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INVESTIGATION OF THE Q-FACTOR OF OPTICAL RESONATORS IN PHOTONIC CRYSTALS AND PRINCIPLES OF DESIGNING HIGHLY SELECTIVE FILTERS ON THEIR BASIS

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Designs of optical resonators comprising a half-wavelength dielectric layer bounded from both ends by quarter-wavelength layers with permittivity higher or lower than that of the half-wavelength layer are investigated. Electrodynamic analysis demonstrates a significant increase in the Q-factor of the multilayered resonator in comparison with the conventional single-layer half-wavelength resonator. Principles of designing highly selective passband filters based on photonic crystal structures are considered.

Keywords: photonic crystal, Q-factor of the resonator, passband filter.

Multilayered structures consisting of alternating dielectric layers with different refractive indices and having the period commensurable with the electromagnetic wavelength are often called 1D photonic crystals. They are used to develop various optoelectronic devices: filters, polarizers, and mirrors [1–4]. In essence, the photonic crystals are systems of interacting (coupled) resonators; therefore, they have periodically alternating transparency windows and rejection bands (photonic band gaps) [5].

Coupled resonators in narrow band filters should have weak coupling with each other, and the end resonators should be weakly coupled with the input and output; in this case, they have high external Q-factor [6]. However, the external Q-factors of input and output resonators in photonic crystals, as well as the coupling coefficients of the neighboring resonators, depend on the difference (contrast) between the refractive indices of the contacting media: the greater the difference, the smaller the coupling coefficient and hence the higher the Q-factor of the resonator [7, 8]. Considering that the refractive indices of materials in optics are not so high, the external Q-factor of the half-wavelength dielectric layer in free space will also be low. Therefore, nonresonant quarter-wavelength separating layers are used to design narrow gap photonic crystal devices to weaken coupling of the resonant half-wavelength layer with free space or with the adjacent resonant half-wavelength layer [1].

In this regard, of great interest is investigation of the external Q-factor of the half-wavelength dielectric layer bounded by quarter-wavelength multilayers in free space together with the selective properties of some photonic crystal structures of passband filters based on half-wavelength resonators.

To calculate wave propagation through a multilayered dielectric structure, we take advantage of the 1D model in which the field strengths depend only on the *z* coordinate perpendicular to the layers, and the layers themselves are considered as cascaded sections of transmission lines. The normalized *ABCD* transmission matrix [9]

$$\begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} = \begin{pmatrix} \cos\theta_k & -iZ_k \sin\theta_k \\ -iZ_k^{-1} \sin\theta_k & \cos\theta_k \end{pmatrix} \tag{1}$$

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is convenient for a description of electrodynamic behavior of the kth section of the transmission line. Here Z_k is the normalized wave impedance and θ_k is the electric length of the kth section. It is assumed that the field components are oscillated following a harmonic law exp $(-i\omega t)$. In this case,

$$Z_k = 1/n_k\,,\ \theta_k = \frac{\omega}{c} n_k\, T_k\,, \tag{2}$$

where n_k is the refractive index and T_k is the thickness of the kth dielectric layer.

Transmission matrix (1) together with the formula

$$\begin{pmatrix} E_1 \\ Z_0 H_1 \end{pmatrix} = \begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix} \begin{pmatrix} E_2 \\ Z_0 H_2 \end{pmatrix} \tag{3}$$

enables one to calculate the electric and magnetic field strengths on one surface of the dielectric layer for the given field strengths on another surface. Here $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the characteristic impedance of free space.

For cascaded line sections, their transmission matrices are multiplied in the same order. Therefore, the $ABCD$ matrix of the n-layer structure can be calculated from the formula

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \prod_{k=1}^{n} \begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix}. \tag{4}$$

We characterize the power transmitted through the multilayered structure by the element of the scattering matrix $|S_{21}|^2$, measured in decibels, which is expressed through the $ABCD$ matrix by the formula [9]

$$S_{21} = \frac{2\sqrt{Z_2 Z_1}}{A Z_2 + B + C Z_2 Z_1 + D Z_1}\,, \tag{5}$$

where Z_1 and Z_2 are the characteristic impedances of the media before and behind the structure. Having the transmission spectrum and neglecting dielectric loss, we can estimate the external Q-factor at resonance from the formula

$$Q = f_1/\Delta f\,, \tag{6}$$

where Δf is the bandwidth at a level of 3 dB and f_1 is the resonant frequency.

