Journal of Magnetism and Magnetic Materials 398 (2016) 121–126

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Flat and conical incommensurate magnetic structures in the two-subsystem partially frustrated Heisenberg ferrimagnet

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ARTICLE INFO

Article history:
Received 21 April 2015
Received in revised form
3 September 2015
Accepted 9 September 2015
Available online 11 September 2015

Keywords:
Incommensurate magnetic structures
Phase transitions
Ferrimagnets

ABSTRACT

The phase transitions into flat and conical incommensurate magnetic structures are considered for a ferrimagnet with the dominant nonfrustrated exchange between the spins in one crystallographic position, competing exchanges between the spins in another position and frustrated exchange between the spins in different positions. The appearance conditions and the temperatures of the second order phase transitions are analytically obtained in the mean field approximation. The first order phase transition between these states is studied and the phase diagrams of temperature vs frustrated exchanges are calculated by the numerical minimization of free energy.

1. Introduction

The frustration of the exchange interactions is the main reason for a great variety of magnetic structures. In ferrimagnets, systems with magnetic ions in two and more nonequivalent crystal positions (magnetic subsystems), the type of magnetic ordering is determined by the spin dimension, spatial distribution of the exchange bonds, type of the exchanges both within each position and between them and their anisotropies. In this case the number of parameters determining the state of the total magnetic system (and the number of the problem variables) is increased as compared with single-subsystem ferro- and antiferromagnets. As a result, the number of possible magnetic states is increased. Different temperature dependences of subsystem contributions into the total state lead to the phase transitions, depending on the temperature and exchange parameters. The frustration (competition) of the exchange interaction results in the degeneracy of the ground state with the collinear orientation of magnetic moments. A classical example of such a ferrimagnet firstly considered by Yafet and Kittel [1] is a two-subsystem ferrite structure with antiferromagnetic exchanges within each subsystem and a frustrated antiferromagnetic exchange between the spins in different subsystems. The authors show that in such a system a long range magnetic order can be formed by means of successive phase transitions through: (a) antiferromagnetic ordering in the subsystem with a dominant exchange (AF phase); (b) noncollinear triangular Yafet–Kittel structure (YK phase); (c) collinear ferrimagnetic ordering (F phase); and (d) antiferromagnetic ordering in both subsystems. In the latter case the ground state remains degenerate relative to the mutual orientation of antiferromagnetic systems and, therefore, such a state becomes unstable towards the collinearity distortion of sublattices in the subsystems. In [2] it is shown that under antiferromagnetic exchange between the spins in each subsystem this instability is global – an incommensurate flat helical structure with the locally orthogonal orientation of the subsystem antiferromagnet vectors removes the degeneracy of the ground state with the value of the frustrated intersubsystem exchange being arbitrarily small. The transition from the AF phase into the antiferromagnetic flat helix (AFH) phase occurs with the appearance of magnetization in the subsystem with a weak exchange as a second order phase transition. The ferromagnetic exchange within this subsystem leads either to the incommensurate structure with the locally triangular spin orientation (triangular helix) [3] or to the ferrimagnetic helix, depending on the fulfillment of threshold conditions on the frustrated and competing interactions. All the above-mentioned structures were calculated assuming the flat spin arrangement. The incommensurate structure with the three-dimensional spin configuration – conical ferrimagnet – in the cubic spinel lattice was thoroughly considered by Kaplan et al. [4,5] in another limit case of the dominant antiferromagnetic intersubsystem and frustrated intrasubsystem exchanges. It was shown that the instability of the collinear magnetic structure with respect to a small distortion of the helical type is a result of the enhancement of the frustrated intrasubsystem antiferromagnetic exchange. When the threshold condition on the ratio between the exchanges is fulfilled, the ferrimagnetic cone helix can appear from collinear ferrimagnetic state. The collinear ferrimagnetic structure evolves into an incommensurate conical one also in the case of dominating ferromagnetic (i.e.

