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ORIGINAL PAPER

Uniform Impurity Scattering in Two-Band $s_\pm$ and s_{++} Superconductors

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Abstract The $s_\pm$ and s_{++} models for the superconducting state are subject of intense studies regarding Fe-based superconductors. Depending on the parameters, disorder may leave intact or suppress T_c in these models. Here, we study the special case of disorder with equal values of intra- and interband impurity potentials in the two-band $s_\pm$ and s_{++} models. We show that this case can be considered as an isolated point and T_c there has maximal damping for a wide range of parameters.

Keywords Multiband superconductivity · Impurity scattering · Fe-based superconductors

1 Introduction

Fe-based materials—pnictides and chalcogenides—represent a new class of unconventional superconductors with high transition temperatures [1–8]. While the mechanism of superconductivity is still a mystery, the main candidates are spin or orbital fluctuations. Except for the extreme hole and electron dopings, the Fermi surface consists of two or three hole pockets around the $\Gamma = (0, 0)$ point and two electron pockets around the $M = (\pi, \pi)$ point in the 2-Fe Brillouin zone. Scattering between them with the large wave vector results in the enhanced antiferromagnetic fluctuations, which promote the $s_\pm$ type of the superconducting order parameter that change sign between electron and hole pockets [2–8]. On the other hand, bands near the Fermi level have mixed orbital content and orbital fluctuations enhanced by the electron-phonon interaction may lead to the sign-preserving s_{++} state [9, 10]. However, most experimental data including observation of a spin-resonance peak in inelastic neutron scattering, the quasiparticle interference in tunneling experiments, and the NMR spin-lattice relaxation rate are in favor of the $s_\pm$ scenario [2–8].

The $s_\pm$ and s_{++} states are expected to behave differently subject to the disorder [11–21]. In general, s_{++} ($s_\pm$) state should be stable (fragile) against a scattering on a nonmagnetic impurities [11–15]. Detailed studies revealed that T_c stays finite in the presence of nonmagnetic disorder in the following cases: (i) s_{++} state [16, 17], (ii) $s_\pm \to s_{++}$ transition for the sizeable intraband attraction in the two-band $s_\pm$ model in the strong-coupling $\mathcal{T}$-matrix approximation [18] and via the numerical solutions of the Bogoliubov-de Gennes equations [22, 23], and (iii) an unitary limit [24]. Magnetic impurities leave T_c finite [21] in the case of (1) $s_\pm$ superconductor with the purely interband impurity scattering, (2) s_{++} state with the purely interband scattering due to the $s_{++} \to s_\pm$ transition, and (3) the unitary limit for both s_{++} and $s_\pm$ states independent on the exact form of the impurity potential. But even if T_c is suppressed, its behavior may differ from the Abrikosov-Gor'kov (AG) theory for the single-band superconductors [11, 12], which state that T_c is determined by

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the expression $\ln T_{c0}/T_c = \Psi(1/2+\Gamma/2\pi T_c) - \Psi(1/2)$, where $\Psi(x)$ is the digamma function, Γ is the impurity scattering rate, and T_{c0} is the critical temperature in the absence of impurities [11, 12].

The choice of the "proper" theory for disorder effects in iron-based materials is severely complicated by the fact that the exact form of the impurity potential is not known. In such a situation, it is instructive to theoretically explore as many situations as possible. Here, we focus on a special case of a uniform impurity potential with equal intra- and interband components. We consider two-band models for the isotropic $s_\pm$ and s_{++} superconductors with either nonmagnetic or magnetic impurities within the self-consistent $\mathcal{T}$-matrix approximation following approach from Refs. [18, 21].