Now we derive the formulas for the external Q-factor of the half-wavelength dielectric layer placed in the middle of the layered structure formed by quarter-wavelength dielectric layers. We assume that the external layers have the refractive indices n_{H}, and the refractive indices of all internal layers alternately take values n_{L} and n_{H}, where $n_{\mathrm{L}} < n_{\mathrm{H}}$.

The external Q-factor of any oscillatory system without intrinsic loss is related to the complex frequency of free oscillations by the formula [10]

$$Q_e = -\operatorname{Re}\omega/(2\operatorname{Im}\omega)\,. \tag{7}$$

Since the electric thickness θ of dielectric layers is proportional to the frequency ω, formula (7) can be written as follows:

$$Q_e = -\operatorname{Re}\theta/(2\operatorname{Im}\theta)\,. \tag{8}$$

The electric thickness of the quarter-wavelength dielectric layer at frequency f_1 is $\operatorname{Re}\theta = \pi/2$.

To simplify calculations, we now consider the presence of the symmetry plane of the layered structure. We place the origin of coordinate z in the centre of the resonant half-wavelength layer. Then it is sufficient to consider the field strength distribution only for $z > 0$. The half-wavelength layer is equivalent to the section of the transmission line close-circuited at both ends, when its refractive index (n_R) is much lower than the refractive indices of the surrounding layers (n_H); otherwise (for $n_R >> n_L$), it is equivalent to the open-circuited section of the transmission line. We note that the electric field strength $E(z)$ in the half-wavelength layer is distributed by a cosine law if $n_R = n_L$ and by a sine law if $n_R = n_H$ irrespective of the contrast degree of the refractive indices.

Thus, the distribution functions for the three-layer structure ($n_R = n_L$) have the form

$$E_x(z) = \begin{cases} A_1 \cos(k_1 z) & \text{for } 0 < z < z_1, \\ A_2 e^{ik_2(z-z_1)} + A_3 e^{-ik_2(z-z_1)} & \text{for } z_1 < z < z_2, \\ A_4 e^{ik_0(z-z_2)} & \text{for } z_2 < z < \infty, \end{cases}$$

$$Z_0 H_y(z) = \begin{cases} i n_L A_1 \sin(k_1 z) & \text{for } 0 < z < z_1, \\ n_H A_2 \, e^{ik_2(z-z_1)} - n_H A_3 \, e^{-ik_2(z-z_1)} & \text{for } z_1 < z < z_2, \\ A_4 e^{ik_0(z-z_2)} & \text{for } z_2 < z < \infty, \end{cases} \tag{9}$$

where z_1 and z_2 are the coordinates of the layer boundaries, A_m (m = 1, 2, 3, and 4) are unknown wave amplitudes, $k_1 = \omega n_L/c$, $k_2 = \omega n_H/c$, and $k_0 = \omega/c$.

According to the electrodynamic boundary conditions, functions (9) should be continuous at points z_1 and z_2. From here we obtain the system of equations

$$\begin{aligned} \cos\theta A_1 - A_2 - A_3 &= 0, \\ i n_L \sin\theta A_1 - n_H A_2 + n_H A_3 &= 0, \\ e^{i\theta} A_2 + e^{-i\theta} A_3 - A_4 &= 0, \\ n_H \, e^{i\theta} \, A_2 - n_H \, e^{-i\theta} \, A_3 - A_4 &= 0. \end{aligned} \tag{10}$$

