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FEATURES OF THE ELLIPSOMETRIC INVESTIGATION OF MAGNETIC NANOSTRUCTURES

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The technique for interpreting magneto-ellipsometric measurements is proposed. The model of a homogeneous semi-infinite medium for reflecting layered magnetic structures in the presence of the magnetic field in the configuration of the magneto-optical equatorial Kerr effect is considered. Based on the analysis of the Fresnel coefficients with regard to the magneto-optical parameter Q appearing in the off-diagonal elements of the permittivity tensor, the expressions are obtained using which the refraction (n) and absorption (k) coefficients, the real (Q_1) and imaginary (Q_2) parts of the magneto-optical parameter can be found from the ellipsometric (ψ_0 and Δ_0) and magneto-ellipsometric ($\psi_0 + \delta\psi$ and $\Delta_0 + \delta\Delta$) measurements. The results will allow to measure and analyze the magnetic characteristics such as hysteresis loops and the coercitive force of layered nanostructures using the conventional ellipsometric equipment.

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INTRODUCTION

At present, the ellipsometric method is been developing for its application to the investigation of magnetic materials. This seems to be very promising for the complete determination of all elements of the permittivity tensor of the material, in which the diagonal elements are responsible for the conventional refraction and absorption parameters and the off-diagonal elements are associated with magneto-optical effects.

The permittivity tensor of magnetized ferromagnetic metal is based on the forced anisotropy and looks as follows [1]:

$$[\varepsilon] = \begin{bmatrix} \varepsilon & -i\varepsilon Q & 0 \\ i\varepsilon Q & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}, \tag{1}$$

where $\varepsilon = \varepsilon_1 - i\varepsilon_2$ is the complex dielectric permittivity of the medium; ε_1 is the real part of the dielectric permittivity of the medium; $\varepsilon_2 = 4\pi\sigma/\omega$ is the imaginary part of the dielectric permittivity of the medium; Q is the magneto-optical parameter

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depending on the object magnetization; σ is the specific electric conductivity; ω is the circular frequency. With zero magnetization meaning that the magneto-optical parameter Q is zero, the off-diagonal elements of the permittivity tensor become zero.

In order to reveal the relationship between classical ellipsometry and magnetooptics, a series of studies and calculations are required to develop a mathematical apparatus allowing us to interpret experimental ellipsometric and magneto-ellipsometric measurements of layered magnetic nanostructures and examine their optical and magnetic properties within one experiment.

The ellipsometric methods for measuring the parameters of ferromagnetics and magnetic films based on the equatorial Kerr effect (the experiments are often carried out in the configuration of the magneto-optical equatorial Kerr effect due to the design features of a high-vacuum chamber and electromagnets used for sample remagnetization) were previously considered in [2, 3]. However, these articles do not contain the information of interest on the reliable magneto-ellipsometric technique.

In [4], we reported the analytical expressions for the Fresnel coefficients with regard to the magneto-optical parameter Q appearing in the off-diagonal elements of the permittivity tensor. Different models of the reflecting optical systems in the presence of the magnetic field in the configuration of the magneto-optical equatorial Kerr effect were considered making possible to interpret the experimental data in the ellipsometric and magneto-ellipsometric studies of layered magnetic nanostructures. We got the relationship between the ellipsometric parameters ψ and Δ and the magnetization-proportional magneto-optical parameter Q; for the first time, we assessed $\delta\psi$ and $\delta\Delta$ corrections for ellipsometric angles associated with the surface equatorial magneto-optical Kerr effect.

A new research subject is a dependence of the ellipsometric parameters ψ and Δ on the density of the applied magnetic field. Using the magneto-optical parameter Q, it would be possible to pass from this dependence to the magnetization dependence on the density of the applied magnetic field. The solution of this problem will allow the measurement and analysis of magnetic characteristics such as hysteresis loops and the coercitive force of layered nanostructures using the conventional ellipsometric equipment.