2 General Equations and their Analysis

We employ the Eliashberg approach for multiband superconductors [25] and calculate the ξ-integrated Green's functions $\hat{\mathbf{g}}(\omega_n) = \int d\xi \hat{\mathbf{G}}(\mathbf{k},\omega_n) = \begin{pmatrix} \hat{g}_{an} & 0 \\ 0 & \hat{g}_{bn}\end{pmatrix}$, where $\xi_{\alpha,\mathbf{k}} = \mathbf{v}_{\alpha,F}(\mathbf{k}-\mathbf{k}_{\alpha,F})$ is the linearized dispersion, $\mathbf{k}_{\alpha,F}$ is the Fermi momentum, $\hat{g}_{\alpha n} = g_{0\alpha n}\hat{\tau}_0\otimes\hat{\sigma}_0 + g_{2\alpha n}\hat{\tau}_2\otimes\hat{\sigma}_2$, indices a and b correspond to two distinct bands, index $\alpha = a, b$ denote the band space, Pauli matrices define Nambu ($\hat{\tau}_i$), and spin ($\hat{\sigma}_i$) spaces, $\hat{\mathbf{G}}(\mathbf{k},\omega_n) = \left[\hat{\mathbf{G}}_0^{-1}(\mathbf{k},\omega_n) - \hat{\Sigma}(\omega_n)\right]^{-1}$ is the matrix Green's function for a quasiparticle with momentum $\mathbf{k}$ and the Matsubara frequency $\omega_n = (2n+1)\pi T$ defined in the band space and in the combined Nambu and spin spaces, $\hat{G}_0^{\alpha\beta}(\mathbf{k},\omega_n) = [\mathrm{i}\omega_n\hat{\tau}_0\otimes\hat{\sigma}_0 - \xi_{\alpha\mathbf{k}}\hat{\tau}_3\otimes\hat{\sigma}_0]^{-1}\delta_{\alpha\beta}$ is the bare Green's function, $\hat{\Sigma}(\omega_n) = \sum_{i=0}^{3}\Sigma_{\alpha\beta}^{(i)}(\omega_n)\hat{\tau}_i$ is the self-energy matrix, $g_{0\alpha n}$ and $g_{2\alpha n}$ are the normal and anomalous ξ-integrated Nambu Green's functions,

$$g_{0\alpha n} = -\frac{\mathrm{i}\pi N_\alpha \tilde{\omega}_{\alpha n}}{\sqrt{\tilde{\omega}_{\alpha n}^2 + \tilde{\phi}_{\alpha n}^2}}, \quad g_{2\alpha n} = -\frac{\pi N_\alpha \tilde{\phi}_{\alpha n}}{\sqrt{\tilde{\omega}_{\alpha n}^2 + \tilde{\phi}_{\alpha n}^2}}, \tag{1}$$

depending on the density of states per spin of the corresponding band at the Fermi level $N_{a,b}$ and on renormalized (by the self-energy) order parameter $\tilde{\phi}_{\alpha n}$ and frequency $\tilde{\omega}_{\alpha n}$,

$$\mathrm{i}\tilde{\omega}_{an} = \mathrm{i}\omega_n - \Sigma_{0a}(\omega_n) - \Sigma_{0a}^{\mathrm{imp}}(\omega_n), \tag{2}$$
$$\tilde{\phi}_{an} = \Sigma_{2a}(\omega_n) + \Sigma_{2a}^{\mathrm{imp}}(\omega_n). \tag{3}$$

It is also convenient to introduce the renormalization factor $Z_{\alpha n} = \tilde{\omega}_{\alpha n}/\omega_n$ that enters the gap function $\Delta_{\alpha n} = \tilde{\phi}_{\alpha n}/Z_{\alpha n}$. The self-energy due to the spin fluctuation interaction is then given by

$$\Sigma_{0\alpha}(\omega_n) = T\sum_{\omega_n',\beta}\lambda_{\alpha\beta}^z(n-n')\frac{g_{0\beta n}}{N_\beta}, \tag{4}$$
$$\Sigma_{2\alpha}(\omega_n) = -T\sum_{\omega_n',\beta}\lambda_{\alpha\beta}^\phi(n-n')\frac{g_{2\beta n}}{N_\beta}, \tag{5}$$

The coupling functions $\lambda_{\alpha\beta}^{\phi,z}(n-n') = 2\lambda_{\alpha\beta}^{\phi,z}\int\limits_0^\infty \frac{d\Omega\Omega B(\Omega)}{(\omega_n-\omega_{n'})^2+\Omega^2}$ depend on the normalized bosonic spectral function $B(\Omega)$ used in Refs. [18, 19]. While the matrix elements $\lambda_{\alpha\beta}^\phi$ can be positive (attractive) as well as negative (repulsive) due to the interplay between spin fluctuations and electron-phonon coupling [26, 27], the matrix elements $\lambda_{\alpha\beta}^z$ are always positive. For simplicity, we set $\lambda_{\alpha\beta}^z = |\lambda_{\alpha\beta}^\phi| \equiv |\lambda_{\alpha\beta}|$ and neglect possible $\mathbf{k}$-space anisotropy in each order parameter $\tilde{\phi}_{\alpha n}$.

We use the $\mathcal{T}$-matrix approximation to calculate the average impurity self-energy $\hat{\Sigma}^{\mathrm{imp}}$:

$$\hat{\Sigma}^{\mathrm{imp}}(\omega_n) = n_{\mathrm{imp}}\hat{\mathbf{U}} + \hat{\mathbf{U}}\hat{\mathbf{g}}(\omega_n)\hat{\Sigma}^{\mathrm{imp}}(\omega_n), \tag{6}$$

where n_{imp} is the impurity concentration.