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http://dx.doi.org/10.1016/j.jmmm.2015.09.046
0304-8853/

nonfrustrating) intrasubsystem exchange between the Cr^{2+} ions in spinel $FeCr_2S_4$ where the frustration originates from the competition between nearest and next nearest exchanges coupling Fe^{3+} ions in A-positions [6–8]. Increasing interest in incommensurate magnetic structures in the last decade is primarily due to the discovery of their multiferroic properties. The conical helixes with a low magnetic anisotropy are perspective from practical view point for the low-magnetic-field control of electrical polarization at the room temperature [9–11]. The definition of the origin of spatial anisotropy at the conical ordering in the framework of isotropic Heisenberg model with different frustrated interactions is necessary for the correct mathematical description of the effects observed. The possibility of conical incommensurate magnetic ordering in the ferrimagnet with the dominant nonfrustrated antiferromagnetic exchange in one subsystem and the frustrated (competing) exchanges in another one and between the subsystems has not been considered so far. The aims of the work are to determine the conditions of the incommensurate structures appearance in such a system and to study the type of possible phase transition between them.

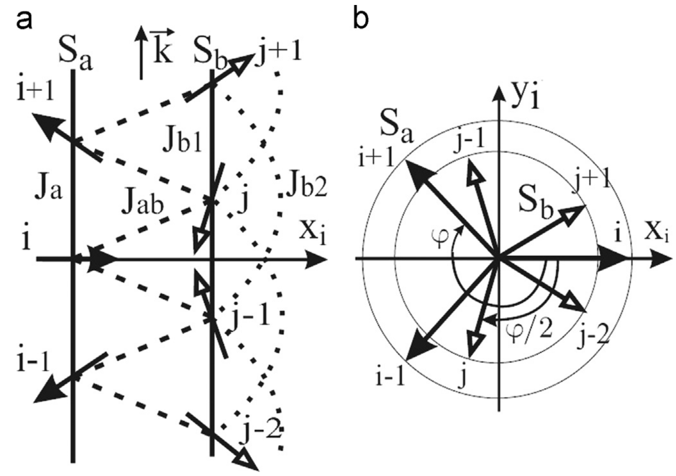

Fig. 1. (a) Scheme of the exchange interactions: solid lines – intrasubsystem exchanges between nearest neighbours; dashed lines – intersubsystem exchange; dot lines – exchange between second magnetic neighbours in the B-subsystem. (b) Mutual orientation of the spins for the flat helix.

2. Model and approach

The number of the variables describing a certain state depends firstly on the spin dimensionality and on the number of magnetic positions with various local distributions of neighbours. The latter is determined both by the crystal structure and by the type of the considered magnetic order. For example, in the cubic spinel AB_2O_4 with two crystallographic positions of the magnetic A- and B-ions the incommensurate magnetic ordering with the magnetic structure wave vector along the [110] crystal direction results in two positions of the B-ions which are nonequivalent relative to the orientation of the neighboring spins. As a result, the helix structure consists of three spin cones with various angles [5,12]. In the present work consideration is given to the model of a ferrimagnet with the coincident direction of the frustrated and competing exchanges and equivalent position of the spins within both subsystems in relation to this direction. In this case, with the contributions from different mechanisms of incommensurability in the total state being changed with the temperature, the direction of the vector of the magnetic structure modulation **k** is not varied and the number of nonequivalent magnetic positions coincides with the number of subsystems in all the considered phases. The scheme of the exchange interactions and the variation of the mutual orientation of the spins at the spatial displacement in the direction of the frustrating and competing exchanges (along the vector of the magnetic structure modulation **k**) is shown in Fig. 1.

