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Frequency comb generation for wave transmission through the nonlinear dimer

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Frequency comb generation for wave transmission through the nonlinear dimer

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Abstract. We study dynamical response of the nonlinear dimer relative to monochromatic wave injected via the waveguide. We show existence of a domain in space of frequency and injected amplitude where the stationary solutions of the time evolution equations do not exist. We present time dependent solutions which show that scattering waves carry multiple harmonics with frequencies spaced equidistantly.

In a series of publications [1, 2, 3, 4, 5, 6] time evolution of quantum nonlinear dimer was explicitly considered. The dimer is the simplest realization of discrete nonlinear Schrödinger equation and consists of two nonlinear sites. That case is exactly solvable with the solutions in the form of elliptic functions [2]. Observation of the closed nonlinear dimer implies application

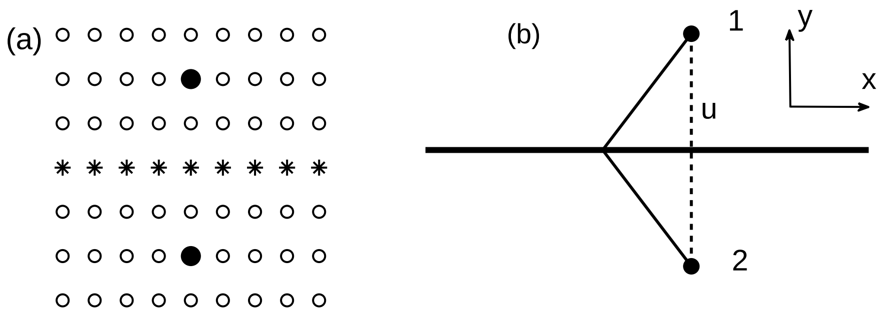

Figure 1. (a) Two identical microcavities made from a Kerr media marked by filled circles are inserted into the square lattice photonic crystal of dielectric rods with dielectric constant ϵ_0. The 1D waveguide is formed by substitution of linear chain of rods by the rods with dielectric constant $\epsilon_W + \epsilon_0$ marked by stars. (b) Schematic system consisting of a waveguide coupled with two off-channel microcavities.

of probing signal onto the system that opens the dimer. Respectively, dynamical equations describing the opened nonlinear dimer become non integrable that constitutes a main difference between the closed and opened nonlinear dimer. The opened quantum nonlinear dimer can be realized in the form of directional linear photonic crystal waveguide with two in-channel or off-channel Kerr microcavities as shown in Fig. 1 [7]. As dependent on engineering of openness the stationary states of the opened nonlinear dimer were studied in Refs. [8, 9, 10, 11, 12] to show a symmetry breaking phenomenon. There is a domain in space of frequency and injected power where the stationary solutions of the time evolution equations do not exist [7, 10]. Then one can expect that dynamical response of the nonlinear dimer will display features which can not be described by the stationary scattering theory. In particular injection of a monochromatic symmetric wave onto the nonlinear plaquette gives rise to emission of anti-symmetric satellite waves with frequencies different from the frequency of the incident wave [10]. This phenomenon is determined as frequency comb (FC) generation and studied in many linear and nonlinear systems [13, 14].

We write the equations for the dynamical evolution of the microcavity modes in the framework of the coupled mode theory (CMT) [7, 15] for the dimer shown in Fig. 1 (b):

$$\begin{aligned} i\dot{A}_1 &= (\omega_0 + \lambda|A_1|^2)A_1 - i\gamma A_1 + (u - i\gamma)A_2 + i\sqrt{\gamma}E_{in}e^{i\omega t}, \\ i\dot{A}_2 &= (\omega_0 + \lambda|A_2|^2)A_2 - i\gamma A_2 + (u - i\gamma)A_1 + i\sqrt{\gamma}E_{in}e^{i\omega t}. \end{aligned} \tag{1}$$

where $I_1 = |A_1|^2, I_2 = |A_2|^2$, $\sqrt{\gamma}$ is the coupling constant of the microcavities with probing wave with the amplitude E_{in} and frequency ω injected through the directional waveguide, u is the coupling constant between the microcavities due to overlapping. In the present paper we consider the case of uncoupled microcavities $u = 0$. Even in that simplest case dynamics of the nonlinear dimer effected by injected monochromatic wave remains cardinally different from the case of closed dimer because of interaction of the microcavities via the continuum. An amplitude of outgoing (transmitted) wave is given by equation [7, 15]

$$E_{out} = E_{in} - \sqrt{\gamma}(A_1 + A_2). \tag{2}$$

If to assume that the solution is stationary $A_j(t) = A_{j0}e^{i\omega t}$ the CMT equations (1) are simplified to be equal