2.1 Nonmagnetic Impurities

First, we consider the nonmagnetic disorder. Impurity potential matrix entering equation (6) is defined as $\hat{\mathbf{U}} = \mathbf{U}\otimes\hat{\tau}_3$, where $(\mathbf{U})_{\alpha\beta} = \mathcal{U}_{\mathbf{R}_i}^{\alpha\beta}$ with $\mathbf{R}_i = 0$ is the impurity site. For simplicity, we set intra- and interband parts of the potential equal to v and u, respectively, so that $(\mathbf{U})_{\alpha\beta} = (v-u)\delta_{\alpha\beta} + u$. The relation between the two will be controlled by the parameter η: $v = u\eta$.

Apart from the general case, later we are going to examine the two important limiting cases: Born limit (weak scattering) with $\pi u N_{a,b} \ll 1$ and the opposite case of a very strong impurity scattering (unitary limit) with $\pi u N_{a,b} \gg 1$.

It is useful to introduce the generalized scattering cross-section

$$\sigma = \frac{\pi^2 N_a N_b u^2}{1+\pi^2 N_a N_b u^2} \to \begin{cases} 0, \text{Born} \\ 1, \text{unitary}\end{cases} \tag{7}$$

and the impurity scattering rate

$$\Gamma_{a,b} = \frac{2n_{\mathrm{imp}}\sigma}{\pi N_{a,b}} \to \begin{cases} 2n_{\mathrm{imp}}\pi N_{b,a}u^2, \text{Born} \\ 2n_{\mathrm{imp}}/\left(\pi N_{a,b}\right), \text{unitary}\end{cases} \tag{8}$$

Then equations on frequency (2) and order parameter (3) become

$$\tilde{\omega}_{an} = \omega_n + \mathrm{i}\Sigma_{0a}(\omega_n) + \frac{\Gamma_a}{2D}\left[\sigma\frac{\tilde{\omega}_{an}}{Q_{an}}(1-\eta^2)^2 + (1-\sigma)\left(\frac{N_a\tilde{\omega}_{an}}{N_b Q_{an}}\eta^2 + \frac{\tilde{\omega}_{bn}}{Q_{bn}}\right)\right], \tag{9}$$

$$\tilde{\phi}_{an} = \Sigma_{2a}(\omega_n) + \frac{\Gamma_a}{2D}\left[\sigma\frac{\tilde{\phi}_{an}}{Q_{an}}(1-\eta^2)^2 + (1-\sigma)\left(\frac{N_a\tilde{\phi}_{an}}{N_b Q_{an}}\eta^2 + \frac{\tilde{\phi}_{bn}}{Q_{bn}}\right)\right]. \tag{10}$$

where $Q_{\alpha n} = \sqrt{\tilde{\omega}_{\alpha n}^2 + \tilde{\phi}_{\alpha n}^2}$, $D = (1-\sigma)^2 + \sigma(1-\sigma)\left(2\frac{\tilde{\omega}_{an}\tilde{\omega}_{bn}+\tilde{\phi}_{an}\tilde{\phi}_{bn}}{Q_{an}Q_{bn}} + \frac{N_a^2+N_b^2}{N_a N_b}\eta^2\right) + \sigma^2(1-\eta^2)^2$.

Let's consider the main limits. Since in the Born approximation $\sigma \to 0$, then $D = 1$, $\Gamma_a = 2n_{\mathrm{imp}}\pi N_b u^2$, and

$$\tilde{\omega}_{an} = \omega_n + \mathrm{i}\Sigma_{0a}(\omega_n) + \frac{\gamma_{aa}}{2}\frac{\tilde{\omega}_{an}}{Q_{an}} + \frac{\gamma_{ab}}{2}\frac{\tilde{\omega}_{bn}}{Q_{bn}}, \tag{11}$$

$$\tilde{\phi}_{an} = \Sigma_{2a}(\omega_n) + \frac{\gamma_{aa}}{2}\frac{\tilde{\phi}_{an}}{Q_{an}} + \frac{\gamma_{ab}}{2}\frac{\tilde{\phi}_{bn}}{Q_{bn}}, \tag{12}$$

where $\gamma_{aa} = 2\pi n_{\mathrm{imp}} N_a u^2 \eta^2$ and $\gamma_{ab} = 2\pi n_{\mathrm{imp}} N_b u^2$. Obviously, for the finite interband scattering γ_{ab} (i.e., finite η), different bands are mixed in equations. This leads to the AG-like suppression of T_c.

