Switching from normal to anomalous dispersion in photonic crystal with Raman gain defect

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Propagation of a light through a one-dimensional photonic crystal containing a defect layer doped with a Raman gain medium is discussed. We demonstrate all-optically controlled switching from normal to anomalous dispersion in such a structure. A group delay for the transmitted probe (Raman) pulse is investigated. We show that the group velocity of a Raman pulse can be tuned from subluminal to superluminal by varying the intensity of the pump field.

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The search for efficient control of the group velocity of light in dispersive media has been the subject of extensive study both theoretical and experimental [1–5]. Controlling the group velocity of light has attracted much interest owing to its potential applications, such as tunable optical buffers, optical memory, high-speed optical switches, improving sensitivity of sensing systems, and enhancing the nonlinear effect. There are two approaches to controlling the group velocity of light [1]. One makes use of the dispersive properties associated with the resonance structure of a material medium—the material dispersion [2,4]. The other one makes use of structural resonances such as those that occur in photonic crystals (PhCs)—the structural dispersion [1,5]. Both procedures have proved to be useful in a variety of situations. Group velocity $v_g = d\omega/dk$ depends on a dispersion relation [1,2]. One speaks of light being slow (subluminal propagation) under circumstances when $v_g \ll c$ (c is the velocity light in vacuum) [5]. There are circumstances when $v_g > c$ or even $v_g < 0$. This occurrence is referred to as fast light (superluminal propagation) [2]. Slow and fast light are nowadays a growing field of research (see, for example, [4,6–10]).

In most experiments, superluminal propagation was observed in poorly transmitting media [11]. A few experiments have been reported with a small enhancement, when the medium can be considered as transparent [6,7], as well as with a large enhancement (see for example the recent review [10]). It has been shown that by using Raman gain medium (room-temperature and ultracold atoms, molecular gases and solid state), a slow [12–14] and fast [6,7] light, gain-assisted giant Kerr effect [15], and superluminal solitons [16] can be obtained. Hollow-core PhC fibers filled with atoms and molecular gas can be used for significant enhancements of Raman amplification. Recently a fast Kerr phase gate using the active Raman gain method has been experimentally demonstrated where the probe wave travels superluminally [17].

It would be useful to have a system where a group velocity can be controlled from subluminal to superluminal [18,19]. Of the greatest interest are the schemes where superluminal propagation is achieved without significantly reducing or even with amplification of the pulse amplitude at the output. In this Letter we report a method that allows the group velocity of a probe pulse to be controlled by controlling the strength of a pump field during Raman interaction in a medium placed in the defect of a one-dimensional PhC. We show that due to the combination of the structural resonance of the multilayer geometry and the Raman resonance in the defect layer, the group velocity can be switched from subluminal to superluminal. At the same time, the transmitted (or reflected) pulse can be amplified. Note that transmission in PhC with Raman gain can be controlled from enhanced to eliminated one by varying the pump field intensity; that is, this structure can operate as an optical switch [20].

Let two plane waves (the pump and the probe) with frequencies $\omega_{1,2}$ be normally incident on PhC with a $(HP)^p$ HDH $(LH)^p$ structure. Here, H and L refer to different dielectric layers with high and low refractive indices, n_H and n_L, and thicknesses t_H and t_L, respectively; D is the defect layer with t_D thickness and the refractive index n_D; and p is the number of periods. The defect layer contains a Raman gain medium with an energy-level diagram showing in the left inset in Fig. 1. States $|0\rangle$ and $|2\rangle$ are the ground and metastable states, respectively. The pump field E_1 interacts with the $|0\rangle - |1\rangle$ transition and the probe field E_2 interacts with the adjacent transition $|1\rangle - |2\rangle$. The frequency difference $\omega_1 - \omega_2$ is close to the transition frequency ω_{20}.

