2 [6].

In Ref. [5] we explained this result. We showed that the time dependence of FID for the system consisting of quantum spins and one formed by classical magnetic moments $\mu = \gamma\hbar\sqrt{S(S+1)}$ coincides in a limit of a large number of the equivalent nearest neighbors surrounding a probe spin (anyone spin) in a lattice. The deduction has been made on the basis of the analysis of various contributions to the spectral moments of all orders of NMR spectrum. Actually in Ref [5], we demonstrated that if the number of the rather equivalent nearest neighbors for any spin is large enough, then the moment terms with maximum number of the summing indexes on the lattice are carrying the principal contributions to the arbitrary NMR spectral moment. Referred above contributions coincide exactly for classical and quantum spin systems. So it works for ordinary regular three-dimensional lattices (e.g. simple cubic one). Comparison of the values of the exact spectral moments from M_4 to M_8 , performed in Ref. [5] also revealed insignificant discrepancies between results for the systems of quantum and classical spins.

It is interesting to calculate share of quantum correlations under these conditions. One of the approaches to solve the problem of clearing quantum effects (quantum correlations) consists of the reduction of the multispin density matrix to the two-spin matrix with the subsequent analysis of pair correlations [3]. Thus, such approach is applied to the description of one-dimensional XY chain in Refs. [7, 8], and also, in Ref. [9], to investigation of spins in nanopore with equal dipolar interaction between any two spins. In both cases, only nuclei with a spin quantum number $S = 1/2$ were studied. In the present work we consider lattices formed by nuclei with an arbitrary spin S . Any disturbing quadrupole effects are neglected. We will provide a reduction of the multispin density matrix to a two-spin matrix. Then, following the program put forward in Ref. [10], we are going to calculate shares of quantum and classical correlations: for $S = 1/2$ we shall use the von Neumann orthogonal measurement,

whereas for $S > 1/2$ we shall provide generalized POVM measurement (positive-operator-valued measure) [3, 11] using the basis of spin coherent states (SCS) [12].

2 Hamiltonian and Dynamics

In traditional experiments employing NMR, the spin temperature considerably exceeds the energy of the Zeeman and other interactions in the spin system. As a consequence, polarization is very small for nuclear spin in the strong static magnetic field at room temperature T , $\beta = \hbar\omega_0/kT \approx 10^{-5} \ll 1$ (ω_0 is the Larmor frequency), and the equilibrium density matrix has the form [1]:

$$\hat{\rho}_{eq} = (1 + \beta\hat{S}_z)/Z, \tag{1}$$

where Z is the partition function, $\hat{S}_\alpha = \sum_j \hat{S}_{j\alpha}$, $\hat{S}_{j\alpha}$ is the α -component ($\alpha = x, y, z$) of the spin j , and the external magnetic field H_0 is directed along the z -axis. As well known [1] for observation of a FID signal it is preliminary necessary to prepare the spin system using the pulse of the radio-frequency magnetic field causing rotation of spins at $\pi/2$ -angle around the y -axis of the rotating with the Larmor frequency reference frame. So we get

$$\hat{\rho}(0) = \hat{Y}\hat{\rho}_{eq}\hat{Y}^{-1} = (1 + \beta\hat{S}_x)/Z.$$

This initial density matrix evolves in time as

$$\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(0)\hat{U}^{-1}(t) = [1 + \beta\hat{U}(t)\hat{S}_x\hat{U}^{-1}(t)]/Z = [1 + \beta\Delta\hat{\rho}(t)]/Z, \tag{2}$$

where $\hat{U}(t) = \exp(-i\hat{H}t/\hbar)$ is the operator of evolution with the Hamiltonian \hat{H} . An observable signal of FID is proportional to time correlation function:

$$F(t) = \frac{Tr\{\hat{S}_x\hat{\rho}(t)\}}{Tr\{\hat{S}_x\hat{\rho}(0)\}} \tag{3}$$

and it links to the shape of NMR absorption line via the Fourier transform.