This work deals with the interpretation of the experimental ellipsometric (ψ_0 and Δ_0) and magneto-ellipsometric ($\psi_0 + \delta\psi$ and $\Delta_0 + \delta\Delta$) measurements by means of the model of a homogeneous semi-infinite medium for reflecting magnetic nanostructures in the presence of the magnetic field in the configuration of the magneto-optical equatorial Kerr effect, namely, to find the refraction (n) and absorption (k) coefficients, the real (Q_1) and imaginary (Q_2) parts of the magneto-optical parameter from the analysis of the Fresnel coefficients taking into account the magneto-optical parameter Q.

RELATIONSHIP BETWEEN THE ELLIPSOMETRIC AND MAGNETO-OPTICAL MEASUREMENTS

The method of classical ellipsometry is based on the study of a change in the light polarization mode after its interaction with the interface surface. The experiment measures the ellipsometric parameters ψ and Δ which are used to calculate the complex ellipsometric parameter ρ [5] equal to the ratio of complex coefficients of reflection or transmission for two types of light wave polarization: in the incidence plane (index p) and perpendicularly to it (index s). In the case of magneto-ellipsometric measurements, a change in the magnetization of the studied structure leads to changes in the ellipsometric angles owing to the contribution of the equatorial magneto-optical effect in the polarization state [6]. In this work, we consider the visible optical band because at present most ellipsometers work exactly at these frequencies.

Let us denote the ellipsometric angles corresponding to the absence of magnetization ($Q = 0$) by ψ_0, Δ_0. With non-zero magnetization, the magneto-optical parameter $Q = Q_1 - iQ_2$ differs from zero, and some changes in the ellipsometric parameters $\delta\psi$ and $\delta\Delta$ occur, so the ellipsometric angles are $\psi_0 + \delta\psi$, $\Delta_0 + \delta\Delta$ respectively. Let us write down the basic ellipsometry equation and explicitly write the real and imaginary parts of the complex refraction coefficients [4]

$$\mathrm{tg}\psi\exp(i\Delta)=\frac{R_p(Q)}{R_s}=\frac{R'_p-iR''_p}{R'_s-iR''_s},\tag{2}$$

where the imaginary parts of the numerator and the denominator are denoted as R''_p and R''_s, and the real as R'_p and R'_s. Therefore, we have

$$\begin{cases}\mathrm{tg}\psi=\dfrac{\sqrt{(R'_pR'_s+R''_pR''_s)^2+(R''_sR'_p-R''_pR'_s)^2}}{R'^2_s+R''^2_s}\\ \Delta=\mathrm{arctg}\dfrac{R''_sR'_p-R''_pR'_s}{R'_pR'_s+R''_pR''_s}\end{cases}.\tag{3}$$

Let us separate the contribution of the magnetic field and denote it as R''_{p1} and R''_{s1} at the imaginary parts of the refraction coefficients, as R'_{p1} and R'_{s1} at the real parts, and denote the non-magnetic addends as R'_{p0}, R'_{s0}, R''_{p0}, and R''_{s0}. As shown in [4], for the equatorial Kerr effect the magnetization does not affect the intensity of the reflected *s*-component of light, i.e., $R''_{s1}=0$, $R'_{s1}=0$; the magnetic field affects the refraction coefficients only for *p*-polarized component of a light wave

$$R''_s=R''_{s0}+R''_{s1}=R''_{s0},\tag{4}$$

$$R''_p=R''_{p0}+R''_{p1},\tag{5}$$

$$R'_s=R'_{s0}+R'_{s1}=R'_{s0},\tag{6}$$

$$R'_p=R'_{p0}+R'_{p1}.\tag{7}$$

By substituting (4)-(7) in (3) we get that in the absence of magnetism

$$\rho_0=\mathrm{tg}\psi_0\exp(i\Delta_0)=\frac{R_{p0}}{R_{s0}},\tag{8}$$

$$\mathrm{tg}\psi_0=\sqrt{\frac{(R'_{p0}R'_{s0}+R''_{s0}R''_{p0})^2+(R''_{s0}R'_{p0}-R''_{p0}R'_{s0})^2}{R'^2_{s0}+R''^2_{s0}}},\tag{9}$$

$$\Delta_0=\mathrm{arctg}\frac{R''_{s0}R'_{p0}-R''_{p0}R'_{s0}}{R'_{p0}R'_{s0}+R''_{p0}R''_{s0}},\tag{10}$$