A homogeneous system has a nontrivial solution if its determinant is equal to zero. From here we obtain the equation for the frequency of free oscillations:

$$n_L \tan^2\theta + i n_H (n_H + n_L) \tan\theta - n_H = 0 . \tag{11}$$

It has the following solution:

$$\theta = \frac{\pi}{2} - i \operatorname{arc\,coth}\left(\frac{n_H + n_L}{2n_L} n_H + \sqrt{\left(\frac{n_H + n_L}{2n_L} n_H \right)^2 - \frac{n_H}{n_L}} \right). \tag{12}$$

After substitution of Eq. (12) into Eq. (8), we obtain the external Q-factor of the half-wavelength resonator in the three-layer structure [11]

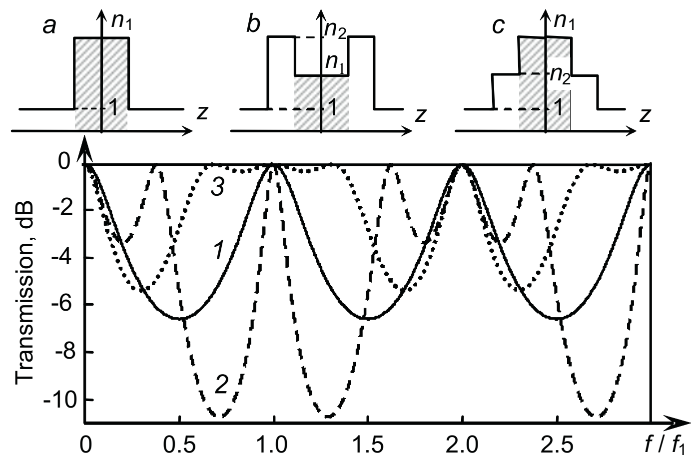

Fig. 1. Frequency dependences of the light transmission through the single half-wavelength resonator (*a*), the resonator with quarter-wavelength layers for $n_2 > n_1$ (*b*), and the resonator with quarter-wavelength layers for $n_2 < n_1$ (*c*) (the inserts show the refractive index profiles; the half-wavelength layers with n_1 are hatched).

$$Q_{e3} = \frac{\pi}{4\,\mathrm{arccoth}\left(\frac{n_\mathrm{H} + n_\mathrm{L}}{2n_\mathrm{L}} n_\mathrm{H} + \sqrt{\left(\frac{n_\mathrm{H} + n_\mathrm{L}}{2n_\mathrm{L}} n_\mathrm{H}\right)^2 - \frac{n_\mathrm{H}}{n_\mathrm{L}}}\right)} . \tag{13}$$

Addition of one more quarter-wavelength layer from the right and from the left of the half-wavelength resonator results in a significant increase of its Q-factor. In this case, the refractive index of the half-wavelength layer should be higher than that of the adjacent quarter-wavelength layer.

The Q-factor of the half-wavelength resonator in the five-layer structure can be calculated from the formula [11]

$$Q_{e5} = \frac{\pi}{4\,\mathrm{arccoth}\, x} , \tag{14}$$

where x is the real root of the cubic equation

$$n_\mathrm{L}^2 x^3 - n_\mathrm{H}\left(n_H^2 + n_\mathrm{L} n_\mathrm{H} + n_\mathrm{L}^2\right) x^2 + n_\mathrm{H}\left(n_\mathrm{L} + n_\mathrm{H} + n_\mathrm{L}\right) x - n_\mathrm{H}^2 n_\mathrm{L} = 0 . \tag{15}$$

These formulas were derived by analogy with formula (13).

Figure 1 shows the frequency dependences of the coefficient of light transmission through the examined structures with the half-wavelength resonator. All dependences have the period f_1. Here solid curve *1* is for the resonator comprising only single half-wavelength layer with the refractive index $n_1 = 4$. As is well known [12], the n_1 values in the optical range are close to the maximum values of the materials. Such single half-wavelength layer even for so high refractive index has relatively low external Q-factor. At the frequency f_1, it is equal only to $Q_e \approx 2.8$, which is testified by wide passband and considerable transmitted power in stopbands. Dashed curve *2* in Fig. 1 shows light transmission through the three-layer structure in which the half-wavelength layer with refractive index $n_1 = 2$ is surrounded by quarter-wavelength layers with $n_2 = 4$. In this case, the Q-factor increased almost 3 times to $Q_e \approx 8.3$. Dotted curve *3* is for the resonator with bleaching quarter-wavelength layers ($n_1 = 4$ and $n_2 = 2$).