In nonmetallic diamagnetic solids a principal cause of the absorption NMR line broadening is a secular part of dipole–dipole interaction between nuclear spins [1]:

$$\hat{H}_d = \sum_{i \neq j} b_{ij}\hat{S}_{zi}\hat{S}_{zj} + \sum_{i \neq j} a_{ij}(\hat{S}_{xi}\hat{S}_{xj} + \hat{S}_{yi}\hat{S}_{yj}) = \hat{H}_{zz} + \hat{H}_{ff}, \tag{4}$$

where, $b_{ij} = \gamma^2\hbar(1 - 3\cos^2\theta_{ij})/2r_{ij}^3$, $a_{ij} = -b_{ij}/2$, \vec{r}_{ij} is the vector connecting spins i and j , θ_{ij} is the angle, formed by vector \vec{r}_{ij} with the static external magnetic field. From here on, energy is expressed in frequency unities ($\hbar = 1$). So this interaction completely specifies the dynamics of the nuclear spin system.

Interaction between transversal spin components does not allow writing an explicit time dependence of the density matrix. In this situation for finding the appropriate form of the density matrix, we shall decompose it over the complete system of orthogonal operators $[k]$ following the line outlined in Ref. [13–15]. In this representation,

$$\hat{S}_x(t) = \hat{U}(t)\hat{S}_x\hat{U}^{-1}(t) = \sum_{k=0}^{\infty} A_k(t)[k]. \tag{5}$$

The initial operator $[0] = \hat{S}_x$. Each subsequent operator of the basis is obtained from the previous one after the procedure of commutation with the Hamiltonian according to the recursion relations:

$$[1] = i[\hat{H}_d, [0]], \quad [k + 1] = i[\hat{H}_d, [k]] + v_{k-1}^2[k - 1] \quad (\text{if } k \geq 1),$$

$$v_k^2 = Tr\{[k + 1]^+[k + 1]\} / Tr\{[k]^+[k]\}.$$

where $[k]^+$ is Hermitian adjoint to $[k]$. For amplitudes $A_k(t)$ the system of the differential equations [13, 14] has been revealed

$$\dot{A}_0(t) = v_0^2 A_1(t), \quad \dot{A}_k(t) = A_{k-1}(t) - v_k^2 A_{k+1}(t) \quad (\text{if } k \geq 1). \tag{6}$$

To avoid confusion, a certain difference in the definition of amplitudes $A_k(t)$ in references [13] and [14] should be noticed. The difference is in the factor $(i)^k$. We have chosen a variant used in Ref. [14] at which functions $A_k(t)$ contain no imaginary part, because the factor $(i)^k$ is included into definition of operators $[k]$. The parameters $\{v_k\}$ which values determine the solution of the system Eq. (6), are expressed unequivocally through the moments of the NMR absorption line [13]. In particular

$$v_0^2 = M_2 = 3S(S + 1) \sum_j b_{ij}^2, \quad v_1^2 = (M_4 - M_2^2) / M_2, \quad v_2^2 = (M_2 M_6 - M_4^2) / (M_4 - M_2^2) M_2, \tag{7}$$

where M_2, M_4, M_6 are the second, fourth and sixth moments of the NMR absorption line.

Let us substitute decomposition Eq. (5) to Eq. (2) and then execute the reduction. As it means we have to choose two spins at sites i and j and then to calculate a trace in Eq. (2) over all other spin variables. Thus we have

$$\hat{\rho}_{ij}(t) = \frac{1}{d^2} \left\{ 1 + \beta \sum_{k=0}^{\infty} A_k(t) \frac{d^2}{Z} Tr_{\neq i,j}[k] \right\}, \tag{8}$$

where $d = 2S + 1$. So for the first two orthogonal operators of the complete set we get

$$\frac{1}{Z} Tr_{\neq i,j}[0] = \frac{1}{Z} Tr_{\neq i,j} \sum_f \hat{S}_{xf} = \frac{1}{d^2} (\hat{S}_{xi} + \hat{S}_{xj}), \tag{9}$$

$$\frac{1}{Z} Tr_{\neq i,j}[1] = \frac{i}{Z} Tr_{\neq i,j} [\hat{H}_d, \hat{S}_x] = \frac{-2}{d^2} (b_{ij} - a_{ij}) (\hat{S}_{yi} \hat{S}_{zj} + \hat{S}_{yj} \hat{S}_{zi}). \tag{10}$$