In the unitary limit $\sigma \to 1$, $\Gamma_a = 2n_{\mathrm{imp}}/(\pi N_a)$, and we have to consider two cases.

I). Uniform impurity potential with $\eta = 1$:

$$\tilde{\omega}_{an} = \omega_n + \mathrm{i}\Sigma_{0a}(\omega_n) + \frac{n_{\mathrm{imp}}}{\pi N_a N_b D_{uni}}\left[N_a\frac{\tilde{\omega}_{an}}{Q_{an}} + N_b\frac{\tilde{\omega}_{bn}}{Q_{bn}}\right],$$

$$\tilde{\phi}_{an} = \Sigma_{2a}(\omega_n) + \frac{n_{\mathrm{imp}}}{\pi N_a N_b D_{uni}}\left[N_a\frac{\tilde{\phi}_{an}}{Q_{an}} + N_b\frac{\tilde{\phi}_{bn}}{Q_{bn}}\right],$$

where $D_{uni} = 2\frac{\tilde{\omega}_{an}\tilde{\omega}_{bn}+\tilde{\phi}_{an}\tilde{\phi}_{bn}}{Q_{an}Q_{bn}} + \frac{N_a^2+N_b^2}{N_a N_b}$. Again, different bands are mixed so we have a suppression of T_c.

II). All other cases with $\eta \neq 1$:

$$\tilde{\omega}_{an} = \omega_n + \mathrm{i}\Sigma_{0a}(\omega_n) + \frac{n_{\mathrm{imp}}}{\pi N_a}\frac{\tilde{\omega}_{an}}{Q_{an}}, \tag{13}$$

$$\tilde{\phi}_{an} = \Sigma_{2a}(\omega_n) + \frac{n_{\mathrm{imp}}}{\pi N_a}\frac{\tilde{\phi}_{an}}{Q_{an}}. \tag{14}$$

We get the same result, as for the intraband impurities since the other band (b) does not contribute to the equations. Surprisingly, but here the Anderson theorem works independent of the gap signs in different bands. Thus, T_c should be finite for arbitrary impurity concentration.

Here, we conclude that there is a special case of T_c suppression in the unitary limit for the uniform impurity potential $\eta = 1$. Such situation arise due to the structure of the denominator D in equations (9)–(10). It vanishes for $\eta = \sigma = 1$ and one has to accurately take the limit $\eta \to 1$ first and only then put $\sigma \to 1$. It is the $\eta = 1$ case, that was considered in Ref. [10]. For all other values of η (even for a slight difference between intra- and interband potentials), impurities are not going to affect the critical temperature.

2.2 Magnetic Impurities

Now, we switch to the magnetic disorder. Impurity potential for the non-correlated impurities can be written as $\hat{\mathbf{U}} = \mathbf{V}\otimes\hat{S}$, where $\hat{S} = \mathrm{diag}\left[\hat{\sigma}\cdot\mathbf{S}, -(\hat{\sigma}\cdot\mathbf{S})^T\right]$ is the 4×4 matrix with $(...)^T$ being the matrix transpose and $\mathbf{S} = (S_x, S_y, S_z)$ being the spin vector [28]. The vector $\hat{\sigma}$ is composed of τ matrices, $\hat{\sigma} = (\hat{\tau}_1, \hat{\tau}_2, \hat{\tau}_3)$. The potential strength is determined by $(\mathbf{V})_{\alpha\beta} = V^{\alpha\beta}_{\mathbf{R}_i=0}$. For simplicity, intraband and interband parts of the potential are set equal to $\mathcal{I}$ and $\mathcal{J}$, respectively, such that $(\mathbf{V})_{\alpha\beta} = (\mathcal{I}-\mathcal{J})\delta_{\alpha\beta} + \mathcal{J}$. Components of the impurity potential matrix $\hat{\mathbf{U}}$ is then $\hat{U}_{aa,bb} = \mathcal{I}\hat{S}$ and $\hat{U}_{ab,ba} = \mathcal{J}\hat{S}$. We introduce the parameter η to control the ratio of intra- and interband scattering potentials, so that $\mathcal{I} = \mathcal{J}\eta$. Coupled $\mathcal{T}$-matrix equations for aa and ba components of the self-energy become

$$\hat{\Sigma}_{aa}^{\mathrm{imp}} = n_{\mathrm{imp}}\hat{U}_{aa} + \hat{U}_{aa}\hat{g}_a\hat{\Sigma}_{aa}^{\mathrm{imp}} + \hat{U}_{ab}\hat{g}_b\hat{\Sigma}_{ba}^{\mathrm{imp}}, \tag{15}$$

$$\hat{\Sigma}_{ba}^{\mathrm{imp}} = n_{\mathrm{imp}}\hat{U}_{ba} + \hat{U}_{ba}\hat{g}_a\hat{\Sigma}_{aa}^{\mathrm{imp}} + \hat{U}_{bb}\hat{g}_b\hat{\Sigma}_{ba}^{\mathrm{imp}}. \tag{16}$$

