DOI 10.1007/s11182-015-0546-1

Russian Physics Journal, Vol. 58, No. 5, September, 2015 (Russian Original No. 5, May, 2015)

DIFFRACTION OF ELECTROMAGNETIC WAVES ON A ONE-DIMENSIONAL STRIP CONDUCTOR GRATING LOCATED AT THE INTERFACE BETWEEN DIELECTRIC MEDIA

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UDC 537.874.6

Analytical formulas are derived for the transmission and reflection coefficients of electromagnetic E- and H-waves incident at an arbitrary angle on a one-dimensional strip conductor grating located at the interface between two dielectric media. The feasibility of adjusting the grating reflectivity within wide limits by varying its design parameters is demonstrated, which allows these structures to be used in multilayered dielectric bandpass filters.

Keywords: electromagnetic waves, dielectric layers, strip conductors, diffraction.

INTRODUCTION

Widely investigated resonant structures based on one-dimensional strip conductor gratings [1–4] are promising for frequency selection of electromagnetic waves. However, one-dimensional non-resonant reflecting strip conductor gratings with small period compared to the wavelength are no less interesting. They are used in microwave devices, such as reflector antenna systems [5, 6], angular filters for side lobe suppression [7], and polarization discriminators [8]. These structures can be used in bandpass filters of the optical and terahertz ranges as well as of the upper microwave range.

As is well known, modern optical filters are multilayered dielectric structures in which layers-resonators with phase thickness π forming passbands are separated by multilayered dielectric mirrors with layer phase thickness $\pi/2$. The number of mirror layers is determined by the filter passband width and by the contrast between the refractive indices of the employed materials; moreover, to reduce the transmission ripple in the filter passband, the materials with different refractive indices are required [9]. However, only three or even two materials with different refractive indices can be used in these filters given that the phase thickness of the mirror layers differs from $\pi/2$ [10, 11]. It is important to note that the multilayered dielectric mirrors substantially reduce the stopband width because of the presence of spurious resonances, thereby reducing the filter selectivity. The use of one-dimensional metal film strip structures deposited from both sides on the dielectric resonator layers instead of the dielectric mirrors eliminates completely the aforementioned disadvantages of the filters.

Different numerical and numerical-analytical methods for calculating electromagnetic wave diffraction on plane strip conductor gratings [2, 4, 12, 13] are very complicated, which limits their application for solving concrete problems. In particular, a strict representation of the one-dimensional structure in the form of two equivalent circuits of inductive or capacitive elements corresponding to the *E*- and *H*-wave incidence was proposed in [12]. Expressions for the elements of the multimodal coupling matrices describing equivalent circuits contain several double definite integrals whose values can be obtained only numerically.

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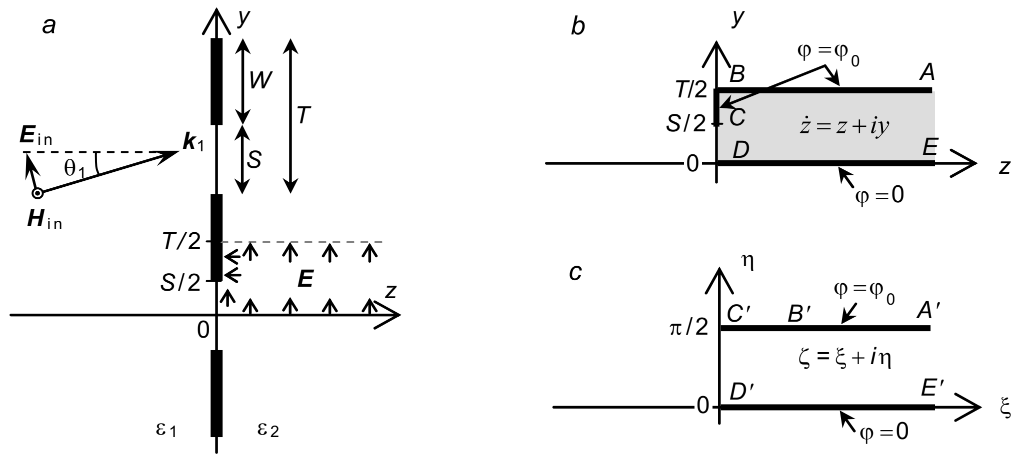

Fig. 1. Transverse cross section of the strip conductor grating (*a*) and conformal mapping of the *ABCDE* area in the $\dot{z}$ plane shown in gray color (*b*) onto the *A′B′C′D′E′* area in the ζ plane (*c*).

Analytical expressions for the transmission and reflection of electromagnetic waves normally incident on a one-dimensional ideal grating of strip conductors having zero thickness surrounded by the same medium were first derived in [14] for the grating period much smaller than the wavelength. The same models, but for arbitrary period of the grating in which the strip conductor width was equal to the spacing were considered in [15, 16]. A comparison of the expressions presented in [16] has shown that the long-wavelength approximation [14] is sufficiently correct when the grating period $T < 0.4\lambda$, where λ is the wavelength.