and in the presence of magnetism

$$tg(\psi_0+\delta\psi)=tg(\psi_0)\times$$
$$\times\sqrt{1+\frac{(R''_{s0}R''_{p1})^2+(R''_{p1}R'_{s0})^2+2R''_{p0}R''_{p1}(R'^2_{s0}+R''^2_{s0})}{(R'_{p0}R'_{s0}+R''_{p0}R''_{s0})^2+(R''_{s0}R'_{p0}-R''_{p0}R'_{s0})^2}+\frac{(R'_{p1}R'_{s0})^2+(R'_{p1}R''_{s0})^2+2R'_{p0}R'_{p1}(R'^2_{s0}+R''^2_{s0})}{(R'_{p0}R'_{s0}+R''_{p0}R''_{s0})^2+(R''_{s0}R'_{p0}-R''_{p0}R'_{s0})^2}}=$$
$$=F\,\mathrm{tg}(\psi_0),\tag{11}$$

where a new designation F is introduced for the multiplier following $\mathrm{tg}(\psi_0)$.

Given (3) and (8)-(11), the changes in the ellipsometric parameters $\delta\psi$ and $\delta\Delta$ at remagnetization are

$$\delta\psi=\psi-\psi_0=\mathrm{arctg}(F\,\mathrm{tg}(\psi_0))-\psi_0,\tag{12}$$

$$\delta\Delta=\Delta-\Delta_0=\mathrm{arctg}\frac{R''_{s0}(R'_{p0}+R'_{p1})-(R''_{p0}+R''_{p1})R'_{s0}}{(R'_{p0}+R'_{p1})R'_{s0}+(R''_{p0}+R''_{p1})R''_{s0}}-\mathrm{arctg}\frac{R''_{s0}R'_{p0}-R''_{p0}R'_{s0}}{R'_{p0}R'_{s0}+R''_{p0}R''_{s0}}.\tag{13}$$

For the case of an insignificant contribution of magnetism, we can introduce two small parameters $\alpha=R''_{p1}/R''_{p0}$ and $\beta=R'_{p1}/R'_{p0}$ and expand the expressions for the changes in the ellipsometric parameters $\delta\psi$ and $\delta\Delta$ in a series in them

$$\delta\psi = \frac{\mathrm{tg}\psi_0 \; R''^{\,2}_{p0}}{(R'^{\,2}_{p0} + R''^{\,2}_{p0})(1+\mathrm{tg}^2\psi_0)}\alpha + \frac{\mathrm{tg}\psi_0 \; R'^{\,2}_{p0}}{(R'^{\,2}_{p0} + R''^{\,2}_{p0})(1+\mathrm{tg}^2\psi_0)}\beta +$$

$$+\frac{\alpha^2}{2}\frac{\mathrm{tg}\psi_0 \, R''^{\,2}_{p0}(R'^{\,2}_{p0} + \mathrm{tg}^2\psi_0(R'^{\,2}_{p0} - 2R''^{\,2}_{p0})}{(R'^{\,2}_{p} + R''^{\,2}_{p0})^2(1+\mathrm{tg}^2\psi_0)^2} +$$

$$+\frac{\beta^2}{2}\frac{\mathrm{tg}\psi_0 \, R'^{\,2}_{p0}(R''^{\,2}_{p0} + \mathrm{tg}^2\psi_0(R''^{\,2}_{p0} - 2R'^{\,2}_{p0})}{(R'^{\,2}_{p} + R''^{\,2}_{p0})^2(1+\mathrm{tg}^2\psi_0)^2} +$$

$$+\alpha\beta\frac{\mathrm{tg}\psi_0 \; R''^{\,2}_{p0} \, R'^{\,2}_{p0} \, (1+3\;\mathrm{tg}^2\psi_0)}{(R'^{\,2}_{p} + R''^{\,2}_{p0})^2(1+\mathrm{tg}^2\psi_0)^2}, \tag{14}$$