Renormalizations of frequencies and gaps come from $\Sigma_{0a}^{\mathrm{imp}} = \frac{1}{4}\mathrm{Tr}\left[\hat{\Sigma}_{aa}^{\mathrm{imp}}\cdot(\hat{\tau}_0\otimes\hat{\sigma}_0)\right]$ and $\Sigma_{2a}^{\mathrm{imp}} = \frac{1}{4}\mathrm{Tr}\left[\hat{\Sigma}_{aa}^{\mathrm{imp}}\cdot(\hat{\tau}_2\otimes\hat{\sigma}_2)\right]$.

Expressions for $\Sigma_{0\alpha}^{\mathrm{imp}}$ and $\Sigma_{2\alpha}^{\mathrm{imp}}$ are proportional to the effective impurity scattering rate $\Gamma_{a,b}$ and as in the case of nonmagnetic impurities contain the generalized cross-section parameter σ that helps to control the approximation for the impurity strength ranging from Born (weak scattering, $\pi\mathcal{J}N_{a,b} \ll 1$) to the unitary (strong scattering, $\pi\mathcal{J}N_{a,b} \gg 1$) limits,

$$\Gamma_{a,b} = \frac{2n_{\mathrm{imp}}\sigma}{\pi N_{a,b}} \to \begin{cases} 2\pi\mathcal{J}^2 s^2 n_{\mathrm{imp}} N_{b,a}, \text{Born} \\ \frac{2n_{\mathrm{imp}}}{\pi N_{a,b}}, \text{unitary} \end{cases} \tag{17}$$

$$\sigma = \frac{\pi^2\mathcal{J}^2 s^2 N_a N_b}{1+\pi^2\mathcal{J}^2 s^2 N_a N_b} \to \begin{cases} 0, \text{Born} \\ 1, \text{unitary} \end{cases} \tag{18}$$

We assume that spins are not polarized and $s^2 = \langle S^2\rangle = S(S+1)$. Since s enters all equations only in conjunction with $\mathcal{I}$ or $\mathcal{J}$, without loosing generality we set $s = 1$ assuming that $\mathcal{I}$ and $\mathcal{J}$ are both renormalized by s.

For the uniform impurity potential $\eta = 1$ in the Born limit $\sigma = 0$, we find

$$\tilde{\omega}_{an} = \omega_n + \mathrm{i}\Sigma_{0a}(\omega_n) + \pi \mathcal{J}^2 n_{\mathrm{imp}} \left(N_a \frac{\tilde{\omega}_{an}}{Q_{an}} + N_b \frac{\tilde{\omega}_{bn}}{Q_{bn}} \right),$$

$$\tilde{\phi}_{an} = \Sigma_{2a}(\omega_n) - \pi \mathcal{J}^2 n_{\mathrm{imp}} \left(N_a \frac{\tilde{\phi}_{an}}{Q_{an}} + N_b \frac{\tilde{\phi}_{bn}}{Q_{bn}} \right).$$

Here, the contribution from both a and b bands is mixed so we expect a suppression of T_c by disorder.

In the unitary limit ($\sigma = 1$) at $T \to T_c$, we have $\tilde{\omega}_{an} = \omega_n + \mathrm{i}\Sigma_{0a}(\omega_n) + \frac{\Gamma_a}{2}\mathrm{sgn}(\omega_n)$ and $\tilde{\phi}_{an} = \Sigma_{2a}(\omega_n) + \frac{\Gamma_a}{2}\frac{\tilde{\phi}_{an}}{|\tilde{\omega}_{an}|}$ for any value of η including the case of intraband-only impurities, $1/\eta = 0$. This form is the same as for nonmagnetic impurities and thus analogously to the Anderson theorem there is no impurity contribution to the T_c equation. The only exception here is the special case of uniform impurities, $\eta = 1$, when

$$\tilde{\omega}_{an} = \omega_n + \mathrm{i}\Sigma_{0a}(\omega_n) + \frac{n_{\mathrm{imp}}}{\pi\,(N_a + N_b)}\mathrm{sgn}(\omega_n),$$

$$\tilde{\phi}_{an} = \Sigma_{2a}(\omega_n) + \frac{n_{\mathrm{imp}}}{\pi\,(N_a + N_b)^2} \left(N_a \frac{\tilde{\phi}_{an}}{|\tilde{\omega}_{an}|} + N_b \frac{\tilde{\phi}_{bn}}{|\tilde{\omega}_{bn}|} \right).$$

Both gaps are mixed in equation for $\tilde{\phi}_{an}$, thus they tend to zero with increasing amount of disorder. That's also true away from the unitary limit and that's why there is a special case of uniform potential of the impurity scattering, $\mathcal{I} = \mathcal{J}$, when the strongest T_c suppression occurs.

