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PHYSICS

One-Dimensional Photonic Crystal Bandpass Filters

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One-dimensional photonic crystals (PCs) are periodic multilayer structures consisting of dielectric layers with different refractive indices and thicknesses comparable with electromagnetic wavelengths in a certain frequency range. In such structures, interference of the waves reflected from interfaces between layers leads to the occurrence of alternating passbands (transparency bands) and reflectance bands (photonic band gaps) in the band structure [1]. However, inside each band gap, one or several additional passbands can be formed by embedding a defect dielectric layer in a PC with a finite number of dielectric layers, which breaks the PC periodic structure [2]. Simultaneous embedding of several defect layers with the same resonant frequency in a PC enables fabrication of highly selective optical bandpass filters [3]. In optics, a set of dielectric layers separating a defect layer from the exterior or the neighboring defect layer is called a dielectric mirror. Such a mirror ensures maximum optical reflection at the frequency for which the thickness of its layers equals the quarter-wavelength ($\lambda/4$) or is multiple to an odd number of $\lambda/4$. The mirror reflectivity increases with the growth of both the refractive index contrast of adjacent layers and the number of the latter.

One of the important drawbacks of optical filters based on the periodic PC structures is the high passband ripple of optical transmittance [4]. This ripple is supposed to be caused by the frequency dispersion of the refractive index of the effective quarter-wavelength layer, which, for the sake of simplicity, often stands for, e.g., trilayer mirrors in multilayer structures [3]. However, the real cause for this ripple is absolutely different.

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The passband ripple of optical transmittance in the PC filters can be equalized with the use of the approaches that have been applied for a long time in microwave bandpass filters [5]. For this purpose, a PC filter should be considered as a system of interacting electrodynamic resonators, which are the defect layers with a thickness multiple to the half-wavelength at the passband center. The coupling coefficient of these resonators and, consequently, the filter passband width, are determined by the multilayer dielectric mirrors between them, the layer thickness in which is multiple to the quarter-wavelength at the passband center.

In this study, we investigate the principles of constructing filters on the basis of one-dimensional PC structures with a specified level of the passband ripple of the optical power within the general theory of resonator filters.

FUNDAMENTALS OF THE THEORY OF RESONATOR FILTERS

The simplest resonance bandpass filter is a chain of m electromagnetically coupled resonators, each having its own resonant frequency f_i and coupling coefficients with two neighboring resonators $k_{i, i-1}$ and $k_{i, i+1}$, where i is the resonator number. The couplings of the input ($i = 1$) and output ($i = m$) resonators with the transmission lines or, in case of a PC filter, with the exterior, are characterized by the external Q factor (Q_e) often called a loaded Q. To obtain a filter with a small passband ripple of transmittance, two conditions must be met. First, resonant frequencies f_i of all the resonators of a filter that form its passband should coincide with the passband center frequency f_0; second, the couplings of the resonators should be matched with each other, and the edge resonators, with the input and output of the device in accordance with its passband.

To meet the second condition, along with the resonators, coupling circuits are used in the filters, which provide the required interaction of adjacent resonators with one another and the edge resonators with the input and output. As a rule, the lowest resonant frequencies of the coupling circuits in real devices are much higher than f_0. In such cases, the frequency dependence of incident power transmittance $|S_{21}|^2$ is expressed by the formula [6]

$$|S_{21}|^2 = \frac{1}{1 + P_m^2(\sin\theta)}, \quad (1)$$

where **S** is the filter scattering matrix, θ is the electrical length of the resonators (thickness of the layers), and $P_m(x)$ is a polynomial of degree m. Coefficients for expansion of $P_m(x)$ in power of sinθ are the functions of only coupling coefficients $k_{i,\,i+1}$ of the resonators and Q_e [5]. It can be seen that at the frequencies at which the polynomial $P_m(\sin\theta)$ turns into zero, total incident power transmission without any reflection occurs, which results in the formation of the filter passband.

When the polynomial is $P_m(x) = \sqrt{\varepsilon}\,T_m(x)$, where $T_m(x)$ is a Chebyshev polynomial of the first kind and ε is the dimensionless normalization parameter, all the reflection maxima $|S_{11}|^2_{\max}$ in the passband will be localized, according to (1), at the level

$$|S_{11}|^2_{\max} = \frac{\varepsilon}{1+\varepsilon}. \quad (2)$$

This statement means that the filter has equalized extrema of the passband ripple of transmittance. Thus, parameter ε determines the passband ripple of not only reflectance but also power transmittance and the greater ε we set, the greater value of this ripple we get. However, it can easily be demonstrated that with decreasing ε the passband slope decreases and the stopband transmission level grows; i.e., the filter selectivity degrades. Therefore, the optimal value of parameter ε is always a compromise of requirements for the frequency response of the designed device.