A similar model was used for describing dynamical properties and incommensurate ordering in the two-subsystem tetragonal magnet CuB_2O_4 [13–16]. As will be shown later, the model has both flat and conical solutions. The model Hamiltonian is

$$H = J_a \sum_{ii'} \mathbf{S}_i \mathbf{S}_{i'} + J_{b1} \sum_{jj'} \mathbf{S}_j \mathbf{S}_{j'} + J_{b2} \sum_{jj''} \mathbf{S}_j \mathbf{S}_{j''} + J_{ab} \sum_{ij} \mathbf{S}_i \mathbf{S}_j, \quad i \in A,$$
$$j \in B,$$
$$z_a J_a > z_{b1} J_{b1}, z_{b2} J_{b2}, z_{ab} J_{ab}, z_{ba} J_{ab} > 0. \tag{1}$$

Here i and j are the indexes of the spins in the A and B subsystems, respectively, and $z_{\alpha\beta}$ are the numbers of the neighboring spins for the corresponding exchange $J_{\alpha\beta}$ within the subsystems and between them. In the mean field approximation (MFA) the Hamiltonian is additive with respect to the spins

$$H_{MFA} = \sum_i \mathbf{h}_i \mathbf{S}_i + \sum_j \mathbf{h}_j \mathbf{S}_j,$$

$$\mathbf{h}_i = \frac{1}{2} J_a \sum_{i'} \mathbf{S}_{a,i'} + \frac{1}{2} J_{ab} \sum_j \mathbf{S}_{b,j},$$

$$\mathbf{h}_j = \frac{1}{2} J_{b1} \sum_{j'} \mathbf{S}_{b,j'} + \frac{1}{2} J_{b2} \sum_{j''} \mathbf{S}_{b,j''} + \frac{1}{2} J_{ab} \sum_i \mathbf{S}_{a,i}, \tag{2}$$

where $\mathbf{S}_{a,i}$, $\mathbf{S}_{b,j}$ are the vectors of the average spins in the A and B subsystems, respectively. The necessary condition of the existence of stationary states in MFA is the collinearity of the average spins and the corresponding total fields [17]. This requirement is equivalent to the constraints imposed on the effective fields acting on the spins: the transverse components of the fields must be equal to zero. In the case of two nonequivalent magnetic positions the fields on the spins $\mathbf{h}_{i,j}$ are the function of four spin orientation angles $\theta_{a,b}$ and $\varphi_{a,b}$ – a polar angle and an azimuthal one in the local spherical coordinates of the corresponding spins and of two average values S_a and S_b. Four constraints of collinearity and two self-consistent equations for the average values of the spins in MFA form a full system of nonlinear equations for the problem variables

$$h^{\theta,\varphi}_{a,b}(\theta_{a,b}, \varphi_{a,b}, S_{a,b}) = 0 \tag{3}$$

$$S_{a,b} = -S^0_{a,b} B_{S^0_{a,b}}\left(\frac{h^{a,b} S^0_{a,b}}{T}\right), \tag{4}$$

where $h^{\theta,\varphi}_{a,b}$ are the transverse fields along the unit vectors $\mathbf{e}^{\theta}_{i,j}$ and $\mathbf{e}^{\varphi}_{i,j}$ of the local coordinate systems, $h^{a,b}$ are the longitudinal fields and $B_{S^0_{a,b}}(x)$ is the Brillouin function for the spins $S^0_{a,b}$. In the local coordinate systems all $S_{a,b} > 0$ and $h^{a,b} < 0$.

The system of equations (3) and (4) determines all the solutions with two nonequivalent positions. To find a solution with the minimal free energy

$$F = -T \ln Z,$$
$$Z_{MFA} = Sp \exp\left(\frac{H_{MFA}}{T}\right) = Z_a^{N_a} Z_b^{N_b}, \tag{5}$$

where $Z_{a,b}$ are the partition functions for the single spins, the energy is varied over the problem variables

$$\delta F = N_a S_a \delta h^a + N_b S_b \delta h^b = 0. \tag{6}$$

Here, N_a and N_b are the numbers of the spins in the subsystems.