The contribution to Eq. (8) from orthogonal operators of the higher order can be obtained in two cases. First case assumes zero direct interaction between the chosen spins i and j . It is a possible case for example, if the angle θ_{ij} between the vector \vec{r}_{ij} and external magnetic field is equal to the “magic” value $54^\circ 44'$. In this situation we have to take into account the contribution from vector [3] which depends on the

constant $b_{ij}b_{jf}^2$ through the intermediate spin f if this constant is distinct from zero. The second case appears if $S > 1/2$ because orthogonal operators of the high order are formed of products of spin operators not only from different sites, but also from the same site. For examples in vector [2] there is a contribution $\hat{S}_{xi}\{\hat{S}_{zj}^2 - S(S+1)/3\}$, and in vector [3] one gets a contribution $\hat{S}_{yi}\{\hat{S}_{zj}^3 - \hat{S}_{zj}(3S^2 + 3S - 1)/5\}$. We shall neglect above mentioned contributions in Eq. (8) because these parts are small corrections to contributions from Eqs. (9) and (10). The trifle of discussing corrections is a consequence of the different time dependence of the different order amplitudes: $A_k(t) \sim t^k$ at small times. Because of the rapid decay of amplitudes $A_k(t)$ at times $t \geq 1/\sqrt{M_2}$, each additional power of t adds only a small factor $|b_{ijt}| \sim \sqrt{b_{ij}^2/M_2} \sim 1/\sqrt{n} < 1$.

Having retained two contributions Eqs. (9) and (10) in Eq. (8) we get

$$\hat{\rho}_{ij}(t) \approx \frac{1}{d^2} \{1 + \beta A_0(t)(\hat{S}_{xi} + \hat{S}_{xj}) + \beta A_1(t)B_{ij}(\hat{S}_{yi}\hat{S}_{zj} + \hat{S}_{yj}\hat{S}_{zi})\}, \tag{11}$$

where $B_{ij} = -2(b_{ij} - a_{ij}) = -3b_{ij}$ for the Hamiltonian Eq. (4). At last, at the further reduction to one spin one gets

$$\hat{\rho}_{i(j)}(t) \approx \frac{1}{d} \{1 + \beta A_0(t)\hat{S}_{xi(j)}\}. \tag{12}$$

Having substituted Eq. (12) in Eq. (3), we get $F(t) = A_0(t)$.

3 Classical and Quantum Correlations

3.1 Mutual information

The information-theoretic measure of correlations between two spins is the mutual information [3, 11],

$$I(\hat{\rho}_{ij}) = S_N(\hat{\rho}_i) + S_N(\hat{\rho}_j) - S_N(\hat{\rho}_{ij}), \tag{13}$$

where $S_N(\hat{\rho}) = -Tr\{\hat{\rho} \log_2 \hat{\rho}\}$ is the von Neumann entropy. We assume to calculate the von Neumann entropy in the lowest order on β [1, 10],

$$S_N(\hat{\rho}) = -Tr\{\hat{\rho} \log_2 \hat{\rho}\} \approx \log_2 Z - \frac{\beta^2}{2Z \ln 2} Tr(\Delta \hat{\rho})^2.$$

In the high-temperature approach accepted the mutual information Eq. (13) is as follows:

$$I(\hat{\rho}_{ij}) = \frac{\beta^2}{2 \ln 2} \left\{ \frac{1}{d^2} Tr(\Delta \hat{\rho}_{ij})^2 - \frac{1}{d} Tr_i(\Delta \hat{\rho}_i)^2 - \frac{1}{d} Tr_j(\Delta \hat{\rho}_j)^2 \right\}. \tag{14}$$

The density matrix Eq. (11) looks like similar expression for isolated pair of the spins, calculated in [10] at small times. Therefore, skipping on intermediate evaluations, we are giving the results at once. By such a way we calculated for the mutual information

$$I(\hat{\rho}_{ij}) \approx \frac{\beta^2}{9 \ln 2} [S(S + 1)B_{ij}A_1(t)]^2 = \frac{\beta^2 b_{ij}^2}{M_2^2 \ln 2} [S(S + 1)\dot{F}(t)]^2. \tag{15}$$

Under the transformations in process of obtaining the Eq. (15) formulas Eqs. (6) and (7) were used.