Filters with the frequency responses being described by the Chebyshev polynomials are called Chebyshev filters. The limit case of the Chebyshev filters are Butterworth filters obtained at $\varepsilon \to 0$. In the Butterworth filter, all m reflection zeroes are located at point f_0 and the function $\varepsilon T_m^2(\sin\theta)$ is proportional to $(\sin\theta)^{2m}$.

To ensure the Chebyshev frequency response with a specified fractional passband width and the acceptable ripple of transmittance, each resonator coupling coefficient $k_{i,\,i+1}$ and external Q_e factors of both edge resonators in the system must take quite definite values. These values can be calculated by approximate generalized formulas of synthesis of the filters designed by Cohn and Matthaei on the basis of equivalent circuits describing the prototype low-pass filters [5]. However, the accuracy of these formulas strongly decreases with increasing fractional passband width. Moreover, for the broadband filters, this accuracy depends on the specific construction of both the filter and its resonators. The error of the generalized synthesis formulas results from the frequency dispersion of coupling coefficients $k_{i,\,i+1}$ [7].

Although the generalized formulas for synthesis of the filters are approximate, at the qualitative level they correctly describe all the basic regularities related to coupling coefficients $k_{i,\,i+1}$ and external Q_e factors. We will mention some of them. The optimal values of coupling coefficients $k_{i,\,i+1}$ between adjacent resonators of a multisection filter monotonically decrease from the edge to central resonators. An increase in the passband width requires a proportional growth of all $k_{i,\,i+1}$ and Q_e^{-1}. A decrease in the passband ripple of power transmittance requires, first of all, a decrease in Q_e and a proper variation in the ratios between the coupling coefficients of the exterior and interior resonator pairs. To equalize the ripple of transmittance, it is convenient to use the developed rules of filter optimization [8, 9]. Knowing the mentioned regularities and optimization rules, one can synthesize any bandpass filters without using the generalized formulas for coupling coefficients.

FEATURES OF THE PC FILTER DESIGN

In photonic-crystal filters, the resonators are defect layers. Their thickness should be multiple to $\lambda_i/2$, where λ_i is the wavelength in the ith layer at central frequency f_0 of the filter passband. In the formation of the passband, one may use both the fundamental and higher modes of the resonator oscillations. In the latter case, in the PC band gap not one but several passbands equidistant from each other are arised. In optics, such multiband filters are called the filters with interleaving passbands or interleave filters [10].

The role of coupling circuits in the PC filters is played by multilayer dielectric mirrors. They weaken the interaction of the resonators with their surroundings. As was mentioned above, the mirrors exhibit the highest reflectivity when the thickness of their layers is $\lambda_i/4$. An increase in the reflectivity of the mirror between the neighboring resonators or the edge resonator and the exterior leads to the corresponding weakening of the coupling coefficient or the growth of the external Q_e factor. The values of $k_{i,\,i+1}$ and Q_e^{-1} decrease with both an increasing number of layers in the mirror and decreasing contrast of the refractive indices of adjacent layers [11]. Since the maximum refractive index contrast in the mirrors is limited by the parameters of the existing optical materials, the main parameter of coarse tuning of the resonator couplings in a PC filter is the number of dielectric layers in each mirror. The refractive index contrast in the quarter-wavelength layers is used for fine tuning of the resonator couplings. It is not necessary to introduce asymmetry in the mirror structure in the form of the deviation of their layer thickness from the quarter-wavelength, as was proposed in [10], as this can significantly degrade the frequency selectivity of the filter in the stopbands. As is known, with increasing order of the resonator operating oscillation mode in the filter, the coupling between them $k_{i,\ i+1}$ and the coupling between the edge resonators with the exterior Q_e^{-1} decrease. This fact is used in the interleave filters for fine tuning of the resonator couplings that form a specified passband by selecting an order of the higher oscillation modes for each of them [12]. However, in this case, numerous passbands close to each other arise, which prevent the formation of the bandpass filters with a broad stopband.