A complex refractive index of the defect layer $n_D = n_2$ for a probe field in the presence of a pump wave is given by $n_2 = 1 + 2\pi N \chi_R(\omega_2)|E_1|^2$, where E_1 is the complex amplitude of the pump wave, N is the concentration of atoms, and $\chi_R(\omega_2)$ is the Raman susceptibility

$$\chi_R(\Delta_2) = \frac{1}{4\hbar^3}\frac{d_{21}^2 d_{10}^2}{\Delta_1^2[\Delta_{20} + i\gamma_{20}]}. \tag{1}$$

Here, $\Delta_{20} = \Delta_1 - \Delta_2 = \omega_{20} - (\omega_1 - \omega_2)$ is the Raman detuning, $\Delta_1 = \omega_{10} - \omega_1$, $\Delta_2 = \omega_{21} - \omega_2$ are the one-photon detuning, ω_{10} and ω_{21} are the frequencies of atomic transitions, γ_{20} is the Raman transition half-width, d_{ij} is the matrix dipole moment of the transition, and $\hbar$ is the

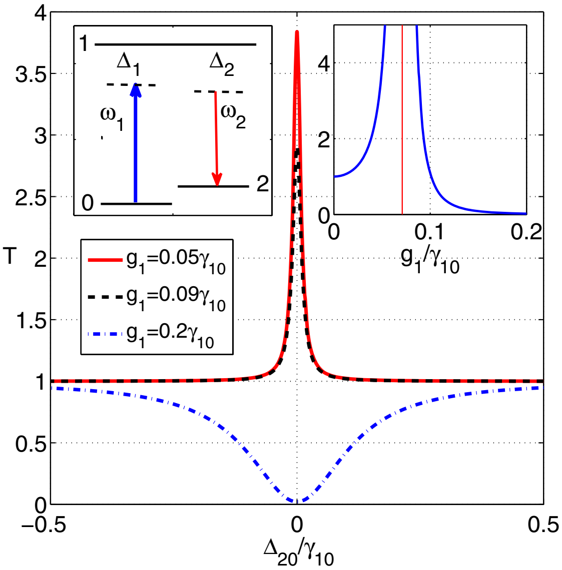

Fig. 1. Transmission spectra T of PhC with Raman gain defect as a function of detuning from the Raman resonance (ω_2 changes at $\Delta_1 = 30\gamma_{10}$) for various Rabi frequencies g_1 at the input of PhC. Left inset: energy-level diagram of a three-level atom in a Raman gain scheme. Right inset: transmission coefficient for a probe field versus Rabi frequency of a pump g_1. The vertical line refers to the threshold value of g_1.

Planck constant. The formula (1) is valid under the following conditions: $|\Delta_1| \gg G_1$, γ_{10} and $G_1 \gg G_2$, where $2G_{1,2} = d_{10,12}E_{1,2}/\hbar$ are the Rabi frequencies of the pump and probe wave and γ_{10} is the half-widths of the $|0\rangle - |1\rangle$ transition. Since the probe field is assumed to be very weak, the population of the lower state $|0\rangle$ can be considered unaffected under these conditions. Note that $\mathrm{Im}\chi_R$ is negative in the vicinity of the Raman resonance, which implies the probe wave enhancement due to energy transfer from the pump to the probe field, and $\mathrm{Re}\,\chi_R$ has normal dispersion $d\chi_R/d\Delta_2 > 0$ in this region.

In a steady-state approximation, a field in an arbitrary jth layer ($j = H, L, D$) can be treated as a superposition of counter-propagating waves $E_j(z) = A_j \exp[ik_j(z-z_j)] + B_j \exp[-ik_j(z-z_j)]$, where A_j and B_j are amplitudes of the forward (incident) and backward (reflected) waves, and $k_j = n_j\omega_i/c (i = 1, 2)$, where n_j is the refractive index of a jth layer. Amplitudes A_j and B_j for each layer were found from wave equations by means of recurrent relations [21] using the continuity of tangential components of the electric and magnetic fields at the interface of adjacent layers. The transmission and reflection spectra were determined as

$$T(\omega) = |A_2(L)/A_{02}|^2, \qquad R(\omega) = |B_2(0)/A_{02}|^2, \qquad (2)$$

where A_{02} and $A_2(L)$ are the input ($z = 0$) and the output ($z = L$ is the PhC length) amplitudes of the probe wave, respectively, and $B_2(0)$ is the amplitude of the probe wave reflected from the input face of the PhC.