The purpose of this paper is analytical solution of the problem of diffraction of electromagnetic *E*- and *H*-waves incident at an arbitrary angle on a one-dimensional plane strip conductor grating and investigation of regularities in the behavior of the transmission (reflection) coefficients as functions of the angle of electromagnetic wave incidence attendant to changes in the permittivity of the media and in the period and spacing between the strip conductors.

1. *E*-WAVE INCIDENCE ON A CONDUCTOR GRATING

Let us consider a one-dimensional grating of strip conductors parallel to the ***x*** axis (Fig. 1*a*) located in the ***xy*** plane at the interface between two media with relative permittivities ε_1 and ε_2. The grating has the period $T = S + W$, where W is the strip conductor width and S is the spacing between them. Let the electromagnetic *E*-wave with the circular frequency ω, the wave vector k_1, and the vectors of the electric field (E_{in}) and magnetic field (H_{in}) be incident from the first medium perpendicularly to the direction of strip conductors at the angle θ_1 to the z axis normal to the grating plane. The *E*-waves are also called *TM* modes or *p*-polarized light.

In this case, the nonzero tangential components of the incident wave are given by the formulas

$$E_y(y,z,t) = E_{in}\cos\theta_1 \exp\left(ik_y y + ik_{1z} z - i\omega t\right), \;\; H_x(y,z,t) = -E_{in}\frac{\sqrt{\varepsilon_1}}{Z_0}\exp\left(ik_y y + ik_{1z} z - i\omega t\right), \tag{1}$$

where

$$Z_0 = \sqrt{\mu_0/\varepsilon_0}, \quad k_0 = \omega\sqrt{\varepsilon_0\mu_0} = \omega/c, \quad k_1 = k_0\sqrt{\varepsilon_1}, \quad k_y = k_1\sin\theta_1, \quad k_{1z} = k_1\cos\theta_1 . \tag{2}$$

Here Z_0 is the wave impedance of the free space, μ_0 and ε_0 are the permeability and permittivity of vacuum, and c is the velocity of light in vacuum.

The incident wave with components (1) excites the transverse surface currents $J_y(y)$ on the strip conductors which, in turn, participate in the formation near the grating of the electromagnetic field uniform along the $\boldsymbol{x}$ axis. The nonzero tangential components of the reflected wave at great distance from the grating ($|z| >> T/2$) are given by the formulas

$$E_y(y,z) = E_{\text{ref}} \cos\theta_1 \exp\left(ik_y y - ik_{1z} z\right),\ \ H_x(y,z) = E_{\text{ref}} \frac{\sqrt{\varepsilon_1}}{Z_0} \exp\left(ik_y y - ik_{1z} z\right), \tag{3}$$

and those of the transmitted wave are

$$E_y(y,z) = E_{\text{tr}} \cos\theta_2 \exp\left(ik_y y + ik_{2z} z\right),\ \ H_x(y,z) = -E_{\text{tr}} \frac{\sqrt{\varepsilon_2}}{Z_0} \exp\left(ik_y y + ik_{2z} z\right), \tag{4}$$

where

$$k_{2z} = \sqrt{k_2^2 - k_y{}^2}\ , \tag{5}$$

$$\sin\theta_2 = \sqrt{\varepsilon_1/\varepsilon_2}\,\sin\theta_1\ . \tag{6}$$

In the examined case, the electric component $\boldsymbol{E}$ in the near field of the plane grating must obey the two-dimensional Helmholtz equation

$$\left(\partial^2/\partial y^2 + \partial^2/\partial z^2 + k_\alpha^2\right) E(y,z) = 0\,, \tag{7}$$

where the subscript α takes the value 1 for $z < 0$ and the value 2 for $z > 0$. It is sufficient to solve Eq. (7) only for a semi-infinite area shown in gray color in Fig. 1:

$$0 \le y \le T/2, \quad z \ge 0\,. \tag{8}$$

In accordance with the Floquet–Lyapunov theorem [17], the electric field in all other areas will be determined by the symmetry of $\boldsymbol{E}$ in the planes $y = 0$ and $z = 0$ by means of periodic continuation along the $\boldsymbol{y}$ axis and multiplication by $\exp(ik_y y)$.

Since the problem is solved in the long-wavelength approximation ($\lambda >> T$), the Helmholtz equation [Eq. (7)] in the vicinity of the strip conductors ($|\,z\,| << \lambda$) can be replaced by the Laplace equation

$$\left(\partial^2/\partial y^2 + \partial^2/\partial z^2\right) \boldsymbol{E}(y,z) = 0\,. \tag{9}$$

This means that the electric field near the strip conductors is given by the formula

$$\boldsymbol{E} = -\operatorname{grad}\varphi\,, \tag{10}$$

where φ is the electrostatic potential that obeys the two-dimensional Laplace equation

$$\left(\partial^2/\partial y^2 + \partial^2/\partial z^2\right) \varphi(y,z) = 0 \tag{11}$$

with boundary conditions

$$\varphi(y,z)\big|_{y=0}=0,\quad \varphi(y,z)\big|_{z=0,\,S/2<y<T/2}=\varphi_0,\quad \varphi(y,z)\big|_{y=T/2}=\varphi_0\,. \tag{12}$$

According to boundary conditions (12), the local directions of the vector $\boldsymbol{E}$ in Fig. 1*a* are indicated by the arrows.