$$\delta\Delta = -\alpha\frac{R'_{p0}R''_{p0}}{R'^{\,2}_{p0} + R''^{\,2}_{p0}} + \beta\frac{R'_{p0}R''_{p0}}{R'^{\,2}_{p0} + R''^{\,2}_{p0}} + \alpha^2\frac{R'_{p0}R''^{\,3}_{p0}}{(R'^{\,2}_{p0} + R''^{\,2}_{p0})^2} -$$

$$-\beta^2\frac{R''_{p0}R'^{\,3}_{p0}}{(R'^{\,2}_{p0} + R''^{\,2}_{p0})^2} + \alpha\beta\frac{R''_{p0}R'_{p0}(R'^{\,2}_{p0} - R''^{\,2}_{p0})}{(R'^{\,2}_{p0} + R''^{\,2}_{p0})^2}. \tag{15}$$

We hold the quadratic contributions to be able to describe the second order magneto-optical effects.

Thus, we got all formulas needed for the subsequent calculations that can be divided into three stages:

1. The determination of the refraction n and absorption k coefficients from equation (8).
2. The determination of R''_{p0}, R''_{s0}, R'_{s0}, and R'_{p0} using n and k found at the first stage.
3. By expressing R''_{p1} and R'_{p1} from (14) and (15), to find the dependence on $\delta\psi$ and $\delta\Delta$ of the real Q_1 and imaginary Q_2 parts of the magneto-optical parameter Q determining the off-diagonal elements of the permittivity tensor.

DETERMINATION OF THE REFRACTION *n* AND ABSORPTION *k* COEFFICIENTS

For the semi-infinite medium model, the complex refraction coefficients of the medium ($N_0 = n_0 - ik_0$) and the material under study ($N = n - ik$) are related as follows [7]:

$$N = N_0 \sin\phi_0 \sqrt{1 + tg^2\phi_0 \left(\frac{1-\rho_0}{1+\rho_0}\right)^2}. \tag{16}$$

It is necessary to find the n and k coefficients characterizing the studied ferromagnetic; i.e., to separate the real and imaginary parts in (16) in the explicit form. To this end, we turn to the algebraic form of the complex number notation. Let us use equation (8)

$$\rho_0 = \mathrm{tg}\psi_0 \exp(i\Delta_0) = \frac{\mathrm{tg}\psi_0 + i\,\mathrm{tg}\psi_0\mathrm{tg}\Delta_0}{\sqrt{1+\mathrm{tg}^2\Delta_0}}. \tag{17}$$

Accordingly, we convert the radical expression in (16)

$$\sqrt{1 + \mathrm{tg}^2\varphi_0 \left(\frac{1-\rho_0}{1+\rho_0}\right)^2} =$$

$$= \frac{1}{(\sqrt{1+\mathrm{tg}^2\Delta_0} + \mathrm{tg}\psi_0)^2 + (\mathrm{tg}\psi_0 tg\Delta_0)^2}\Big(((\sqrt{1+\mathrm{tg}^2\Delta_0} + \mathrm{tg}\psi_0)^2 + (\mathrm{tg}\psi_0\, tg\Delta_0)^2)^2 +$$

$$+\mathrm{tg}^2\varphi_1\Big((1+\mathrm{tg}^2\Delta_0 - \mathrm{tg}^2\psi_0 - \mathrm{tg}^2\Delta_0\,\mathrm{tg}^2\psi_0)^2 - 4\,\mathrm{tg}^2\Delta_0\,\mathrm{tg}^2\psi_0(1+\mathrm{tg}^2\Delta_0)\Big) -$$

$$-4i\,\mathrm{tg}^2\varphi_1\mathrm{tg}\psi_0\mathrm{tg}\Delta_0(1+\mathrm{tg}^2\Delta_0 - \mathrm{tg}^2\psi_0 - \mathrm{tg}^2\Delta_0\mathrm{tg}^2\psi_0)\sqrt{1+\mathrm{tg}^2\Delta_0}\Big)^{1/2}. \tag{18}$$

As is seen, in (18) the expression in the 1/2 power is a complex number. Let us denote this expression by letter $z = a + ib$, i.e., write down that

$$\sqrt{1+tg^2\phi_0\left(\frac{1-\rho_0}{1+\rho_0}\right)^2} = \frac{\sqrt{z}}{(\sqrt{1+tg^2\Delta_0}+tg\psi_0)^2+(tg\psi_0 tg\Delta_0)^2} \tag{19}$$

and take root from z according to the rule $\mathrm{Re}(z) = a^2 - b^2$, $\mathrm{Im}(z) = 2ab$.