We introduce the dimensionless exchange parameters of the model and fields normalized with regard to the total exchange interaction between the A-spins, as well as the parameter of the exchange frustration in the B-subsystem R and temperature normalized with regard to the Neel one in the A-subsystem

$$j_b = \frac{z_{b1}J_{b1}}{z_a J_a}, \quad j_{ab} = \frac{z_{ab}J_{ab}}{z_a J_a},$$

$$j_{ba} = \frac{z_{ba}J_{ab}}{z_a J_a}, \quad h_{a,b} = \frac{h^{a.b}}{z_a J_a},$$

$$R = \frac{z_{b2}J_{b2}}{z_{b1}J_{b1}}, \quad t = \frac{T}{T_N} = \frac{6T}{S_a^0(S_a^0+1)z_a J_a},$$
$$j_b, j_{ab}, j_{ba}, R, t \in \{0, 1\}. \tag{7}$$

In these notations the self-consistent equations (4) take the form

$$S_{a,b} = -S^0_{a,b} B_{S^0_{a,b}}\left(\frac{6h_{a,b}S^0_{a,b}}{S_a^0(S_a^0+1)t}\right). \tag{8}$$

3. Antiferromagnetic flat helix (AFH)

For the antiferromagnetic flat helix with the symmetrical orientation of the neighboring spins in each subsystem (see Fig. 1b)) the condition of equality to zero for the transverse fields (3) is always fulfilled. The longitudinal fields on the spins have the form

$$h_a^{AFH} = \frac{S_a}{2}\cos\varphi + \frac{j_{ab}S_b}{2}\cos\frac{\varphi}{2},$$

$$h_b^{AFH} = \frac{j_b S_b}{2}(\cos\varphi + R\cos 2\varphi) + \frac{j_{ba}S_a}{2}\cos\frac{\varphi}{2}. \tag{9}$$

All the AFH-solutions of the model are parametrized by one independent problem variable – the pitch of helix φ. After substituting the variable

$$y = \cos\frac{\varphi}{2}, \tag{10}$$

one obtains the equation of the free energy minimization

$$\frac{dF}{dy} = z_a J_a N_a\left(S_a\frac{dh_a^{AFH}}{dy} + S_b\frac{dh_b^{AFH}}{dy}\right) = 0, \tag{11}$$

where $n = N_a/N_b = z_{ab}/z_{ba}$. In the vicinity of the transition from the AF – phase $t < 1$ the linearization of self-consistent equation (8) for small values S_b and y leads to the following relation:

$$S_b = \frac{y j_{ba} S_a S_b^0(S_b^0+1)}{j_b(1-R)S_b^0(S_b^0+1) - tS_a^0(S_a^0+1)}. \tag{12}$$

So, the appearance of the average spins S_b is caused by the intersubsystem exchange field from the canting antiferromagnetic sublattices of the A-spins. Eq. (9) which is accurate up to the linear terms over y takes the form

$$y \times \frac{2z_a J_a N_a S_a^2 t}{(t - C(t))(t - B(1-R))} \times \Big((t - B(1-R))(t - B(1-R) - A)$$
$$-\frac{A}{2}\Big(t - C(t)\Big)\Big) + O(y^3) = 0, \tag{13}$$

where the following notations are used:

$$A = \frac{j_{ab}j_{ba}S_b^0(S_b^0+1)}{2S_a^0(S_a^0+1)},$$
$$B = \frac{j_b S_b^0(S_b^0+1)}{S_a^0(S_a^0+1)},$$
$$C(t) = \frac{3S_a^0}{S_a^0+1}B'_{S_a^0}(x). \tag{14}$$

The left part of Eq. (13) corresponds to the derivative of the first term in the thermodynamic potential expansion in the Landau theory of the second order phase transition [18]. It changes the sign at the transition temperature. At $t < 1$ the derivative of the Brillouin function in $C(t)$ decreases faster than t and at $t \ll 1$ it tends to zero exponentially [19]. In this limit case

$$\frac{dF}{dy} = y\frac{2z_a J_a N_a S_a^2}{(t - B(1-R))^2}(t - t_{AFH})(t - t_2) + O(y^3),$$

where the temperature of the transition into the AFH phase has the form

$$t_{AFH} = B(1-R) + \frac{3}{4}A + \sqrt{\frac{9}{4}A^2 + \frac{1}{2}AB(1-R)}$$
$$= t_B + \frac{3}{4}A + \sqrt{\frac{9}{4}A^2 + \frac{1}{2}At_B},$$

$$t_2 = B(1-R) + \frac{3}{4}A - \sqrt{\frac{9}{4}A^2 + \frac{1}{2}AB(1-R)}. \tag{15}$$