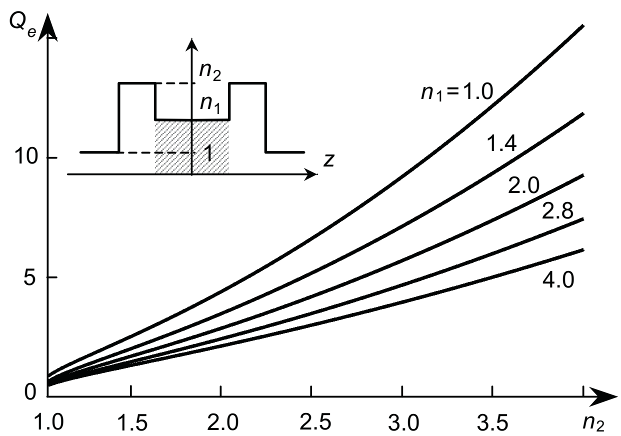

Fig. 2

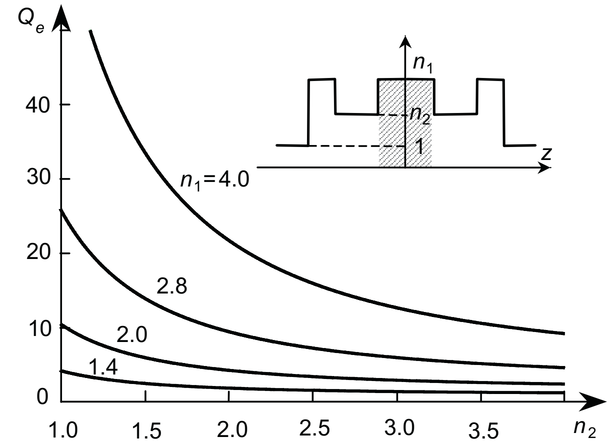

Fig. 3

Fig. 2. Dependences of the Q-factor of the half-wavelength resonator in the three-layer structure on the refractive index of the quarter-wavelength layers for the indicated values of n_1.

Fig. 3. Dependences of the Q-factor of the half-wavelength resonator in the five-layer structure on the refractive index of the quarter-wavelength layer for the indicated values of n_1.

Figure 2 shows dependences of the Q-factor of the half-wavelength resonator in three-layer structure on the refractive index of quarter-wavelength layers [11]. The curves were drawn using formula (13) for the indicated values of the refractive index of the half-wavelength layer. It can be seen that the Q-factor of the resonator rapidly increases with increasing n_2 and decreasing n_1. We note that the Q-factor of the air resonator increases for the three-layer structure more than 5 times in comparison with the single layer half-wavelength resonator.

Figure 3 shows analogous dependences for the five-layer structure calculated using formula (14). It can be seen that the Q-factor in this case, on the contrary, rapidly decreases when n_2 increases or n_1 decreases. However, we note that in comparison with the single layer half-wavelength resonator with $n_1 = 4$, the Q-factor for the five-layer structure increases more than 20 times.

Formulas (1)–(6) can be used to calculate the Q-factor of the half-wavelength layer for any arbitrary number N of quarter-wavelength layers surrounding it with alternating high and low optical densities. These dependences are shown for the indicated refractive indices in Fig. 4. They demonstrate almost exponential increase in the Q-factor of the resonator with increasing number of quarter-wavelength layers surrounding it.