After changing the variables $\xi = \mathrm{tg}\psi_0$, $\eta = \mathrm{tg}\Delta_0$ and $\varsigma = (1 + \eta^2)^1/^2$ we have the following set of equations:

$$\begin{cases} a^2 - b^2 = (\varsigma^2(1+\xi^2)+2\xi\varsigma)^2 + tg^2\phi_0(\varsigma^4(1-\xi^2)^2 - 4\xi^2\varsigma^2(\varsigma^2-1)) \\ ab = -2tg^2\phi_0\xi\varsigma^3(1-\xi^2)\sqrt{\varsigma^2-1} \end{cases}. \tag{20}$$

Let us denote the first part of the first equation in (20) by the symbol k, and the right part of the second equation by the symbol m

$$\begin{cases} a^2 - b^2 = k \\ ab = m \end{cases}. \tag{21}$$

We find b from the second equation and insert it in the first equation, which, as a result, becomes biquadratic

$$a^4 - ka^2 - m^2 = 0. \tag{22}$$

Thus, we get that

$$a_{1,2,3,4} = \pm\sqrt{\frac{k\pm\sqrt{k^2+4m^2}}{2}}, \tag{23}$$

$$b_{1,2,3,4} = \frac{-2tg^2\phi_0\xi\varsigma^3(1-\xi^2)\sqrt{\varsigma^2-1}}{\pm\sqrt{\frac{k\pm\sqrt{k^2+4m^2}}{2}}}. \tag{24}$$

Therefore, the refraction coefficient of the studied ferromagnetic metal and its absorption coefficient are

$$n = \frac{\sin\phi_0(n_0a+k_0b)}{(\sqrt{1+tg^2\Delta_0}+tg\psi_0)^2+(tg\psi_0 tg\Delta_0)^2}, \tag{25}$$

$$k = \frac{\sin\phi_0(k_0a-n_0b)}{(\sqrt{1+tg^2\Delta_0}+tg\psi_0)^2+(tg\psi_0 tg\Delta_0)^2}. \tag{26}$$

Mathematically, we get four pairs of the refraction n and absorption k coefficients, however, there are physical restrictions. Finally, we should obtain real numbers, so, a and b should also be real, consequently, a^2 and b^2 should be positive real numbers. Therefore, two pairs of possible n and k values remain, for which it is true that

$$a^2 = \frac{k+\sqrt{k^2+4m^2}}{2}, \tag{27}$$

$$a_{1,2} = \pm\sqrt{\frac{k+\sqrt{k^2+4m^2}}{2}}, \tag{28}$$

$$b_{1,2} = \frac{-2tg^2\phi_0\xi\varsigma^3(1-\xi^2)\sqrt{\varsigma^2-1}}{\pm\sqrt{\frac{k+\sqrt{k^2+4m^2}}{2}}} \tag{29}$$

and the refraction coefficient n can be negative, while the absorption coefficient k cannot be negative.

DETERMINATION OF THE R''_{p0}, R''_{s0}, R'_{s0}, AND R'_{p0} VALUES

The analytical expressions for the Fresnel coefficients with regard to the magneto-optical parameter appearing in the off-diagonal elements of the permittivity tensor, were reported in [4]. Thus, it was shown that for the semi-infinite medium model, the following expressions for the Fresnel coefficients hold:

$$R_p = \frac{N\cos\varphi_0 - N_0\cos\varphi_1}{N\cos\varphi_0 + N_0\cos\varphi_1} - i\frac{2QN_0^2\sin\varphi_0\cos\varphi_0}{(N\cos\varphi_0 + N_0\cos\varphi_1)^2}, \tag{30}$$

$$R_S = \frac{N_0\cos\varphi_0 - N\cos\varphi_1}{N_0\cos\varphi_0 + N\cos\varphi_1}. \tag{31}$$