It is convenient to calculate the frequency response of the PC filters with the use of characteristic matrices **M**. In microwave engineering, they are called ABCD matrices. They relate electric field E_i and magnetic field H_i strengths on both surfaces ($i = 1, 2$) of the multilayer structure by the matrix equation

$$\begin{pmatrix} E_1 \\ Z_0 H_1 \end{pmatrix} = \mathbf{M} \begin{pmatrix} E_2 \\ Z_0 H_2 \end{pmatrix}, \qquad (3)$$

where $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ is the characteristic impedance of the exterior. The matrix of the m-layer structure is

$$\mathbf{M} = \mathbf{M}_1 \mathbf{M}_2 \cdot \ldots \cdot \mathbf{M}_k \cdot \ldots \cdot \mathbf{M}_m, \qquad (4)$$

where $\mathbf{M}_k$ is the transmission matrix of the kth layer, the elements of which in the case of normal light incidence are expressed as

$$\mathbf{M}_k = \begin{pmatrix} \cos\theta_k & -in_k^{-1}\sin\theta_k \\ -in_k\sin\theta_k & \cos\theta_k \end{pmatrix}. \qquad (5)$$

Here, θ_k is the phase thickness of the kth layer and n_k is its refractive index. The sign before imaginary unit i corresponds to the case when the electromagnetic components of the light wave change in time by the law $\exp(-i\omega t)$.

In the case when a PC filter is surrounded by free space on both sides, its optical transmittance and reflectance are related to the characteristic matrix elements as

$$S_{21} = \frac{2}{M_{11} + M_{12} + M_{21} + M_{22}}, \qquad (6)$$

$$S_{11} = \frac{M_{11} + M_{12} - M_{21} - M_{22}}{M_{11} + M_{12} + M_{21} + M_{22}}. \qquad (7)$$

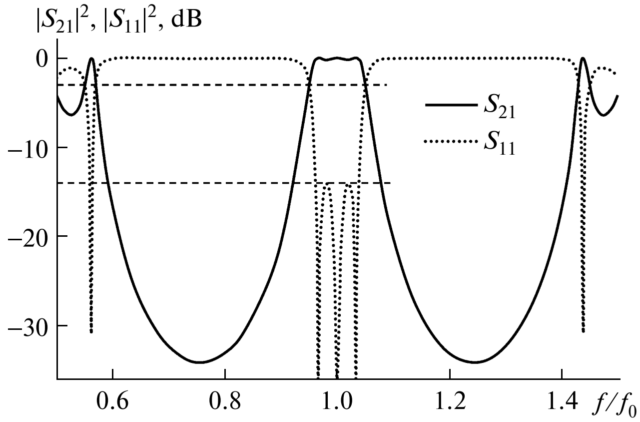

Fig. 1. Frequency dependences of the transmission (solid line) and reflection (dots) in the three-resonator bandpass filter based on the eleven-layer photonic crystal structure.

These formulas allow us to calculate the frequency responses of the PC structures, including those of the bandpass filter described below.

CONSTRUCTION EXAMPLE OF THE THREE-RESONATOR FILTER

The synthesized filter contains three half-wavelength resonators and four mirrors with quarter-wavelength layers. The interior mirrors consist of three layers each, and the exterior ones, of one layer. Thus, the filter comprises 11 layers. Since all the resonators of the filter are air, there is no dielectric loss in them; however, for simplicity, we did not take into account the loss in the dielectric layers of the mirrors as well.

The frequency dependences of transmittance $|S_{21}|^2$ and reflectance $|S_{11}|^2$ are presented in Fig. 1. The filter has a fractional passband width of 10% measured at a level of –3 dB of the transmitted power and the maximum reflection level in the passband of –14 dB (dashed lines in the figure). In the filter passband, three reflectance zeroes are observed, which amounts to the number of resonators. The zeroes are located at the frequencies $f_1 = 0.967 f_0$, $f_2 = f_0$, and $f_3 = 1.033 f_0$.

The refractive index profile for the dielectric layers of the PC filter is shown on the top of Fig. 2. In this structure, the refractive index of the dielectric layers of exterior single-layer mirrors ($n = 2.59$) is somewhat different from that of the other trilayer mirrors ($n = 2.51$). Therefore, two reflection maxima between frequencies f_1, f_2, and f_3 have the level $|S_{11}|^2_{\max} = -14$ dB. Such a level of the reflectivity corresponds to the passband ripple of transmittance $\Delta|S_{21}|^2 = 0.18$ dB or ±2% of the mean transmitted power level.