3.2 Orthogonal projective measurement of von Neumann

The mutual information Eq. (13) is used to measure the total correlations, which are sums of the classical and quantum correlations. The classical correlations can be calculated by the measurement, described in [3]. To perform a von Neumann measurement we must project the state $\hat{\rho}_{ij}(t)$ Eq. (11) on the complete basis of orthogonal wave functions $|\Psi_m\rangle$ by means of a complete set of projectors,

$$\hat{\Pi}_m = |\Psi_m\rangle\langle\Psi_m|, \quad \sum_m \hat{\Pi}_m = 1. \tag{16}$$

In the case of system with $S = 1/2$ the complete set of orthogonal projectors of the spin i consists of two projectors

$$\hat{\Pi}_{i\pm} = \left| \pm \frac{1}{2} \right\rangle \left\langle \pm \frac{1}{2} \right| = \frac{1}{2} \pm \hat{S}_{iz},$$

where $|\pm \frac{1}{2}\rangle_i$ are eigenstates of the operator \hat{S}_{iz} with eigenvalues $\pm \frac{1}{2}$. In general, if the direction of the quantization axis does not coincide with one of the axis z , and specified by the direction cosines n_α ($\alpha = x, y, z$), these projectors can be written as follows:

$$\hat{\Pi}_{i\pm} = \frac{1}{2} [1 \pm (n_x \hat{\sigma}_{ix} + n_y \hat{\sigma}_{iy} + n_z \hat{\sigma}_{iz})], \tag{17}$$

where σ are the Pauli matrices. The density matrix $\hat{\rho}_{ij}(t)$ is transformed after projecting on the states of the spin i to

$$\hat{\Pi}_i(\hat{\rho}_{ij}) = \frac{1}{Z} [1 + \beta \hat{\Pi}_i(\Delta \hat{\rho}_{ij}(t))], \tag{18}$$

where we have

$$\hat{\Pi}_i(\Delta \hat{\rho}_{ij}(t)) = \sum_m (\hat{\Pi}_{im} \otimes \hat{E}_j) \Delta \hat{\rho}_{ij}(t) (\hat{\Pi}_{im} \otimes \hat{E}_j),$$

and where \hat{E}_j is the unit matrix. After carrying out calculations we have

$$\begin{aligned} \hat{\Pi}_i(\Delta \hat{\rho}_{ij}) &= A_0(t) \hat{S}_{jx} + \frac{1}{2} \hat{\Pi}_{i+} \{n_x A_0(t) + A_1(t) B_{ij} (n_z \hat{S}_{jy} + n_y \hat{S}_{jz})\} \\ &\quad - \frac{1}{2} \hat{\Pi}_{i-} \{n_x A_0(t) + A_1(t) B_{ij} (n_z \hat{S}_{jy} + n_y \hat{S}_{jz})\} \end{aligned} \tag{19}$$

As usual let us choose the mutual information $I(\hat{\Pi}_i; \hat{\rho}_{ij})$ calculated using formulas Eqs. (13), (14) and (19) for this matrix, as a measure of classical correlations

$$I(\hat{\Pi}_i(\hat{\rho}_{ij})) = \frac{\beta^2}{32 \ln 2} [B_{ij}A_1(t)]^2 (n_z^2 + n_y^2). \tag{20}$$

Unfortunately the gained value Eq. (20) will depend on the chosen basis Eq. (17). It was proposed [3] to search all bases and to take the maximum value of correlation $I(\hat{\Pi}_i(\hat{\rho}_{ij}))$ as the universal measure. If we subtract the classical part from all correlations Eq. (15), then we obtain the quantum part of the correlations

$$D_{ij} = I(\hat{\rho}_{ij}) - \max_{n_x} I(\hat{\Pi}_i(\hat{\rho}_{ij})). \tag{21}$$