For numerical simulation, we used atomic parameters of sodium as the Raman medium. Wavelengths of the probe and pump fields were chosen close to the D_1 line and $\omega_{20}/2\pi$ was taken to be 1.8 GHz. The PhC had the following parameters: $p = 5$, $n_H d_H = n_L d_L = \lambda/4$, where λ corresponds to the center of the first photonic bandgap, $d_D = \lambda/2$, $n_H = 2.35$, and $n_L = 1.45$. The rest of the parameters were as follows: $\gamma_{10}/2\pi = 5.7$ MHz, $\Delta_1 = 30\gamma_{10}$, $\gamma_{20}/2\pi = 100$ kHz, $d_{10} \approx d_{12} \simeq 6 \times 10^{-18}$ esu (electrostatic units) [22] and $N = 10^{12}$ cm^{-3}.

The parameters of the PhC have been chosen such that in the absence of the Raman gain medium the defect mode is located in the center of a gap and its spectral width is broad enough for both waves to fall within this transmission band. The resonance frequency of the defect mode coincides with that of the probe wave under Raman resonance $\Delta_{20} = 0$. For the specified parameters, the calculation of field distribution in an empty defect layer yields a virtually complete spatial overlapping of the pump and the probe fields. The Rabi frequency in the defect $G_1(z)$ is related to the Rabi frequency g_1 at the entrance into the PhC as follows: $G_1(z) = g_1 F(z)$, where $F(z) = |E_1(z)/E_{\mathrm{in}}|$ is the amplification factor of the field amplitude in the defect, $E_1(z)$ is the spatially dependent field strength in the defect, and E_{in} is the strength of the incident pump field. The fields are distributed inhomogeneously across the defect, and therefore the gain factor and the refractive index in the defect are functions of the z coordinate.

Figure 1 shows typical PhC transmission spectra for a probe field at different pump intensities. Narrow structures (a peak or a dip) due to the Raman resonance can be observed in the center on the background of a broad transmission band. The transmittance can be larger or less than unity, and therefore, it can be interpreted as the transmission gain. In the right inset in Fig. 1 transmittance maxima are plotted as a function of the Rabi frequency of the incident pump field. The transmission coefficient grows when the pump intensity increases, yet remaining below a certain threshold value. Beyond this critical value, the transmission decreases with growing intensity (the peak in transmission turns into a dip).

Consider now propagation of a probe pulse assuming that the pump field is continuous monochromatic wave. The spectrum of the transmitted pulse can be written as $E_{2t}(\omega) = T(\omega)E_{2i}(\omega)$, where $T(\omega)$ is the transmission coefficient and $E_{2i}(\omega)$ is the spectrum of the incident probe pulse. Applying a reverse Fourier transform, the intensity of the transmitted probe pulse can be written as [23,24]

$$I_{2t} \propto \left| \frac{1}{\sqrt{2\pi}} \int_{\infty}^{\infty} T(\omega)E_{2i}(\omega) \exp(-i\omega t) \mathrm{d}\omega \right|^2. \qquad (3)$$

In terms of this approach, the duration of the probe pulse τ_2 must satisfy the requirement that $\tau_2 \gg |\Delta_{20}|^{-1}$ [7].

Figure 2(a) illustrates a transmitted Gaussian probe pulse. From Fig. 2(a) it follows that depending on a pump field intensity, the transmitted probe pulse may either lag behind (dashed line) or lead (dash-dotted line) the reference pulse. In the first case, the group velocity of the pulse is less than the velocity of light in a vacuum, whereas in the latter case we deal with superluminal propagation. The shape of transmitted pulses remains almost unchanged. The same applies to the case of a reflection.