To solve Eq. (11), let us perform a conformal mapping of semi-infinite strip (8), indicated by gray color and denoted *ABCDE* in Fig. 1*b*, onto the other semi-infinite strip denoted *A′B′C′D′E′* in Fig. 1*c*. Let us perform mapping so that the plane $y = 0$ (segment *DE* in Fig. 1*b*) carrying the potential $\varphi = 0$ is mapped onto the lower boundary of the new semi-infinite strip (segment *D′E′* in Fig. 1*c*), and the plane $y = T/2$ together with the half of the strip conductor (the broken line *ABC* in Fig. 1*b*) carrying the potential $\varphi = \varphi_0$ is mapped onto the upper boundary of the new semi-infinite strip (the line segment *A′B′C′* in Fig. 1*c*). To perform the conformal mapping, a complex number $\dot z = z + iy$ must be put in correspondence to each point with coordinates x and y lying inside the semi-infinite strip *ABCDE* (Fig. 1*b*), and a complex number $\zeta = \xi + i\eta$ must be put in correspondence to each point with coordinates ξ and η lying inside the rectangle *A′B′C′D′E′* (Fig. 1*c*).

With the help of the Schwarz–Christoffel integral, we find that the conformal mapping of the $\dot z$ area onto the ζ area is performed by the analytic function

$$\zeta(\dot z) = \operatorname{arcsinh}\left(\sinh(\pi\dot z/T)\big/\sin\left(\pi S/(2T)\right)\right). \tag{13}$$

We note that the function $\zeta(\dot z)$ like any other analytical function, is a solution of Eq. (11). So the real potential of the electric field $\varphi(y, z)$ that obeys boundary conditions (12) is expressed by the formula

$$\varphi(y,z) = \frac{2\varphi_0}{\pi}\operatorname{Im}\zeta(\dot z)\,. \tag{14}$$

Recall that the potential $\varphi(y, z)$ is an even function of the coordinate z, and the function $\zeta(\dot z)$ is determined by Eq. (13) only for $z \ge 0$.

Solution (14) taking into account Eq. (10) obeys the boundary condition

$$E_y(y,z)\big|_{\substack{z=0\\ \alpha=1}} = E_y(y,z)\big|_{\substack{z=0\\ \alpha=2}} = 0 \tag{15}$$

on the conductor surface and the boundary condition

$$\varepsilon_1 E_z(y,z)\big|_{\substack{z=0\\ \alpha=1}} = \varepsilon_2\, E_z(y,z)\big|_{\substack{z=0\\ \alpha=2}} \tag{16}$$

on the interface between the media free of conductors. Let us express the component of the magnetic field H_x alone in terms of the potential φ (y, z). From the Maxwell equation it follows that

$$\partial H_x/\partial y = i\omega\varepsilon_0\varepsilon_\alpha E_z\,. \tag{17}$$

Substituting Eqs. (10) and (14) into Eq. (17), we obtain

$$\frac{\partial}{\partial y}H_x = (-1)^\alpha i\,\omega\,\varepsilon_0\varepsilon_\alpha\frac{2\varphi_0}{\pi}\frac{\partial}{\partial y}\operatorname{Re}\zeta(\dot z)\,. \tag{18}$$

The factor $(-1)^\alpha$ has been introduced here to complement the definition of Eq. (13) for the function $\zeta(\dot z)$ for negative values of the coordinate z proceeding from the field symmetry.

After integration of Eq. (18) over the argument y, we have

$$H_x(y,z) = H_0 + i\frac{4\varepsilon_\alpha}{\lambda_0 Z_0}\varphi_0 \operatorname{Re}\left[(-1)^\alpha \zeta(\dot{z}) - \pi\dot{z}/T\right], \tag{19}$$

where λ_0 is the wavelength in the free space, and the integration constant (to be more exact, the function of the coordinate z alone) is chosen so that the field $H_x(y,z)$ becomes uniform in the limit $|z|\to\infty$. We note that solution (19) in the segment of the interface between the media free of conductors, i.e., in the segment CD (see Fig. 2a) obeys the boundary condition

$$H_x(y,z)\big|_{\substack{z=0\\ \alpha=1}} = H_x(y,z)\big|_{\substack{z=0\\ \alpha=2}} . \tag{20}$$

To determine the magnetic field at an arbitrary distance from the grating, function (19) must be expanded in a Fourier series in functions $\cos(\pi ny/T)$ for fixed z and each harmonic so obtained must be continued along the z axis so that to obey Eq. (7). The amplitudes of all Fourier harmonics at a sufficiently large distance from the grating, except of the zero harmonic ($n = 0$), tend to zero by an exponential law. Therefore, it is suffice to join the near field of the grating with the incident, reflected, and transmitted wave fields only for the zero harmonic

$$\langle H_x \rangle = \frac{2}{T}\int_0^{T/2} H_x(y,z)\,dy . \tag{21}$$

Substituting Eqs. (13) and (19) into Eq. (21), we obtain

$$\langle H_x \rangle = H_0 - (-1)^\alpha i\frac{4\varepsilon_\alpha}{\lambda_0 Z_0}\varphi_0 \ln\sin\left(\pi S/(2T)\right). \tag{22}$$