3 Numerical Results

Following results were obtained by solving self-consistently frequency and gap equations (2)–(3) for both finite temperature and at T_c with the impurity self-energy as in (9)–(10) for the nonmagnetic disorder or from the solution of (15)–(16) for the magnetic impurities. For definiteness, we choose $N_b/N_a = 2$ and coupling constants to be $(\lambda_{aa}, \lambda_{ab}, \lambda_{ba}, \lambda_{bb}) = (3, 0.2, 0.1, 0.5)$ for the s_{++} state and $(3, -0.2, -0.1, 0.5)$ for the $s_\pm$ state with $\langle\lambda\rangle < 0$.

Typical results [18, 21] of the dependence on the impurity scattering rate Γ_a for the critical temperature T_c and gaps $\Delta_{a,bn}$ for the first Matsubara frequency $\omega_{n=1} = 3\pi T$ are shown in Fig. 1 (nonmagnetic) and in Fig. 2 (magnetic disorder). Scattering on magnetic impurities suppress both $s_\pm$ and s_{++} states due to the finite interband scattering component. The s_{++} state initially transforms to the $s_\pm$ state, but then follows its fate with increasing Γ_a. The only exception is the unitary limit. On the other hand, both states survive the nonmagnetic disorder but for different reasons: the s_{++} due to the Anderson theorem, while the $s_\pm$ state transforms to the s_{++}. The unitary limit, again, gives constant result.

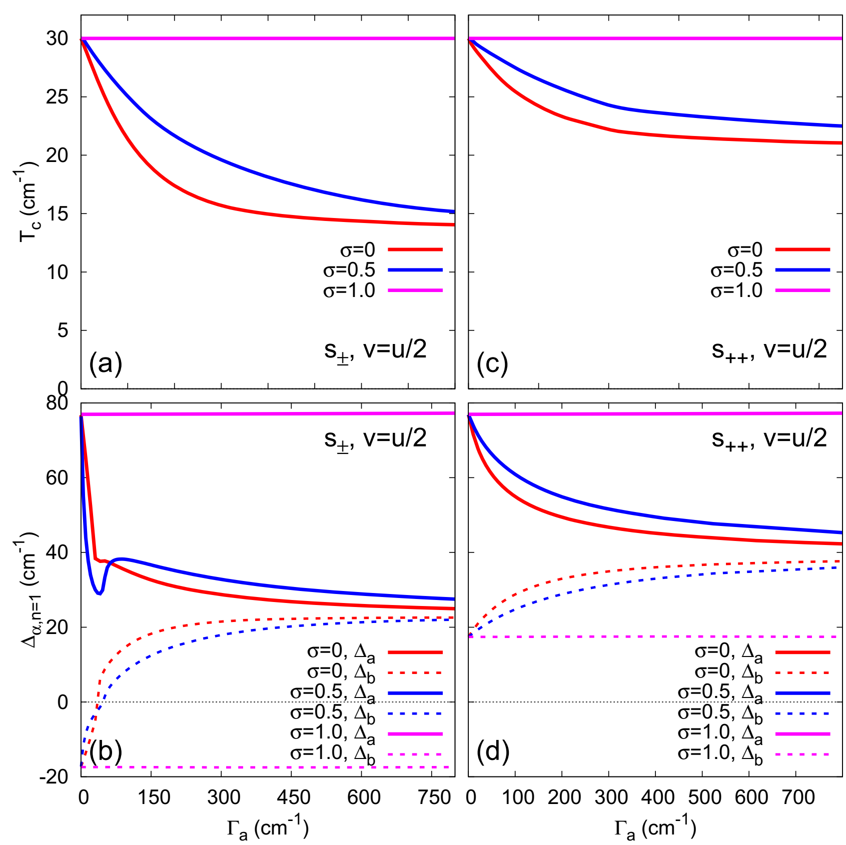

Fig. 1 T_c (**a** and **c**) and Matsubara gap $\Delta_{\alpha n=1}$ (**b** and **d**) dependence on the nonmagnetic scattering rate Γ_a for the $s_\pm$ (**a** and **b**) and the s_{++} (**c** and **d**) superconductors with $\eta = 1/2$