Thus, the coupling of the half-wavelength resonant layer with space can be regulated via the contrast and the number of quarter-wavelength dielectric layers strongly reflecting electromagnetic waves at resonant frequencies of the half-wavelength layer. As is well known, the number of resonators m in the passband filter determines the order of the filter and hence its selective properties. It is obvious that the coupling between the filter resonators can also be regulated via the quarter-wavelength layers. Figure 5 shows the frequency response that demonstrates light transmission through the 7$^{\text{th}}$-order filter with fractional bandwidth of 10%. It can be seen that the frequency response is the periodic function with the period $2f_1$. The structure of layers in the filter is symmetric about the central (IV) half-wavelength layer (in the inset of Fig. 5, the dependence of the refractive index $n(z)$ is shown for the left half of the structure). To minimize the losses in the passband filter, it is important that the resonant half-wavelength layers had minimal dielectric loss; therefore, all seven resonators in the filter (I, II, III, IV, ...) forming the passband are air ones ($n = 1$). The neighboring resonators of the filter are separated from each other by three quarter-wavelength layers, and the end resonators are separated from free space by single quarter-wavelength layer. As a result, the filter comprises 27 dielectric layers.

It is important to note that not all quarter-wavelength layers of the filter shown in Fig. 5 have identical refractive indices. In addition to two values $n = 1$ and 2.86, they also take values $n = 1.06$, 1.35, and 2.45. This means

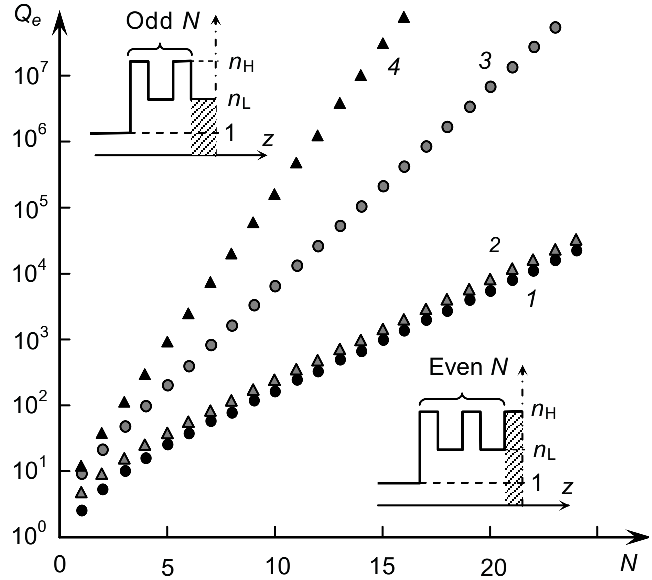

Fig. 4. Dependences of the Q-factor of the half-wavelength resonator on the number of quarter-wavelength layers surrounding it. Here curve *1* is for $n_H = 4$ and $n_L = 2$, curve *2* is for $n_H = 8$ and $n_L = 4$, curve *3* is for $n_H = 16$ and $n_L = 4$, and curve *4* is for $n_H = 16$ and $n_L = 2$.