The second multiplier of the first term of the free energy expansion $(t - t_2)$ is positive in the vicinity of the transition temperature. This provides the local minimum of the free energy for $t > t_{AFH}$ and maximum for $t < t_{AFH}$ at the zero value of the order parameter. The transition temperature (15) is independent of the sign of the intersubsystem interaction J_{ab} and exceeds the temperature of collinear ordering in the B-subsystem in the absence of this interaction $t_B = B(1-R)$. As is firstly shown by numerical minimization of the free energy [16], AFH can appear even without any exchange interaction between the B-spins (the case of the paramagnetic “weak” subsystem). In this case the transition temperature is determined by the intersubsystem exchange only

$$t_{AFH} = \frac{3}{2}A.$$

4. Antiferromagnetic conical helix (ACH)

The conical structure having a three-dimensional orientation of the magnetic moments and retaining the number of nonequivalent magnetic positions equal to two is a structure with two cones in one subsystem located symmetrically relative to the plane of the spins in the second subsystem (Fig. 2). For the symmetrical distribution of the A-spin projections on the B-spin plane (see Figs. 2 and 1b) the condition of equality to zero for two transverse field components on the spins $\mathbf{S}_{b,j}$ and for the component h_a^{φ} on the spins $\mathbf{S}_{a,i}$ is fulfilled. The requirement of equality to zero for the component h_a^{θ} imposes an additional constraint on the angles and average spins

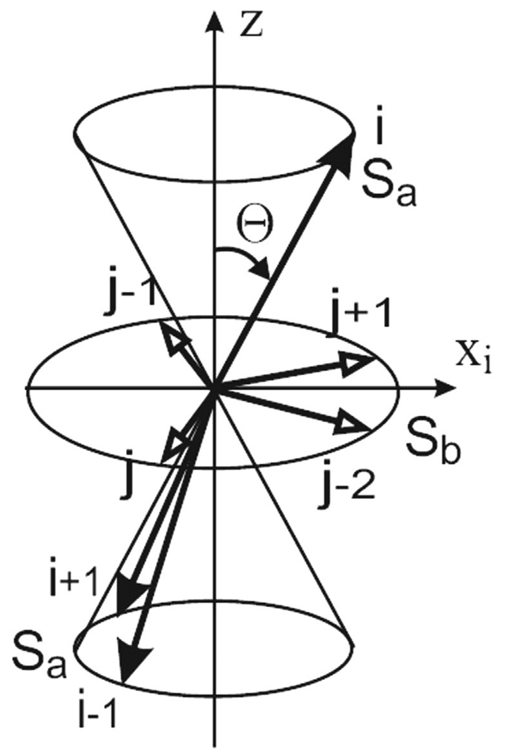

Fig. 2. Spin orientation in the symmetrical (antiferromagnetic) conical helix.

$$h_a^\theta = \cos\frac{\varphi}{2}\cos\theta\left(S_a\sin\theta\cos\frac{\varphi}{2} + S_b\frac{j_{ab}}{2}\right) = 0, \tag{16}$$

resulting in the following three solutions:

1. $\cos(\varphi/2) = 0$ – the antiferromagnetic cross-type ordering in both subsystems [1] with the ground state which is degenerate relative to the mutual orientation of the subsystems antiferromagnetic vectors. As mentioned in Introduction, this state is globally unstable at the antiferromagnetic exchanges of the model (1).
2. $\cos\theta = 0$ – the antiferromagnetic flat helix, calculated in Section 3.
3. The antiferromagnetic helix with the cone angle

$$\sin\theta = -\frac{j_{ab}S_b}{2S_a\cos(\varphi/2)}. \tag{17}$$

For the ACH solution the longitudinal fields on the spins have the form

$$h_a^{ACH} = -\frac{S_a}{2}\cos^2\theta + \frac{S_a}{2}\sin^2\theta\cos\varphi + \frac{S_b}{2}j_{ab}\sin\theta\cos\frac{\varphi}{2},$$

$$h_b^{ACH} = \frac{S_b}{2}(j_b\cos\varphi + R\cos 2\varphi) + \frac{S_a}{2}j_{ba}\sin\theta\cos\frac{\varphi}{2}. \tag{18}$$

Substituting Eq. (17) into the latter expressions allows one to exclude the angle θ from the number of independent variables:

$$h_a^{ACH} = -\frac{S_a}{2},$$

$$h_b^{ACH} = \frac{S_b}{2}\left(j_b(\cos\varphi + R\cos 2\varphi) - \frac{j_{ab}j_{ba}}{2}\right). \tag{19}$$

In the conical phase the decrease of the exchange field on the A-spins due to their noncollinearity is exactly compensated by the field from the B-spins. As a result, its value remains equal to the field at the antiparallel orientation of the antiferromagnetic A-sublattices and is dependent neither on the average value S_b nor on the helix pitch φ. For the B-subsystem the interaction with the A-spins leads to an additional effective exchange between the B-spins. So, the free energy minimization (6) is reduced to the variation of the longitudinal field h_b^{ACH} (19) over the helix pitch which gives a standard expression for $R > 1/4$

$$\cos\varphi = -(4R)^{-1}. \tag{20}$$

At $R < 1/4$, $\varphi = \pi$ and the solution of (17) for the finite S_b is absent. Upon substituting (20) into (17) and (19) one has

$$\sin\Theta = \frac{j_{ab}S_b}{S_a}\left(2 - \frac{1}{2R}\right)^{-1/2} \tag{21}$$

$$h_b^{ACH} = -\frac{S_b}{2}\left(j_b\left(R + \frac{1}{8R}\right) + \frac{j_{ab}j_{ba}}{2}\right). \tag{22}$$

Thus, when the threshold condition is fulfilled, the conical phase arises from the AF phase at the appearance of magnetization on the B-sites. At further temperature decrease the cone angle grows continuously – the A-spins tend to the B-plane. The temperature of the second order phase transition $AF \Rightarrow ACH$ is determined by the linearization of the self-consistent equation for S_b (8) taking into account Eq. (22):

$$t_{ACH} = \frac{S_b^0(S_b^0+1)}{S_a^0(S_a^0+1)}\left(j_b\left(R + \frac{1}{8R}\right) + \frac{j_{ab}j_{ba}}{2}\right) = B\left(R + \frac{1}{8R}\right) + A, \tag{23}$$

where A and B are determined in Eqs. (14).

To determine the appearance order of the incommensurate states from the AF phase for different values of the model exchange parameters a comparison is made of the temperatures of the AFH and ACH phase transitions (Eqs. (15) and (23)). In Fig. 3 the boundary of exchange parameters (14) and (7) is shown when either the AFH or ACH phase arises from the AF state. In the area of small R with the temperature decreases a flat helix is first to arise and the intersubsystem exchange frustration is partially lifted. The phase separation boundary at $A/B \to 0$ tends to the value $R = 1/4$ asymptotically. At $R > 1/4$ and small intersubsystem exchange a conical helix with the finite pitch (20) (wave vector) is the first to arise. The pitch remains constant at further lowering the temperature. The mechanism of the helix appearance is the lift of the competing exchange frustration in the B-subsystem.