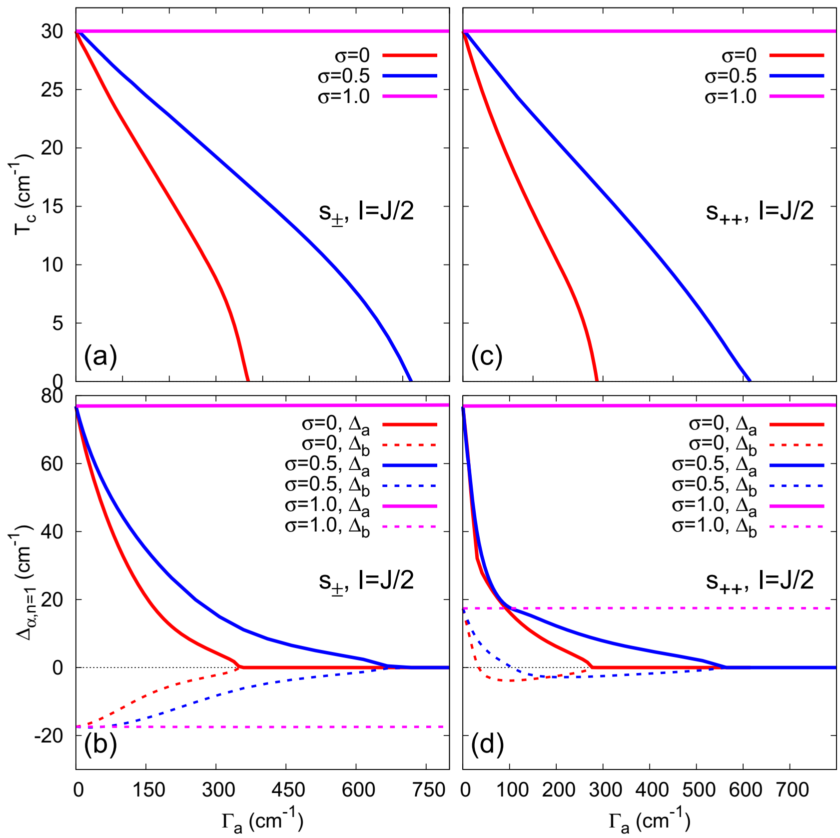

Fig. 2 T_c (**a** and **c**) and Matsubara gap $\Delta_{\alpha n=1}$ (**b** and **d**) dependence on the magnetic scattering rate Γ_a for the $s_\pm$ (**a** and **b**) and the s_{++} (**c** and **d**) superconductors with $\eta = 1/2$

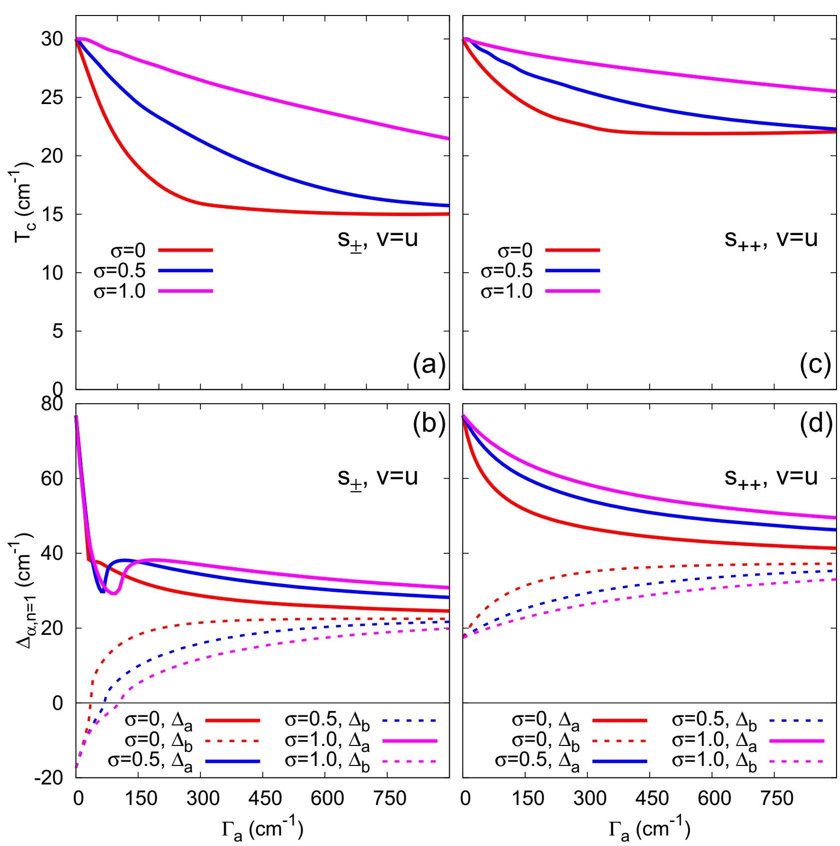

Fig. 3 Uniform nonmagnetic impurity potential $\eta = 1$: T_c (**a** and **c**) and Matsubara gap $\Delta_{\alpha n=1}$ (**b** and **d**) dependence on the scattering rate Γ_a for the $s_\pm$ (**a** and **b**) and the s_{++} (**c** and **d**) superconductors