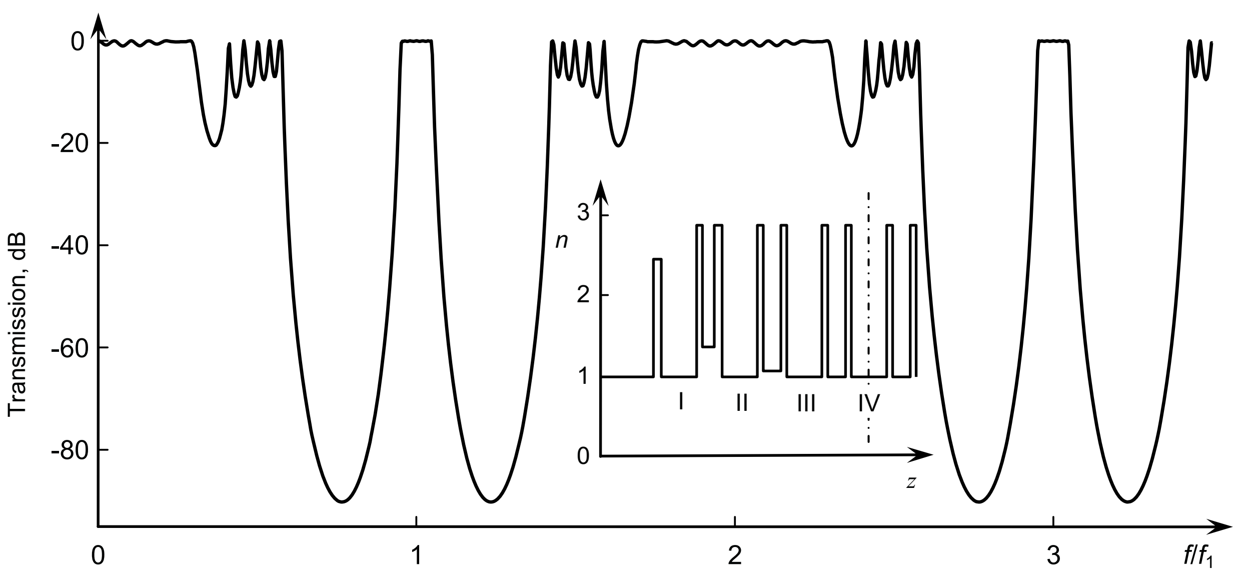

Fig. 5. Calculated frequency response of the 7th-order synthesized filter (in the inset, the refractive index profile is shown for the left half of the filter).

that the photonic crystal structure of the examined optical filter is not strictly periodic. It differs significantly by this feature from all well-known optical filters [1, 13]. A disadvantage of periodic photonic crystal structures is high unequalized transmission ripples in the filter passband [14] exceeding 3 dB, that is, 50% (Fig. 6). In the examined matched optical filter these ripples are insignificant (about 3%), and if necessary, can be made even lower. In microwave filters, these ripples are easily eliminated by increasing the resonator couplings toward the filter end [6]. The same is true when the examined optical filter is matched.

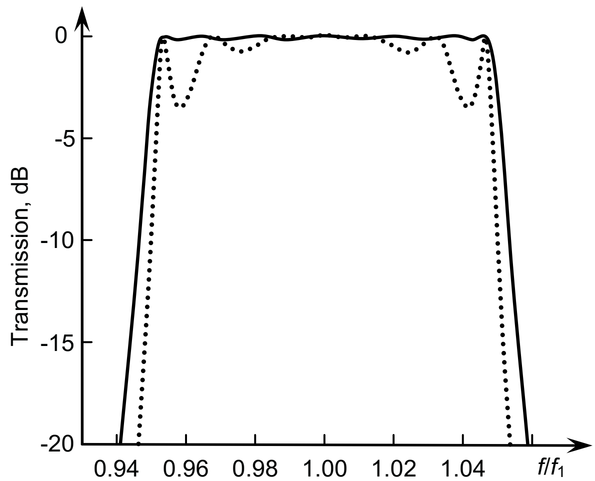

Fig. 6

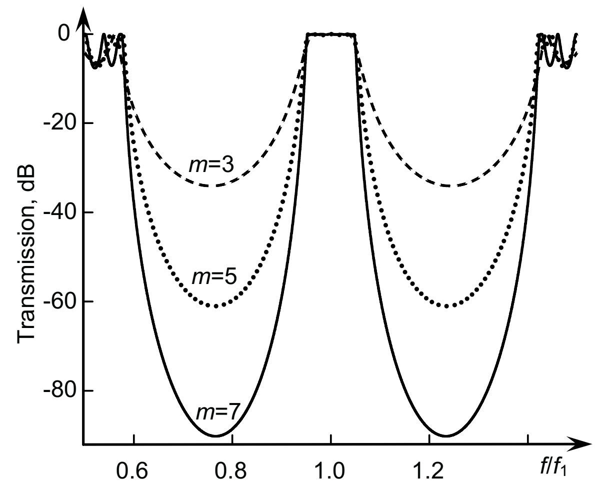

Fig. 7

Fig. 6. Frequency response of the matched filter (the solid curve) and of the periodic photonic crystal structure (dotted curve).

Fig. 7. Frequency response of the 3rd-, 5th-, and 7th-order filters with fractional bandwidth of 10%.