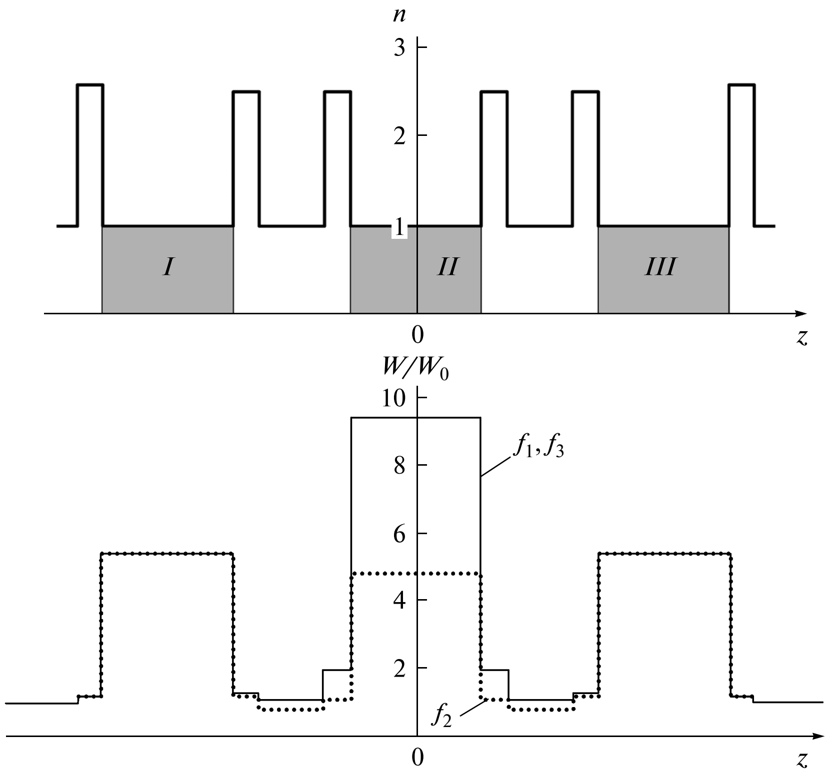

Fig. 2. Refractive index profile of the filter layers and energies accumulated in each filter layer at the total optical transmission frequencies. *I*, *II*, and *III* are the resonators forming the passband, and z is the transverse coordinate.

In the bottom of Fig. 2, diagrams of energy W are presented, which is accumulated in each layer of the filter at three frequencies f_1, f_2, and f_3 of the total optical transmission. Energy W is normalized to energy W_0 supplied to the filter input from the incident power for the half-period of an electromagnetic wave. It can be seen that at all the frequencies the layers of the half-wavelength resonators accumulate energies exceeding by far those accumulated by the quarter-wavelength layers of the mirrors. This means that the Q factor of a material of the mirrors affects the energy loss in the filter passband much more weakly as compared to the material of the resonators. In other words, to minimize the loss in the filter, the intrinsic Q_0 factor of its resonators should be maximally high. For this reason, in the proposed filter construction, air resonators with the infinite intrinsic Q factor are used.

In the investigated filter consisting of three resonators, pairs of adjacent resonators are exterior; therefore, all the interresonator couplings in the filter are the same by virtue of the mirror symmetry of the construction relative to the center. As a result, the reflection maxima in the passband are always equal and their level is determined only by the external Q factor of the resonators, which, as was mentioned above, depends on characteristics of mirrors at the input and output of the filter.

CONSTRUCTION EXAMPLE OF THE SEVEN-RESONATOR FILTER

As is known, an increase in the number of resonators in bandpass filters significantly improves their frequency selectivity. For comparison with the three-resonator filter discussed above (its frequency response is shown by the dashed line in Fig. 3), we synthesized a filter with seven air resonators (its frequency response is shown by the solid line in Fig. 3) with the same relative passband width (10%) and the reflection maxima in the passband at a level of −14 dB. The insert in Fig. 3 presents the profile of refractive indices of the layers in the left half of the synthesized symmetric PC structure, which consists already of 27 layers. The exterior mirrors in the seven-resonator filter also have one quarter-wavelength layer each, and the interior mirrors, three quarter-wavelength layer each. The obtained optimal refractive indices of the layers numbered from the structure edge to its center, including the central 14th layer, are $n_1 = 2.449$, $n_2 = 1$, $n_3 = 2.864$, $n_4 = 1.353$, $n_5 = 2.864$, $n_6 = 1$, $n_7 = 2.864$, $n_8 = 1.063$, $n_9 = 2.864$, $n_{10} = 1$, $n_{11} = 2.864$, $n_{12} = 1$, $n_{13} = 2.864$, and $n_{14} = 1$.