It is necessary to realize that given expressions (2)-(7), expressions (30) and (31) can be written as

$$R_p = R'_{p0} + R'_{p1} - i(R''_{p0} + R''_{p1}), \tag{32}$$

$$R_S = R_{S0} = R'_{S0} - iR''_{S0}. \tag{33}$$

Remembering that $N_0 = n_0 - ik_0$, $N = n - ik$, $Q = Q_1 - iQ_2$, let us compare expressions (30) and (31) with (32) and (33), thereby getting the expressions for R''_{p0}, R''_{s0}, R'_{s0}, and R'_{p0} and for R''_{p1} and R'_{p1}.

$$R'_{p0} = \frac{n^2\cos^2\varphi_0 - n_0^2\cos^2\varphi_1 + k^2\cos^2\varphi_0 - k_0^2\cos^2\varphi_1}{(n\cos\varphi_0 + n_0\cos\varphi_1)^2 + (k\cos\varphi_0 + k_0\cos\varphi_1)^2}, \tag{34}$$

$$R''_{p0} = \frac{2\cos\varphi_0\cos\varphi_1(n_0k - nk_0)}{(n\cos\varphi_0 + n_0\cos\varphi_1)^2 + (k\cos\varphi_0 + k_0\cos\varphi_1)^2}, \tag{35}$$

$$\begin{aligned} R'_{p1} = {} & \frac{2\sin\varphi_0\cos\varphi_0\left\{(2Q_1(n_0^2 - k_0^2) - 4n_0k_0Q_2)(n\cos\varphi_0 + n_0\cos\varphi_1)(k\cos\varphi_0 + k_0\cos\varphi_1) - \right.}{((n\cos\varphi_0 + n_0\cos\varphi_1)^2 + (k\cos\varphi_0 + k_0\cos\varphi_1)^2)^2} \\ & \frac{\left. -(Q_2(n_0^2 - k_0^2) + 2n_0k_0Q_1)((n\cos\varphi_0 + n_0\cos\varphi_1)^2 - (k\cos\varphi_0 + k_0\cos\varphi_1)^2)\right\}}{((n\cos\varphi_0 + n_0\cos\varphi_1)^2 + (k\cos\varphi_0 + k_0\cos\varphi_1)^2)^2}, \end{aligned} \tag{36}$$

$$\begin{aligned} R''_{p1} = {} & \frac{2\sin\varphi_0\cos\varphi_0\left\{(2Q_2(n_0^2 - k_0^2) + 4n_0k_0Q_1)(n\cos\varphi_0 + n_0\cos\varphi_1)(k\cos\varphi_0 + k_0\cos\varphi_1) + \right.}{((n\cos\varphi_0 + n_0\cos\varphi_1)^2 + (k\cos\varphi_0 + k_0\cos\varphi_1)^2)^2} \\ & \frac{\left. +(Q_1(n_0^2 - k_0^2) - 2n_0k_0Q_2)((n\cos\varphi_0 + n_0\cos\varphi_1)^2 - (k\cos\varphi_0 + k_0\cos\varphi_1)^2)\right\}}{((n\cos\varphi_0 + n_0\cos\varphi_1)^2 + (k\cos\varphi_0 + k_0\cos\varphi_1)^2)^2}, \end{aligned} \tag{37}$$

$$R'_{S0} = \frac{n_0^2\cos^2\varphi_0 - n^2\cos^2\varphi_1 + k_0^2\cos^2\varphi_0 - k^2\cos^2\varphi_1}{(n_0\cos\varphi_0 + n\cos\varphi_1)^2 + (k_0\cos\varphi_0 + k\cos\varphi_1)^2}, \tag{38}$$

$$R''_{S0} = -\frac{2\cos\varphi_0\cos\varphi_1(n_0k - nk_0)}{(n_0\cos\varphi_0 + n\cos\varphi_1)^2 + (k_0\cos\varphi_0 + k\cos\varphi_1)^2}. \tag{39}$$

Inserting the above found refraction n and absorption k coefficients in (34), (35) and (38), (39), we get the sought values of R''_{p0}, R''_{s0}, R'_{s0}, and R'_{p0}.