According to Eqs. (10) and (14), the zero Fourier harmonic for the electric field is

$$\langle E_y \rangle = -2\varphi_0/T . \tag{23}$$

Let us write equations relating the components E_y and H_x of the incident, reflected, and transmitted waves with the corresponding components of the zero harmonics of the near field of the grating:

$$\begin{aligned} E_{\text{in}}\cos\theta_1 + E_{\text{ref}}\cos\theta_1 &= \langle E_y\rangle, & -E_{\text{in}}\sqrt{\varepsilon_1}/Z_0 + E_{\text{ref}}\sqrt{\varepsilon_1}/Z_0 &= \langle H_x\rangle_{\alpha=1}, \\ E_{\text{tr}}\cos\theta_2 &= \langle E_y\rangle, & -E_{\text{tr}}\sqrt{\varepsilon_2}/Z_0 &= \langle H_x\rangle_{\alpha=2}. \end{aligned} \tag{24}$$

This system of equations has a solution

$$\frac{E_{\text{ref}}}{E_{\text{in}}} = \frac{\dfrac{\sqrt{\varepsilon_1}}{\cos\theta_1} - \dfrac{\sqrt{\varepsilon_2}}{\cos\theta_2} - i\,2T\dfrac{\varepsilon_1+\varepsilon_2}{\lambda_0}\ln\sin\left(\dfrac{\pi S}{2T}\right)}{\dfrac{\sqrt{\varepsilon_1}}{\cos\theta_1} + \dfrac{\sqrt{\varepsilon_2}}{\cos\theta_2} + i\,2T\dfrac{\varepsilon_1+\varepsilon_2}{\lambda_0}\ln\sin\left(\dfrac{\pi S}{2T}\right)}, \quad \frac{E_{\text{tr}}}{E_{\text{in}}} = \frac{2\sqrt{\varepsilon_1}/\cos\theta_2}{\dfrac{\sqrt{\varepsilon_1}}{\cos\theta_1} + \dfrac{\sqrt{\varepsilon_2}}{\cos\theta_2} + i\,2T\dfrac{\varepsilon_1+\varepsilon_2}{\lambda_0}\ln\sin\left(\dfrac{\pi S}{2T}\right)}. \tag{25}$$

Then we obtain the formulas for the reflection and transmission coefficients of the incident power

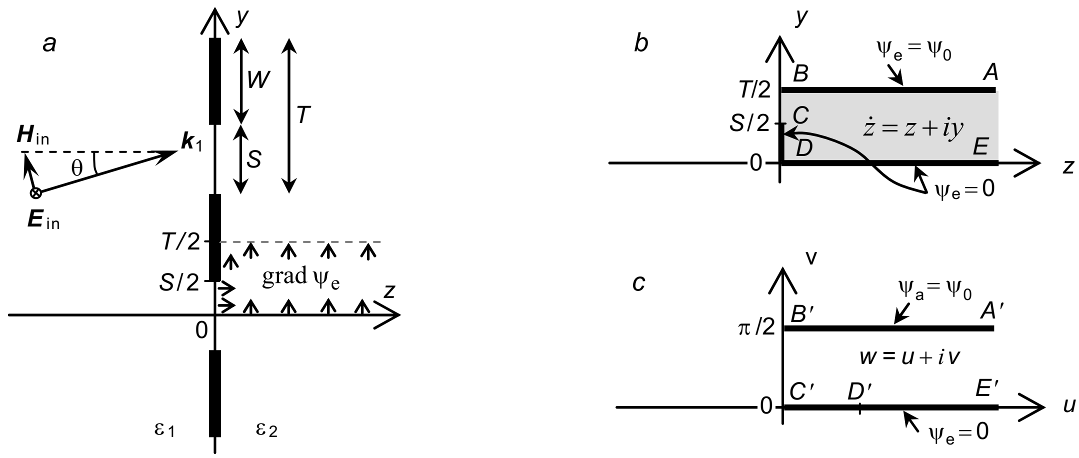

Fig. 2. Cross section of the strip conductor grating (*a*) and conformal mapping of the *ABCDE* area indicated by gray color in the $\dot{z}$ plane (*b*) onto the *A′B′C′D′E′* area in the *w* plane (*c*).

$$K^E_{\text{ref}} = \frac{\left(\dfrac{\sqrt{\varepsilon_1}}{\cos\theta_1} - \dfrac{\sqrt{\varepsilon_2}}{\cos\theta_2}\right)^2 + \left(2\dfrac{\varepsilon_1+\varepsilon_2}{\lambda_0/T}\right)^2 \ln^2 \sin\left(\dfrac{\pi S}{2T}\right)}{\left(\dfrac{\sqrt{\varepsilon_1}}{\cos\theta_1} + \dfrac{\sqrt{\varepsilon_2}}{\cos\theta_2}\right)^2 + \left(2\dfrac{\varepsilon_1+\varepsilon_2}{\lambda_0/T}\right)^2 \ln^2 \sin\left(\dfrac{\pi S}{2T}\right)},$$

$$K^E_{\text{tr}} = \frac{4\sqrt{\varepsilon_1\varepsilon_2}\,/(\cos\theta_1\cos\theta_2)}{\left(\dfrac{\sqrt{\varepsilon_1}}{\cos\theta_1} + \dfrac{\sqrt{\varepsilon_2}}{\cos\theta_2}\right)^2 + \left(2\dfrac{\varepsilon_1+\varepsilon_2}{\lambda_0/T}\right)^2 \ln^2 \sin\left(\dfrac{\pi S}{2T}\right)}. \tag{26}$$

We note that for the orthogonal incidence of the electromagnetic *E*-wave on the strip conductor grating ($\theta_1 = 0°$ and $\theta_2 = 0°$) and for equal relative permittivities of materials ($\varepsilon_1 = \varepsilon_2$), Eqs. (25) coincide with the formulas derived in [19], where the calculation was performed only for this particular case.