After carrying out of minimization of this quantity on the direction cosines n_x one gains an entropy measure of quantum correlations named by quantum discord D_{ij} [3]. We obtain that $n_z^2 + n_y^2 = 1$ and that the quantum discord D_{ij} are related to the mutual information $I(\hat{\rho}_{ij})$ from Eq. (15) by the relations:

$$D_{ij} = I(\hat{\rho}_{ij})/2. \tag{22}$$

3.3 POVM measurement

For $S > 1/2$ we shall provide generalized POVM measurement [3, 11] using the basis of SCS [12]. It is assumed that the SCS (Bloch states) [12]

$$|\theta, \phi\rangle = \hat{R}(\theta, \phi)|S\rangle = \sum_{m=-S}^{m=S} \binom{2S}{S+m}^{1/2} (\cos \theta/2)^{S+m} (e^{i\phi} \sin \theta/2)^{S-m} |m\rangle, \tag{23}$$

are closest to the states of the classical momenta. Here θ and ϕ are the polar and azimuthal angles on the unit sphere (Bloch sphere), $|m\rangle$ is an eigenstate of the operator S_z with eigenvalue m assuming $2S + 1$ values,

$$-S, -S + 1, \dots, S - 1, S.$$

These states Eq. (23) are obtained from the ground state $|S\rangle$ by the rotation operator $\hat{R}(\theta, \phi) = \exp\{-i\theta(-\hat{S}_x \sin \phi + \hat{S}_y \cos \phi)\}$ and are a superposition of states with different m . The average values of spin projections in the state Eq. (23) are as follows

$$\langle \theta, \phi | \hat{S}_z | \theta, \phi \rangle = S \cos \theta, \quad \langle \theta, \phi | \hat{S}_x | \theta, \phi \rangle = S \sin \theta \cos \phi, \quad \langle \theta, \phi | \hat{S}_y | \theta, \phi \rangle = S \sin \theta \sin \phi$$

and are the same as for classical momentum. The completeness property

$$\frac{2S + 1}{4\pi} \int |\theta, \phi\rangle \langle \theta, \phi| \sin \theta d\theta d\phi = 1$$

is satisfied for the SCS basis, but this basis is not orthogonal. If one wants to use the generalized POVM measurement, he must recognize that the functions $|\Psi_m\rangle$ in operators Eq. (16) can now be no orthogonal, and these operators strictly speaking are already not projectors [11].

We take the SCS system as the measurement basis of Eq. (16), to perform the POVM measurement at the spin i , which reduces by multiplying the SCS and calculating the trace, and one obtains the classical density function for the probability distribution of the angle values

$$\begin{aligned} \hat{\Pi}_i(\hat{\rho}_{ij}) &= \hat{\rho}_j(\theta_i\phi_i;t) = \frac{2S+1}{4\pi} \text{Tr}_i\{\hat{\rho}_{ij}(t)|\theta_i, \phi_i\rangle\langle\theta_i, \phi_i| \otimes E_j\} \\ &= \frac{2S+1}{4\pi} \langle\theta_i, \phi_i|\hat{\rho}_{ij}(t)|\theta_i, \phi_i\rangle \\ &= \frac{1}{4\pi(2S+1)} \{1 + \beta A_0(t)(S_i \sin \theta_i \cos \phi_i + \hat{S}_{xj}) \\ &\quad + \beta A_1(t)B_{ij}(S_i \sin \theta_i \sin \phi_i \hat{S}_{zj} + \hat{S}_{yj}S_i \cos \theta_i)\} \end{aligned} \tag{24}$$

Now to calculate the Shannon entropy we have to calculate the integral over the Bloch sphere

$$S_{ShN}(\hat{\rho}_j(\theta_i\phi_i;t)) = - \int \text{Tr}_j\{\hat{\rho}_j(\theta_i\phi_i;t) \log_2 \hat{\rho}_j(\theta_i\phi_i;t)\} \sin \theta_i d\theta_i d\phi_i.$$

As a measure of classical correlations the mutual information $I(\hat{\Pi}_i(\hat{\rho}_{ij}))$ calculated using formulas Eqs. (13), (14) and (24) for this matrix is

$$I(\hat{\Pi}_i(\hat{\rho}_{ij})) \approx \frac{\beta^2}{9 \ln 2} S^3(S+1)[B_{ij}A_1(t)]^2. \tag{25}$$

If we subtract the classical part from all correlations Eq. (15), then we obtain the quantum part of the correlations

$$Q_{ij} = I(\hat{\rho}_{ij}) - I(\hat{\Pi}_i(\hat{\rho}_{ij})). \tag{26}$$

Measure Eq. (26) without optimization was called a measurement dependent discord [3]. We obtain the quantum part of correlations Q_{ij} (if $S > 1/2$) are related to the mutual information $I(\hat{\rho}_{ij})$ from Eq. (15) by the relations:

$$Q_{ij} \approx I(\hat{\rho}_{ij})/(S+1). \tag{27}$$

We should note that whether $S = 1/2$ the expression Eq. (27) equal to $2/3$ whereas from Eq. (22) one gets $1/2$. The discrepancy relates with the distinctions in the methods of measurement.