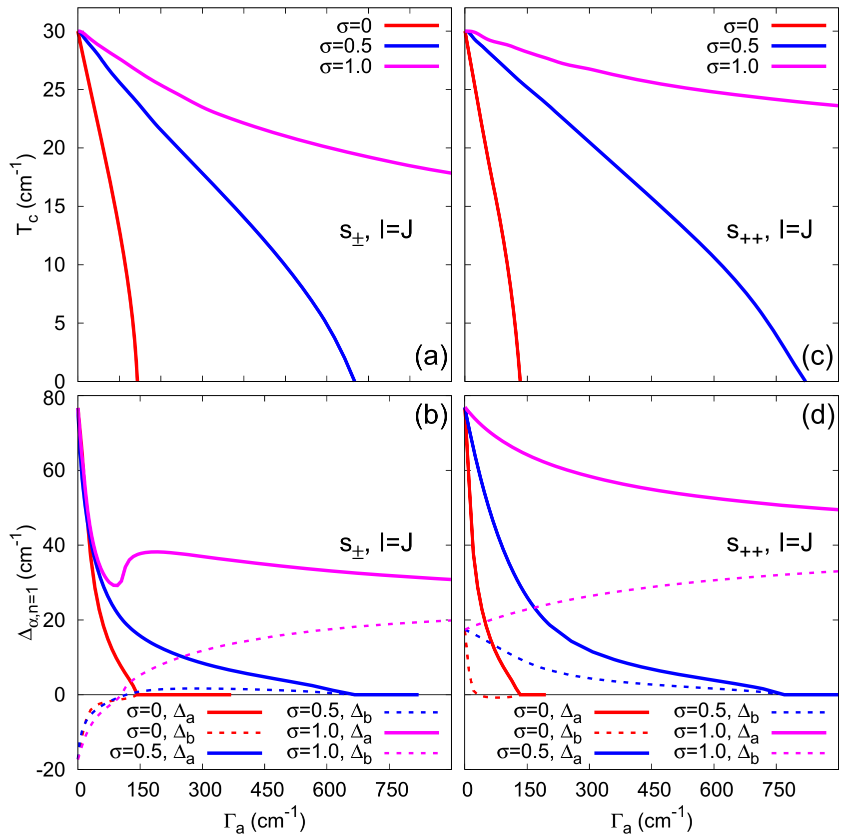

Fig. 4 Uniform magnetic impurity potential $\eta = 1$: T_c (**a** and **c**) and Matsubara gap $\Delta_{\alpha n=1}$ (**b** and **d**) dependence on the scattering rate Γ_a for the $s_\pm$ (**a** and **b**) and the s_{++} (**c** and **d**) superconductors

For the uniform impurity potentials, the situation, however, becomes different. Results for T_c and $\Delta_{\alpha n=1}$ is shown in Fig. 3 for the nonmagnetic disorder and in Fig. 4 for the magnetic one. While the behavior in the Born and intermediate scattering ($\sigma = 0.5$) limits is in general similar to those for $\eta \neq 1$, critical temperature and gaps in the unitary limit are not independent on disorder any more. Following the analytical results in the previous section, T_c gradually decrease with increasing Γ_a. There is even a $s_\pm \to s_{++}$ transition for the magnetic impurities in the unitary limit, which is not seen for $\eta \neq 1$. On the other hand, there is no transition to the $s_\pm$ state for $\sigma = 0.5$, which appeared for s_{++} state with unequal intra- and interband impurity potentials.

4 Conclusions

We have studied the case of uniform impurity potential, that is, the equal strength of intra- and interband scattering, $u = v$ and $\mathcal{I} = \mathcal{J}$ ($\eta = 1$). It appears to be qualitatively different from the other cases. This is particulary demonstrated in the unitary limit where for $\eta \neq 1$ there is an independence of gaps and T_c on the values of both nonmagnetic and magnetic scattering. On the contrary, for the uniform impurity potential, there is a suppression of gaps and critical temperature due to the disorder.

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