2. *H*-WAVE INCIDENCE ON THE CONDUCTOR GRATING

We now consider the case of *H*-wave incidence on the strip conductor grating also parallel to the ***x*** axis (Fig. 2*a*) located in the ***xy*** plane at the interface between two media. The *H*-waves are also called *TE*-modes or *s*-polarized light. In this case, the nonzero tangential components of the incident wave are expressed by the formulas

$$E_x(y,z) = H_{\text{in}}\frac{Z_0}{\sqrt{\varepsilon_1}}\exp\left(ik_y y + ik_{1z}z\right),\;\; H_y(y,z) = H_{\text{in}}\cos\theta_1\exp\left(ik_y y + ik_{1z}z\right). \tag{27}$$

The incident wave with components (27) excites the longitudinal surface currents $J_x(y)$ on the strip conductors, which, in turn, form the near electromagnetic field of the grating uniform along the ***x*** axis.

The nonzero tangential components of the reflected wave at a great distance from the grating are expressed by the formulas

$$E_x(y,z) = -H_{\text{ref}} \frac{Z_0}{\sqrt{\varepsilon_1}} \exp\left(ik_y y - ik_{1z} z\right), \;\; H_y(y,z) = H_{\text{ref}} \cos\theta_1 \exp\left(ik_y y - ik_{1z} z\right), \tag{28}$$

and the components of the transmitted wave are expressed by the formulas

$$E_x(y,z) = H_{\text{tr}} \frac{Z_0}{\sqrt{\varepsilon_2}} \exp\left(ik_y y + ik_{2z} z\right), \;\; H_y(y,z) = H_{\text{tr}} \cos\theta_2 \exp\left(ik_y y + ik_{2z} z\right). \tag{29}$$

The magnetic component $\boldsymbol{H}$ in the near field of the plane grating must obey the two-dimensional Helmholtz equation

$$\left(\partial^2/\partial y^2 + \partial^2/\partial z^2 + k_\alpha^2\right)\boldsymbol{H}(y,z) = 0\,. \tag{30}$$

Since the problem is solved in the long-wavelength approximation ($\lambda >> T$), the Helmholtz equation [Eq. (30)] in the vicinity of the strip conductors ($|z| << \lambda$) can be replaced by the Laplace equation

$$\left(\partial^2/\partial y^2 + \partial^2/\partial z^2\right)\boldsymbol{H}(y,z) = 0\,. \tag{31}$$

This means that the magnetic field near the strip conductors can be described by the formula

$$H = -\operatorname{grad}\psi\,, \tag{32}$$

where ψ is the magnetostatic potential that obeys the equation

$$\left(\partial^2/\partial y^2 + \partial^2/\partial z^2\right)\psi(y,z) = 0\,. \tag{33}$$

The potential ψ, on the contrary to the potential φ, does not have mirror symmetry relative to the plane $z = 0$. So we seek a solution of Eq. (33) in the form

$$\psi(y,z) = \psi_{\text{e}}(y,z) - H_s z\,, \tag{34}$$

where the even component of the magnetic potential $\psi(y, z)$, denoted by $\psi_{\text{e}}(y, z)$, must obey the boundary conditions

$$\psi_{\text{e}}(y,z)\big|_{y=T/2} = \psi_0, \quad \psi_{\text{e}}(y,z)\big|_{z=0,\,0<y<S/2} = 0, \quad \psi_{\text{e}}(y,z)\big|_{y=0} = 0\,. \tag{35}$$

They are illustrated in Fig. 2 by arrows indicating the local direction of the vector gradψ_{e}.

To solve Eq. (34) with boundary conditions (35), we now perform conformal mapping of the semi-infinite strip shown by grey color and marked by *ABCDE* in Fig. 2*b* onto another semi-infinite strip marked by *A′B′C′D′E′* in Fig. 2*c* and located in the *w* plane. We put the complex number $w = u + iv$ in correspondence to each point with coordinates *u* and *v* in the *w* plane. With the help of the Schwarz–Christoffel integral, we find that the conformal mapping of the $\dot{z}$ area onto the *w* area is performed by the analytical function

$$w(\dot{z}) = \operatorname{arccosh}\left(\frac{\cosh(\pi\dot{z}/T)}{\cos(\pi S/(2T))}\right). \tag{36}$$

Then we obtain that the solution of Eq. (33) that obeys boundary conditions (35) is expressed by the formula

$$\psi(y,z) = (-1)^{\alpha} \frac{2}{\pi} \psi_0 \operatorname{Im} w(\dot{z}) - H_s \operatorname{Im} \dot{z}\,. \tag{37}$$