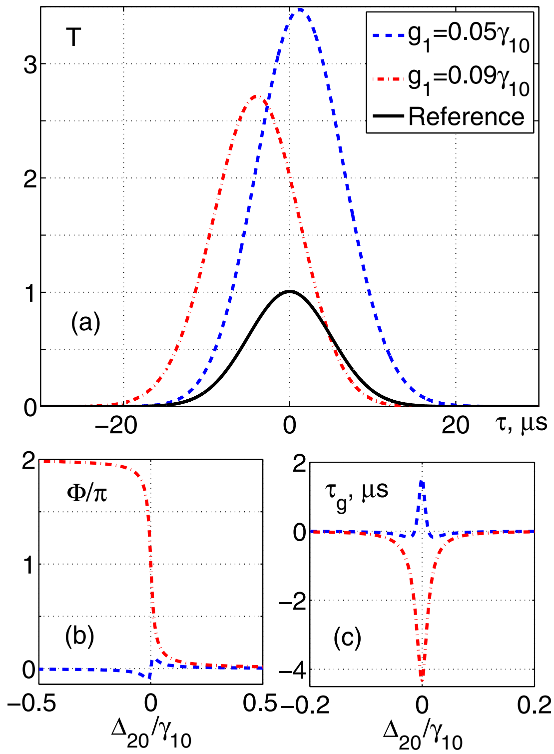

Fig. 2. (a) Time dependence of the probe pulse transmitted through a PhC with a Raman gain medium for different Rabi frequency g_1 of the pump at the input of PhC. The solid curve illustrates the transmitted probe pulse without Raman gain medium in a defect—the reference pulse. The length of the incident Gaussian probe pulse is 20 μs. (b) Spectral dependencies of the phase of the transmitted probe wave. (c) Group delay τ_g as function of detuning from the Raman resonance for different Rabi frequencies g_1.

The results obtained can be qualitatively explained in terms of the effective refractive index for PhC $n(\omega) = c\Phi(\omega)/\omega L$, where $\Phi(\omega)$ is the phase of a probe wave passed through the PC [25]. Obviously, $n(\omega)$ or $\Phi(\omega)$ describe dispersion properties of the PhC structure. In Fig. 2(b) spectral dependencies of the phase $\Phi(\omega)$ of the transmitted probe wave are shown for two Rabi frequencies of the pump field g_1: below (dashed curve) and above (dash-dotted curve) the threshold. In the first case, dispersion is normal and results in subluminal propagation of the probe pulse, whereas in the second case dispersion is anomalous and therefore we deal with superluminal propagation as illustrated in Fig. 2(a). It should be noted that in both cases, the dispersion of medium in the defect is normal. Thus a mechanism of attaining anomalous dispersion in the given case is essentially different from the one reported in [6–9] and is associated with dispersion of the PhC (structural dispersion) rather than with dispersion of the Raman medium (material dispersion).

Group delay (a phase time) for the transmitted pulse can be calculated as [26]

$$\tau_g = \partial\Phi/\partial\omega|_{\omega=\omega_0}, \tag{4}$$

where ω_0 is the carrier frequency of probe pulse. The group delay measures the time difference between appearance of a wave packet at $z = 0$ and at $z = L$. These peaks are not necessarily related by a simple causal translation since incident and transmitted pulses are not an entity and the group delay is therefore not a transit time [4,11]. Figure 2(c) shows the group delay τ_g as a function of the detuning from the Raman resonance for different Rabi frequencies g_1. The calculated group delays are in agreement to those shown in Fig. 2(a). Negative group delay corresponds to the superluminal propagation. In such a situation, the peak of the transmitted pulse will exit the material before the peak of the incident pulse enters the material [27]. Superluminal propagation is not at odds with causality [2,4,11]. While this aspect of the pulse propagation appears to be superluminal, it does not imply superluminal signal propagation.