$$\begin{aligned} (\nu - \lambda|A_{10}|^2)A_{10} + i\gamma(A_{10} + A_{20}) &= i\sqrt{\gamma}E_{in} \\ (\nu - \lambda|A_{20}|^2)A_{20} + i\gamma(A_{10} + A_{20}) &= i\sqrt{\gamma}E_{in} \end{aligned} \tag{3}$$

where $\nu = \omega - \omega_0$. It was shown the system bifurcates from the symmetry preserving state $A_{10} = A_{20}$ to the symmetry breaking state $|A_{10}| \neq |A_{20}|, \theta_1 - \theta_2 = 0, \pi$ where $A_{j0} = |A_{j0}|\exp(i\theta_j), j = 1, 2$ with variation of E_{in} [7], similar to the closed nonlinear dimer [1, 3, 4]. The symmetry preserving solution has the form

$$A_{10} = A_{20} = A_0 = \frac{i\sqrt{\gamma}E_{in}}{\nu - \lambda|A_0|^2 + 2i\gamma} \tag{4}$$

that defines the equation for the stationary intensity $I_0 = |A_0|^2$

$$I_0[(\nu - \lambda I_0)^2 + 4\gamma^2] = \gamma E_{in}^2. \tag{5}$$

However as different from the closed nonlinear dimer the opened dimer can transit into the phase symmetry breaking state with $|A_{10}| = |A_{20}|$ but $\theta_1 - \theta_2 \neq 0, \pi$ [7, 10].

Analysis of stability of the stationary solutions relative to small perturbations was performed in Ref. [7] and revealed a domain in space of parameters ω and E_{in} where *all* stationary solutions are unstable. Similar result was found in the opened nonlinear off-channel microcavity [16] and of four nonlinear microcavities [10]. We apply a standard procedure of small perturbations to find domains in space of physical parameters where the stationary solutions defined by Eq. (3) are stable [17]:

$$A_j(t) = [A_{j0} + a_j(t)]e^{i\omega t}, \quad j = 1, 2. \tag{6}$$

Presenting small perturbations $a_j(t) = (x_j + iy_j)e^{-\mu t}$ we obtain the following equations for eigenfrequencies μ

$$(\mu - 2\gamma)^2 = -(\nu - \lambda I_0)(\nu - 3\lambda I_0), \ \mu^2 = -(\nu - \lambda I_0)(\nu - 3\lambda I_0). \tag{7}$$

Therefore the boundary where the symmetry preserving family of solutions becomes unstable is defined by equations $\nu - \lambda I_c = 0$ and $\nu - 3\lambda I_c = 0$. Substituting these boundary values I_c into Eq. (5) we find a domain where stationary solutions do not exist

$$E_{in}^2 = \frac{4\gamma\nu}{\lambda}, \quad E_{in}^2 = \frac{4\nu}{3\gamma\lambda}[\gamma^2 + \frac{\nu^2}{9}]. \tag{8}$$

shown in Fig. 2.

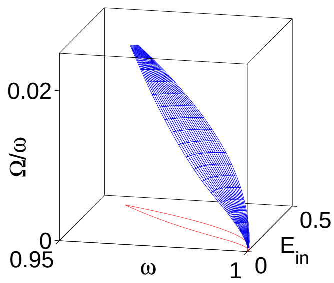

Figure 2. (Color online) The frequency comb generated frequency Ω vs amplitude E_{in} and frequency ω of injected monochromatic wave. The parameters of the dimer: $\omega_0 = 1, \gamma = 0.01, \lambda = -0.01$.

Numerics shows that in this domain the solution of the CMT equations (1) evolves into the periodical one after long time. One can see from Eq. (1) that there is a symmetry relative to a half of period and permutation of the microcavities:

$$A_1(t + T/2) = A_2(t), A_2(t + T/2) = A_1(t). \tag{9}$$

In Fig. 3 (a) and Fig. 3 (b) we present results of numerical simulations of Eq. (1) in the domain of unstable stationary solutions which indeed demonstrate the symmetry (9). Fig. 3 (c) shows that the oscillations of the cavities are nonlinear and include many Fourier harmonics $A_j(t) \approx \exp(i\omega t)\sum_n a_n \exp(in\Omega_c t)$ that demonstrates the FC effect in Fig. 3 (d). Moreover one can see that the time evolution of amplitudes $a_j(t)$ are not symmetrical in time, i.e., there is no any time t_0 that $A_j(t_0 + t) = A_j(t_0 - t)$. Absence of the time reversal symmetry is also demonstrated in Fig. 3 (c) where the oscillations of the real and imaginary parts of the amplitudes $A_j(t)$ show nonsymmetric behavior.