From the Maxwell equation we find the electric field

$$E_x(y,z) = -i\omega\mu_0 \frac{2}{\pi} \psi_0 \operatorname{Re}\left[w(\dot{z}) - \frac{\pi \dot{z}}{T} \right]. \tag{38}$$

Let us write the equations relating the components E_x and H_y of the incident, reflected, and transmitted waves with the corresponding components of the zero harmonics in the near field of the grating. They have the solution

$$\frac{H_{\mathrm{ref}}}{H_{\mathrm{in}}} = \frac{\sqrt{\varepsilon_2}\cos\theta_2 - \sqrt{\varepsilon_1}\cos\theta_1 - \dfrac{i\lambda_0/T}{\ln\cos(\pi S/(2T))}}{\sqrt{\varepsilon_2}\cos\theta_2 + \sqrt{\varepsilon_1}\cos\theta_1 - \dfrac{i\lambda_0/T}{\ln\cos(\pi S/(2T))}},$$
$$\frac{H_{\mathrm{tr}}}{H_{\mathrm{in}}} = \frac{2\sqrt{\varepsilon_2}\cos\theta_1}{\sqrt{\varepsilon_2}\cos\theta_2 + \sqrt{\varepsilon_1}\cos\theta_1 - \dfrac{i\lambda_0/T}{\ln\cos(\pi S/(2T))}}. \tag{39}$$

Then we derive formulas for the reflection and transmission coefficients of the incident power

$$K_{\mathrm{ref}}^{H} = \frac{\left(\sqrt{\varepsilon_1}\cos\theta_1 - \sqrt{\varepsilon_2}\cos\theta_2\right)^2 + (\lambda_0/T)^2 \ln^{-2}\cos(\pi S/(2T))}{\left(\sqrt{\varepsilon_1}\cos\theta_1 + \sqrt{\varepsilon_2}\cos\theta_2\right)^2 + (\lambda_0/T)^2 \ln^{-2}\cos(\pi S/(2T))},$$
$$K_{\mathrm{tr}}^{H} = \frac{4\sqrt{\varepsilon_1\varepsilon_2}\cos\theta_1\cos\theta_2}{\left(\sqrt{\varepsilon_1}\cos\theta_1 + \sqrt{\varepsilon_2}\cos\theta_2\right)^2 + (\lambda_0/T)^2 \ln^{-2}\cos(\pi S/(2T))}. \tag{40}$$

We note that in the case of orthogonal electromagnetic *H*-wave incidence on the strip conductor grating ($\theta_1 = 0°$ and $\theta_2 = 0°$) for equal relative permittivities of the materials ($\varepsilon_1 = \varepsilon_2$), Eqs. (39) coincide with formulas derived in [14] where calculation was performed for this particular case alone.

3. ANALYSIS OF THE RESULTS OBTAINED

Our studies of the behavior of the transmission and reflection coefficients of the *E*- and *H*-waves incident at an arbitrary angle on the one-dimensional strip conductor grating located at the interface between two dielectric media were carried out using analytical formulas (26) and (40) derived by us. Since the problem of scattering of electromagnetic waves has been solved for the grating of ideal conductors and lossless dielectrics, it is obvious that the sum of the reflection, K_{ref}, and transmission, K_{tr}, coefficients at any frequency of the incident wave regardless of its polarization is always equal to unity. Therefore, we present the results of our study considering only the behavior of the transmission coefficients.

The dependences of the transmission coefficients of *E*- and *H*-waves on the ratio of the grating period to the electromagnetic wavelength in the free space T/λ_0, which obviously is the normalized frequency, are shown in Fig. 3. As already mentioned above, the employed long-wavelength approximation provides sufficiently good accuracy of calculations when the grating period $T < 0.4\lambda$ [11]. Exactly this circumstance caused the choice of the limiting normalized frequency in this study. The calculation was performed for the orthogonal wave incidence ($\theta_1 = 0°$) and the

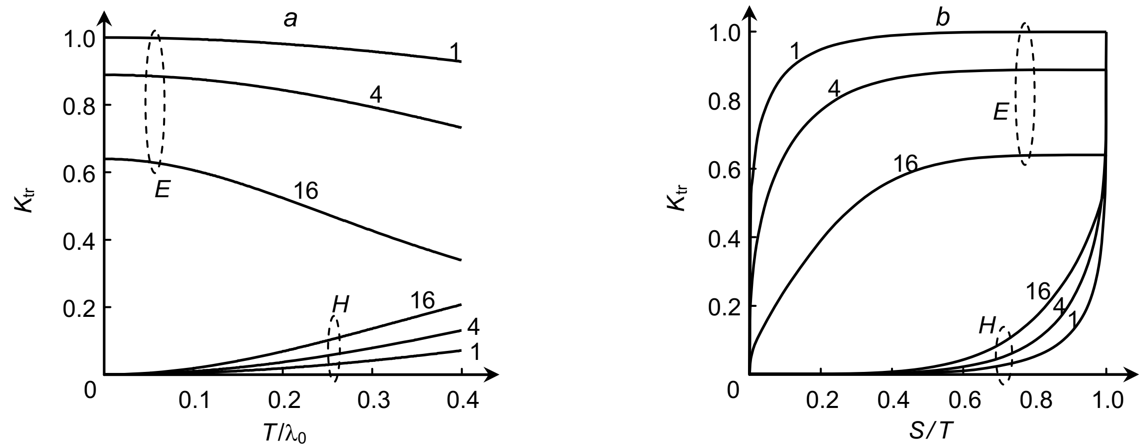

Fig. 3. Dependences of the transmission coefficients of *E*- and *H*-waves on the normalized frequency (*a*) and on the spacing between the conductors in the grating (*b*). The figures near the curves indicate values of the permittivity ε_2.