A qualitative interpretation of group delay features becomes possible if we look at the problem in terms of a Fabry–Perot cavity with the length L equal to the thickness of the defect layer t_D, which is filled with a Raman gain medium. The field transmitted through the cavity is given by

$$E_t = \frac{T_m E_0 e^{ikL}}{(1 - R_m e^{i2kL})} = \frac{T_m E_0 e^{k''L} e^{i\Phi}}{[(1 - R_m e^{2k''L})^2 + 4R_m e^{2k''L}\sin^2(k'L)]^{1/2}}, \tag{5}$$

where T_m and R_m are the energy transmission and reflection coefficients of mirrors, $k = k' - ik''$, $k'' = -(2\pi/\lambda)n''_{\text{eff}} > 0$ is the amplitude Raman gain factor of the probe wave, $k' = (2\pi/\lambda)n'_{\text{eff}}$, $n''_{\text{eff}} = 2\pi N\chi''_R F|E_1|^2$, $n'_{\text{eff}} = 1 + 2\pi N\chi'_R F|E_1|^2$ are the effective imaginary and real parts of the refractive index n_2, $F \simeq 1$ is the spatial overlapping integral of the pump and the probe fields [28], and Φ is the phase of the transmitted field:

$$\Phi(k) = k'L + \tan^{-1}\left[\frac{R_m e^{2k'L}\sin(2k'L)}{1 - R_m e^{2k'L}\cos(2k'L)}\right]. \tag{6}$$

Using Eqs. (4) and (6) one can find the group delay

$$\tau_g = \frac{(1 - R_m^2 e^{4k''L})}{(1 - R_m^2 e^{2k''L})^2 + 4R_m e^{2k''L}\sin^2(k'L)}\frac{L}{v_g}, \tag{7}$$

where v_g is the group velocity in a Raman medium. Quantity $L/v_g = \tau_0$ describes group delay in the absence of a cavity. At a resonance frequency, when $k'L = m\pi$, Eq. (7) can be expressed as

$$\tau_g = \frac{1 + R_m^2 e^{2k''L}}{1 - R_m^2 e^{2k''L}}\frac{L}{v_g}, \tag{8}$$

with the transmission coefficient being

$$T = \frac{|E_t|^2}{|E_0|^2} = \frac{T_m^2 e^{2k''L}}{(1 - R_m^2 e^{2k''L})^2}. \tag{9}$$

We see that at $R_m^2\exp(2k''L) < 1$ group delay $\tau_g > L/v_g$, which corresponds to subluminal propagation. When

$R_m^2 \exp(2k''L) > 1$ or $k''L > T_m$ (when $(1 - R_m) \simeq T_m \ll 1$) the group delay is negative $\tau_g < 0$. Using Eq. (9), it can be readily shown that the maximum transmission coefficient can be $T \gg 1$ [20].

In summary, we investigate the subluminal and superluminal pulse propagation in a one-dimensional PhC with a Raman gain defect. Dispersion of the system can be controlled by a pump field. Occurrence of large positive and negative time delays is the result of the Bragg resonance coupled with Raman resonance. Unlike earlier studies, in this Letter the transmitted probe pulse can be enhanced; its shape, though, hardly changes at all. Similar effects take place in a reflection. The intensity required for these effects to be observed depends on a number of factors (one-photon pump frequency detuning, Raman resonance width, and quality factor of defect modes) and can be anything from 10 to 100 $\mu W/cm^2$. For experimental realization both room-temperature and ultracold atoms and also molecular gases can be used. These experiments are similar to [29] (and references therein), in which atoms have been loaded into hollow-core PhC fibers. Also the proposed scheme can be realized in a macroscopic optical ring cavity for the probe radiation similar to cavity electromagnetically induced transparency [30]. The use of heterostructure semiconductors as nonlinear media with controlled Raman gain is of great interest.

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