We have chosen the parameters listed in caption of Fig. 3 in correspondence to the photonic crystal system considered in Ref. [7]. Fig. 3 (e) and Fig. 3 (f) demonstrate similar FC effect in the transmittance amplitude $E_{out}(t)/E_{in} \approx \exp(i\omega t)\sum_n t_n \exp(2in\Omega_c t)$ calculated by Eq. (2). The Fourier components for the site amplitudes and transmittance in Figs. 3(d) and 3 (f) show that $a_n \neq a_{-n}$ and $t_n \neq t_{-n}$ that is the result of breaking of the time reversal symmetry considered above. Respectively Fig. 3 (c) shows breaking of symmetry relative to the complex conjugation of the site amplitudes.

Thus, probing of the nonlinear quantum dimer by monochromatic wave with the frequency ω and the amplitude E_{in} revealed a domain where the stationary solutions does not exist. In this domain numerical solution of the CMT equations show the following results: (i) the time reversal

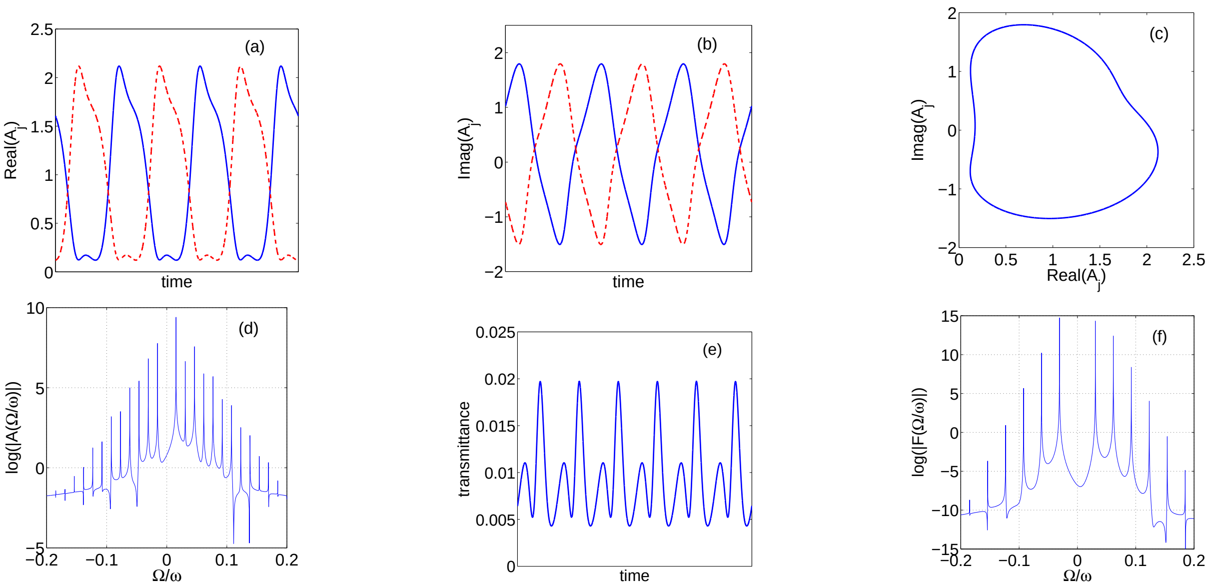

Figure 3. (Color online) Time evolution of site amplitudes $A_j(t), j = 1, 2$ (a), (b) and (c). The Fourier transform $|A(\Omega/\omega)|$ in logarithmic scale (d) with the characteristic frequency $\Omega_c \approx 0.0154\omega$. Time evolution of the transmittance $|E_{out}(t)/E_{in}|^2$ (e) and the Fourier transform $F(\Omega/\omega)$ of the transmittance amplitude $[E_{out}(t)/E_{in}]\exp(-i\omega t)$ in logarithmic scale (f). The parameters: $\omega_0 = 1, \omega = 0.98, \gamma = 0.04, E_{in} = 0.4, \lambda = -0.01$

symmetry is broken to give rise to the ratchet effect, (ii) the time evolution of the sites displays the frequency comb (FC) effect, i.e., probing wave transmitted through the nonlinear dimer acquires many harmonics spaced equidistantly by the frequency $\Omega_c \ll \omega$, (iii) the frequency of harmonics Ω_c is goes down with decreasing of the probing signal amplitude E_{in} as shown in Fig. 2. Therefore the FC frequency generated by the dimer can be governed by injected wave.

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