DETERMINATION OF THE DEPENDENCE OF THE REAL AND IMAGINARY PARTS OF THE MAGNETO-OPTICAL PARAMETER *Q* ON δψ AND δΔ

As stated above, the magnetic-field contribution to the refraction coefficients is denoted as R''_{p1} and R'_{p1}, and in expressions (14) and (15) the small parameters $\alpha = R''_{p1}/R''_{p0}$ and $\beta = R'_{p1}/R'_{p0}$ are responsible for magnetism. Hence, it is necessary to express α and β from (14) and (15), to obtain from them the expressions for R''_{p1} and R'_{p1}, and, in turn, to find from them unknown Q_1 and Q_2.

In (14) and (15), we keep only the first two addends (the terms proportional to first order α and β), because in the experiment [8] the hysteresis loop $\delta\psi(H)$ is observed; i.e., the effect is proportional to the first order of the magneto-optical parameter. Therefore,

$$\delta\psi = \frac{\mathrm{tg}\psi_0\ (R''^{2}_{p0}\alpha + R'^{2}_{p0}\beta)}{(R'^{2}_{p0} + R''^{2}_{p0})(1+\mathrm{tg}^2\psi_0)}, \tag{40}$$

$$\delta\Delta = \frac{R'_{p0}R''_{p0}(\beta-\alpha)}{R'^{2}_{p0} + R''^{2}_{p0}}, \tag{41}$$

hence

$$\alpha = \frac{1+\mathrm{tg}^2\psi_0}{\mathrm{tg}\psi_0}\delta\psi - \frac{R'_{p0}}{R''_{p0}}\delta\Delta, \tag{42}$$

$$\beta = \frac{1+\mathrm{tg}^2\psi_0}{\mathrm{tg}\psi_0}\delta\psi + \frac{R''_{p0}}{R'_{p0}}\delta\Delta. \tag{43}$$

Considering (36) and (37) as a set of equations and solving it for unknown Q_1 and Q_2 given the definitions of α and β, we get

$$Q_1 = \frac{(A\beta R'_{p0} + B\alpha R''_{p0})}{A^2+B^2}G, \tag{44}$$

$$Q_2 = \frac{(A\alpha R''_{p0} - B\beta R'_{p0})}{A^2+B^2}G, \tag{45}$$

where the following designations were introduced:

$$\begin{aligned} A = {} & 2(n_0^2-k_0^2)(n\cos\varphi_0 + n_0\cos\varphi_1)(k\cos\varphi_0 + k_0\cos\varphi_1) - \\ & -2n_0k_0((n\cos\varphi_0 + n_0\cos\varphi_1)^2 - (k\cos\varphi_0 + k_0\cos\varphi_1)^2), \end{aligned} \tag{46}$$

$$\begin{aligned} B = {} & 4n_0k_0(n\cos\varphi_0 + n_0\cos\varphi_1)(k\cos\varphi_0 + k_0\cos\varphi_1) + \\ & +(n_0^2-k_0^2)((n\cos\varphi_0 + n_0\cos\varphi_1)^2 - (k\cos\varphi_0 + k_0\cos\varphi_1)^2), \end{aligned} \tag{47}$$

$$G = \frac{((n\cos\varphi_0 + n_0\cos\varphi_1)^2 + (k\cos\varphi_0 + k_0\cos\varphi_1)^2)^2}{2\sin\varphi_0\cos\varphi_0}. \tag{48}$$

Therefore, we found the unknown dependence of the real and imaginary parts of the magneto-optical parameter Q from the data of ellipsometric and magneto-ellipsometric measurements.

CONCLUSIONS

In the work, the expressions were obtained using which for the semi-infinite medium model, the refraction n and absorption k coefficients of ferromagnetic metal and the real Q_1 and imaginary Q_2 parts of the magneto-optical parameter Q can be found from the ellipsometric (ψ_0 and Δ_0) and magneto-ellipsometric ($\psi_0 + \delta\psi$ and $\Delta_0 + \delta\Delta$) measurements; i.e., the possibility of a simultaneous characterization of the optical and magnetic properties of the studied object by magneto-ellipsometry is demonstrated.

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