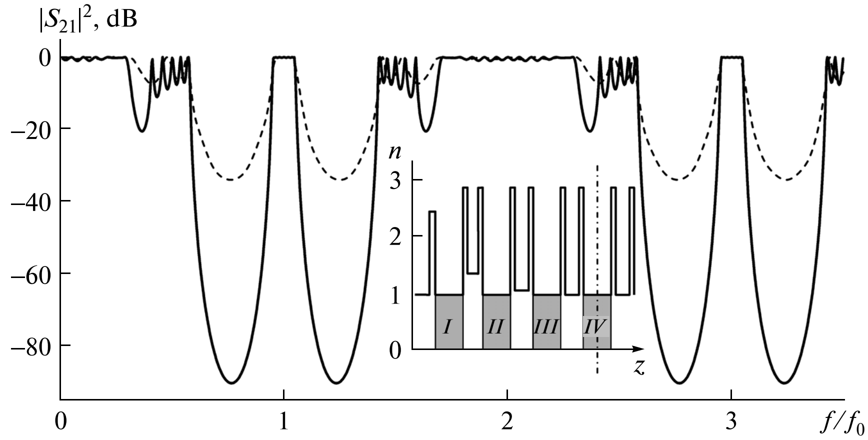

Fig. 3. Frequency response of the synthesized seven-resonator filter (solid line) and the three-resonator filter (dashed line) in a wide frequency range. The inset shows the refractive index profile for the left half of the seven-resonator filter.

It can be seen that the seven-resonator filter has not only a higher slope of the frequency responses but also much larger power attenuation in the passband as compared with the three-resonator filter. Note that the PC structure of such an optimized filter is not strictly periodic. To transform it to the periodic structure, it is sufficient to assign the values $n_1 = 2.864$, $n_4 = 1$, and $n_8 = 1$ to three of the above-mentioned refractive indices. As expected, the filter on the strictly periodic structure exhibits a strong passband ripple of transmittance. This fact is illustrated in Fig. 4, where two frequency dependences of transmittance for the seven-resonator filter are presented. Dots show the frequency response of the periodic PC filter, and the solid line, that of the filter with the optimal structure. The periodic PC filter has the passband ripple of transmittance $\Delta|S_{21}|^2 = 3.5$ dB, and the filter with the optimal structure, $\Delta|S_{21}|^2 = 0.18$ dB; i.e., optimization of the PC structure under the condition $|S_{11}|^2_{\max} = -14$ dB allows decreasing the ripple of transmittance by a factor of almost 20. It is important that the passband ripple of transmittance in the filter can be reduced and made arbitrarily small by optimization of the PC structure by a corresponding decrease in the reflection maxima level $|S_{11}|^2_{\max}$ in the passband.

Thus, in accordance with the fundamentals of the theory of resonator filters, it was shown that the periodic photonic crystal structure used for designing optical filters and consisting of more than three defect layers (resonators) separated by multilayer mirrors has an unacceptably high passband ripple of transmittance. To minimize this ripple, it is necessary to weaken the reflectivity of the mirrors with increasing distance between them and the structure center, simultaneously reducing the refractive index contrast in their layers. In the multi-resonator filters with high frequency selectivity, this requirement should be met, first of all, for the input mirrors ensuring a specified coupling of the edge resonators and the exterior and for the mirrors between the edge resonator pairs in the PC structure.

In addition, it was demonstrated that, at the passband frequencies, the half-wavelength resonator layers accumulate the energies exceeding by far those of the quarter-wavelength mirror layers. Therefore, the Q factor of the mirror material affects the loss in the filter passband much more weakly than the Q factor of the resonator material.

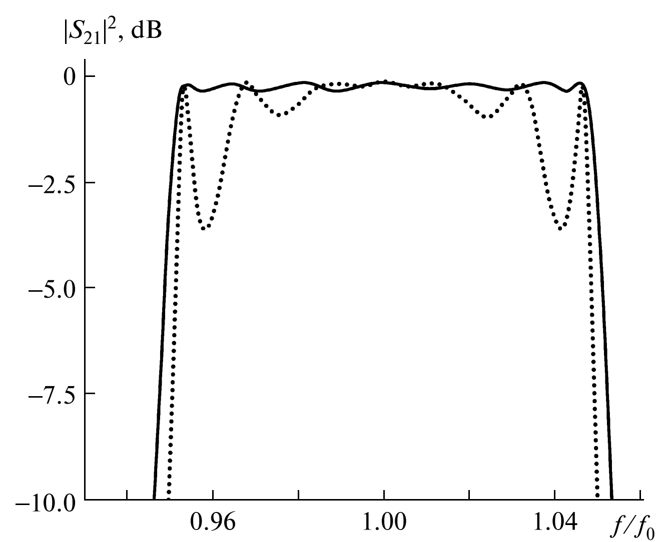

Fig. 4. Frequency response of the tuned seven-resonator filter (solid line) and the traditional periodic photonic crystal structure (dots).

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