spacing between the conductors in the grating equal to the conductor width ($S/T = 0.5$), the relative permittivity of the first medium $\varepsilon_1 = 1$, and three values of the permittivity of the second medium $\varepsilon_2 = 1$, 4, and 16. We note that $\varepsilon_2 = 16$ is close to the maximum permittivity that materials with high Q-factor have in the optical range.

It can be seen that with increasing frequency, a monotonic decrease in the transmission coefficients of the *E*-wave and an increase in the transmission coefficients of the *H*-wave are observed. This is in complete agreement with the fact that the strip conductor grating exhibits the properties of an electric capacity connected in parallel to the transmission line with respect to the *E*-wave and demonstrates the properties of a parallel inductance for the *H*-wave [12]. The observed decrease in the transmission coefficient of *E*-polarized electromagnetic waves with increasing ε_2 is caused by the corresponding increase in their reflection coefficient from the interface between the two dielectric media due to an increase in the contrast between their wave impedances.

Figure 3*b* shows the dependences of the transmission coefficients of the *E*- and *H*-waves passed through the strip conductor grating under study on the relative width of the spacing between the conductors in the grating. Our calculations were performed for fixed normalized frequency $T/\lambda_0 = 0.1$, $\varepsilon_1 = 1$, three values of ε_2, and normal wave incidence ($\theta_1 = 0°$). It can be seen that as the spacing increases, the transmission coefficients of the *E*-waves quickly increase and then reach saturation whose level decreases with increasing ε_2, as already mentioned above, due to the increased wave reflection coefficient from the interface between the two dielectric media. The transmission coefficients of the *H*-waves first slowly increase with the spacing, because the inductance of the *wide* strip conductors is small, but for large spacings, the conductors become narrow, and their inductance quickly increases, thereby leading to the corresponding increase in K_{tr}.

Figure 4*a* shows the dependences of the transmission coefficients of the *E*- and *H*-waves on the relative permittivity of the second medium. Our calculations were performed for three spacings between the strip conductors in the grating ($S/T = 0.25$, 0.50, and 0.75), $T/\lambda_0 = 0.1$, $\varepsilon_1 = 1$, and $\theta_1 = 0$. It seems likely that the observed decrease of the transmission coefficients of the *E*-waves with increasing ε_2 is caused by the increased contrast between the wave impedances of two media, leading to the corresponding increased reflection coefficient from the interface. On the contrary, the increase of the transmission coefficient of *H*-waves is observed with increasing ε_2. It is also explained easily. Indeed, it has already been mentioned above that the strip conductor grating for the *H*-waves is equivalent to the parallel inductivity in the transmission line [12]. In this case, the level of the reflected electromagnetic waves, as is well known, decreases not only with increasing frequency (see Fig. 3*a*), but also with decreasing wave impedance of the transmission line observed with increasing ε_2.

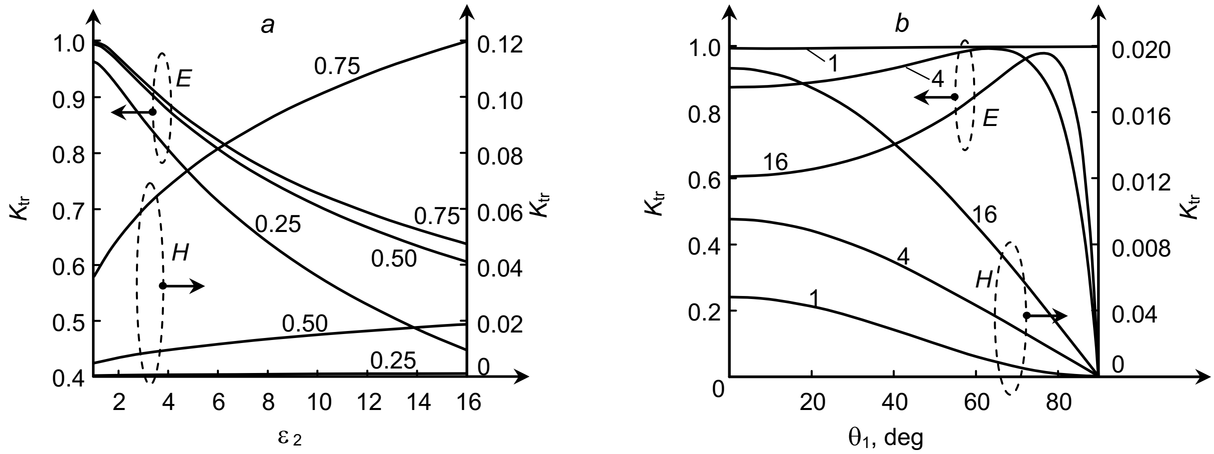

Fig. 4. Dependences of the transmission coefficients on the permittivity (figures near the curves indicate the relative spacings S/T) (*a*) and on the wave incidence angle (*b*) (here figures indicate ε_2).