On the basis of the results derived above, it can be concluded that the time dependence of the mutual information Eq. (15) and the quantum part of correlations (22), (27) is revealing through the derivative of FID shape Eq. (3). Thus rapid exhaustion of pair correlations and reduction of their peak values with the growing up of the number of neighbors n generally speaking do not mean impairment of correlated relations of spins, but mean redistribution of pair correlations to more complicated multispin ones. As a measure of total correlation the total information [3, 16] can serve:

$$T(\hat{\rho}) = \sum_i S_N(\hat{\rho}_i) - S_N(\hat{\rho}) \approx \frac{\beta^2}{3 \ln 2} S(S+1)[1 - F^2(t)]. \tag{28}$$

At the initial moment of time $F(0) = 1$ and $T(\hat{\rho}) = 0$. For a long times $F^2(t)$ is coming to zero and, therefore, $T(\hat{\rho})$ reaches own limiting value only defined by entry conditions: e.g. by polarization β at given temperature and at the fixed strength of the external magnetic field.

4 Conclusion

Our results mean that in spite of coincidence [5] of the FID shapes of both classical and quantum spin systems for a large number n of nearest neighbors, the quantum properties of the system are not lost. For every pair of spins the portion of quantum correlations changes from $1/2$ to $1/(S + 1)$ with S growing up. In reality the quantum properties disappear completely only if $S \rightarrow \infty$ but not in the case when $n \rightarrow \infty$. The similarity of the FID shapes means that measurable classical correlations and “immeasurable” (lost at measurement) quantum correlations are bringing the equal influence at FID. So it implies that unobservable simultaneously spin components $\hat{S}_x, \hat{S}_y, \hat{S}_z$ are capable to give the contribution to dynamics of spins simultaneously. Thereof the time scale dependence is determined by the quantity $S(S + 1)$, instead of S^2 , where S is the maximal value of an observable projection upon any axis.

References

1. A. Abraham, M. Goldman, *Nuclear Magnetism: Order and Disorder* (Univ. Press, Oxford, 1982)
2. J.A. Jones, *Prog. NMR Spectrosc.* **59**, 91 (2011)
3. K. Modi, A. Brodutch, H. Cable, T. Paterek, V. Vedral, *Rev. Mod. Phys.* **84**, 1655 (2012)
4. S.J. Knak Jensen, O. Platz, *Phys. Rev. B* **7**, 31 (1973)
5. A.A. Lundin, V.E. Zobov, *J. Magn. Reson.* **26**, 229 (1977)
6. I.J. Lowe, R.E. Norberg, *Phys. Rev.* **107**, 46 (1957)
7. E.B. Fel'dman, A.I. Zenchuk, *Phys. Rev. A* **86**, 012303 (2012)
8. E.B. Fel'dman, A.I. Zenchuk, *Quantum Inf. Process.* **13**, 201 (2014)
9. E.B. Fel'dman, E.I. Kuznetsova, M.A. Yurishchev, *J. Phys. A: Math. Theor.* **45**, 475304 (2012)
10. V.E. Zobov, *Teor. Math. Phys.* **177**, 1377 (2013)
11. J. Preskill, *Quantum Information and Computation. Lecture Notes for Physics 229*, California Institute of Technology, (1998)
12. F.T. Arrechi, E. Courtens, R. Gilmore, H. Thomas, *Phys. Rev. A* **6**, 2211 (1972)
13. F. Lado, J.D. Memory, G.W. Parker, *Phys. Rev. B* **4**, 1406 (1971)
14. M.H. Lee, *Phys. Rev. Lett.* **52**, 1579 (1984)
15. V.E. Zobov, A.A. Lundin, *JETP* **103**, 904 (2006)
16. B. Groisman, S. Popescu, A. Winter, *Phys. Rev. A* **72**, 032317 (2005)