For a comparison, Fig. 7 shows the frequency dependences of light transmission through 3rd-, 5th-, and 7th-order filters. It can be seen that with increasing order of the filter, that is, with increasing number of half-wavelength layers, the depth of the stopband increases together with the slope steepness of its frequency response. The slope steepness of the microwave filters is characterized by its low-slope coefficient K_L and upper-slope coefficient K_H [15]. In the photonic crystal structures, the passband slopes are symmetric; therefore, they coincide. For symmetric slopes, they are defined by the simple formula

$$K_S = \frac{\Delta f_{-3\text{ dB}}}{\Delta f_{-30\text{ dB}} - \Delta f_{-3\text{ dB}}}, \tag{16}$$

where $\Delta f_{-3\text{ dB}}$ and $\Delta f_{-30\text{ dB}}$ are the passband widths at levels of –3 dB and –30 dB, respectively.

We note that proportional weakening of all resonator couplings leads to proportional narrowing of the filter bandwidth [6]. It is obvious that to weaken the couplings in the examined structure of the optical filter, not only a greater number of quarter-wavelength layers separating the half-wavelength resonators, but also a greater number of the quarter-wavelength layers separating the end resonators from free space are required. We synthesized and investigated narrowband filters with fractional bandwidth of 1.45% comprising from 2 to 7 resonators. For their implementation, three quarter-wavelength layers rather than one were placed between the end resonator and space, and five separating quarter-wavelength layers with the refractive index of the material $n \le 4$ rather than three were placed between other resonators. In this case, the 7th-order filter comprised already 43 layers.

Figure 8 shows the dependences of the slope steepness on the order of the examined filters with fractional bandwidth of 1.45 and 10%. It can be seen that the slope steepness increases with increasing order of the filter slightly faster than a linear function. The slope steepness for the narrowband filters is slightly higher than for broadband filters.

Thus, the electrodynamic analysis of the optical resonator representing the half-wavelength dielectric layer with low or high refractive index bounded from each end by quarter-wavelength layers with alternating high and low refractive indices has been carried out. The formulas for calculation of the external Q-factor of such resonator in free

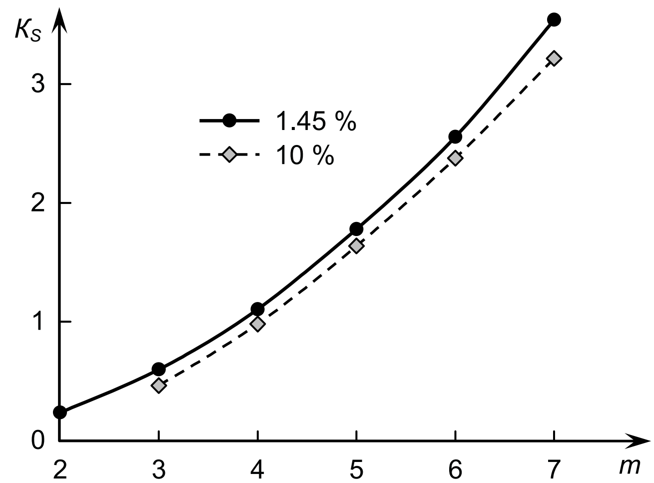

Fig. 8. Dependences of the slope steepness of filters with fractional bandwidth of 1.45 and 10% on the number of resonators comprised in them.

space were derived. The dependences of the Q-factor on the number of dielectric layers in the structure and on their optical properties were investigated. The suggested approach allows the number of the quarter-wavelength dielectric layers covering the dielectric resonator to be determined to obtain the required Q-factor.

The photonic crystal filters from the 2^{nd}- to 7^{th}-order with fractional bandwidth of 10 and 1.45% have been synthesized. Their selective properties were investigated. For the first time it was demonstrated that the photonic crystal structure of the matched optical filter should not be strictly periodic. Otherwise, the transmission ripples in the passband could exceed 50%. To eliminate these ripples, the contrast of the refractive indices of the neighboring quarter-wavelength layers must be decreased from the center of the structure toward its ends. The examined designs of the filters can be fabricated by electron beam lithography and etching with inductively coupled plasma [16].

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