Figure 4*b* shows the dependences of the transmission coefficients of the *E*- and *H*-waves on the angle θ_1 of wave incidence on the strip conductor grating. Our calculations were performed for three values of ε_2, $S/T = 0.50$, $\varepsilon_1 = 1$, and $T/\lambda_0 = 0.1$. The monotonic decrease of the transmission coefficient with increasing angle of *H*-wave incidence is caused by the increased wave impedance Z_1 inversely proportional to $\cos\theta_1$ (as follows from Eq. (27)) leading to the corresponding increase of the reflection coefficient. The increase of the transmission coefficient with ε_2 observed for any arbitrary angle of *H*-wave incidence is caused by the decrease of the reflection coefficient due to the decreased wave impedance of the medium. It can also be seen from Fig. 4*b* that the transmission maximum exists at certain angles of wave incidence, and the amplitude and position of this maximum depend on proportion between ε_1 and ε_2. Our study shows that the amplitude and position of the maximum also depend on the electromagnetic wave frequency and parameters of the grating. The angular dependences calculated for several spacings S/T between the strip conductors, $\varepsilon_1 = 1$, $\varepsilon_2 = 4$, and $T/\lambda_0 = 0.2$ shown in Fig. 5 confirm this fact. The existence of the transmission maximum and the regularities in its behavior are caused by the complicated dependences of the wave impedances of the first medium (Z_1) and of the second medium (Z_2) on the angle of *E*-wave incidence, its frequency, and the design parameters of the grating situated at the interface between the media. It is well known that in the absence of strip conductor grating at the interface between two media, the wave impedances

$$Z_1 = Z_0 \cos\theta_1 \big/ \sqrt{\varepsilon_1}\,,\ \ Z_2 = Z_0 \cos\theta_2 \big/ \sqrt{\varepsilon_2} \qquad (41)$$

depend only on the permittivities of the media and on the angle of *E*-wave incidence, as follows from Eq. (1). In this case, the position of the maximum observed under condition that $Z_1 = Z_2$ corresponds to the Brewster angle θ_B marked by the dashed straight line in Fig. 5. This can be easily found from Eq. (41) taking into account Snell's law (6):

$$\theta_B = \arctan\sqrt{\varepsilon_2/\varepsilon_1}\,. \qquad (42)$$

The dependences shown in Fig. 5 demonstrate that the angle at which the maximum transmission of the *E*-waves incident on the interface between two media in the presence of the strip conductor grating between them is always greater than the Brewster angle.

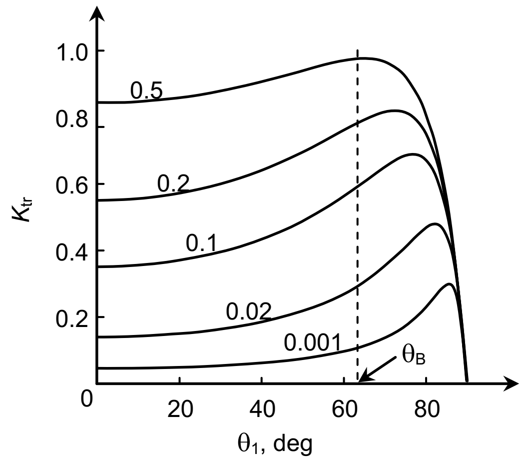

Fig. 5. Dependences of the transmission coefficients on the angle of *E*-wave incidence (figures near the curves indicate the relative spacings *S*/*T* in the grating).

CONCLUSIONS

Thus, the problem of scattering of the plane electromagnetic *E*- and *H*-waves incident at an arbitrary angle on the one-dimensional strip conductor grating located at the interface between two dielectric media has been solved. The analytical formulas were derived for the transmission and reflection coefficients of electromagnetic waves when the grating period is small in comparison with the wavelength. These formulas were used to study the regularities in the behavior of the transmission coefficients of the *E*- and *H*-waves passed through the considered structure attendant to variations of the permittivities of the media and the design parameters of the strip conductor grating.

The established possibilities and the revealed regularities in control over the reflectivity of the one-dimensional strip conductor grating in wide limits by changing the design parameters demonstrated prospects for its application as mirrors in multilayered dielectric bandpass filters. In these filters, the resonant dielectric layers forming the passband are separated by one-dimensional strip conductor gratings possessing the proper reflective characteristics rather than by conventional multilayered dielectric mirrors. In this case, only one high-Q material with arbitrary permittivity can be used, which is undoubtedly an important advantage of the proposed design. In addition, elimination of multilayered dielectric mirrors used, as a rule, in conventional optical filters significantly decreases the number of dielectric layers and hence improves their adaptability to manufacture.

This work was supported in part by the Ministry of Education and Science of the Russian Federation (Agreement No. 14.607.21.0039).

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