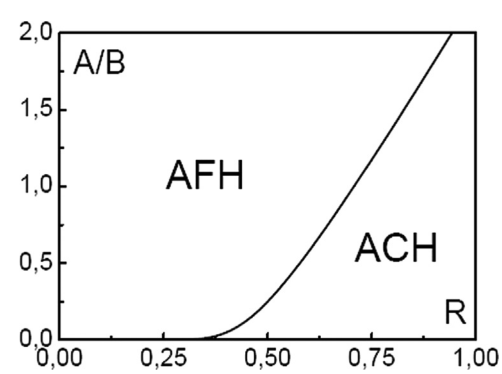

Fig. 3. Boundary between areas of exchange parameters (14) and (17) when either the AFH or ACH phase arise from the AF state. $A/B = j_{ab}j_{ba}/2j_b = z_{ab}z_{ba}J_{ab}^2/2z_{b1}z_aJ_{b1}J_a$, $R = z_{b2}J_{b2}/z_{b1}J_{b1}$.

5. Numerical minimization of the free energy

The exchange energy of the frustrated interactions resulting in the considered incommensurate structures has different dependences on the subsystem average spins S_a and S_b:

$$E_{AFH} \propto -AS_aS_b,$$
$$E_{ACH} \propto -BS_b^2.$$

Different temperature dependences of the subsystem magnetizations lead to different temperature dependences of the free energy in the AFH and ACH states. At $t \ll 1$ the spins S_a are close to saturation and in the case when AFH is the first to arise at further temperature lowering a faster decrease of the ACH free energy can lead to the change of the incommensurate ordering type. To study the peculiarities of such a phase transition the numerical minimization of the free energy (5) is carried out for the particular case $S_a^0 = S_b^0 = 1/2$ and $n=1$ with the fixed parameters j_{ab}, j_b, R. To clarify the role of the collinearity condition imposed on the fields $\mathbf{h}_i$ and spins $\mathbf{S}_{a,i}$ in the ACH state the minimization is carried out both with constraint (16) and without it. In the second case the fields on the spins (18) are used and the minimization is carried out over four variables $S_{a,b}$, θ, φ. In both cases the self-consistent conditions (8) are imposed. In Fig. 4 the free energy temperature dependencies are given for the AFH and ACH states. The energy is normalized over the number of spins N_a and total exchange between the neighboring A-spins z_aJ_a. When the collinearity condition is taken into account at t_c, the type of the incommensurate ordering is changed and the angle θ is changed from $\pi/2$ up to θ_c by jump (Fig. 5). The helix pitch and subsystem magnetizations are changed by jump, too. Further temperature lowering leads to the increase of the angle θ while the helix pitch remains constant.

The phase diagrams of temperature vs frustrated interactions are shown in Figs. 6 and 7. The triple points in the diagrams correspond to the phase boundary of the helix phase appearance in Fig. 3. At large values of the intersubsystem exchange $j_{ab} \geq 1$ the flat helix phase arises from the paramagnetic phase (P) at $t \geq 1$ (Fig. 6). The high-temperature area of the AFH state also arises at large values of j_b and R. For these cases of the dominant intersubsystem exchange or exchanges in the B-subsystem the expression for the temperature of the AFH phase transition (15) is irrelevant. The long range order simultaneously appears in both subsystems and it is necessary to consider the free energy expansion over the smallness of both S_b and S_a. The study of the phase transitions in this area of the exchange parameters is outside the framework of the above stated problem.

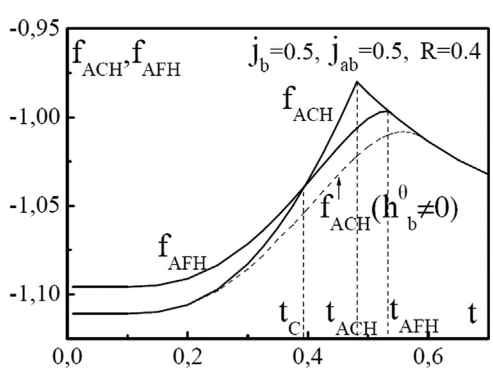

Fig. 4. Temperature dependences of the normalized free energy in the AFH and ACH states (solid lines). The dashed line shows the dependence of the ACH free energy without taking into account constraint (16).

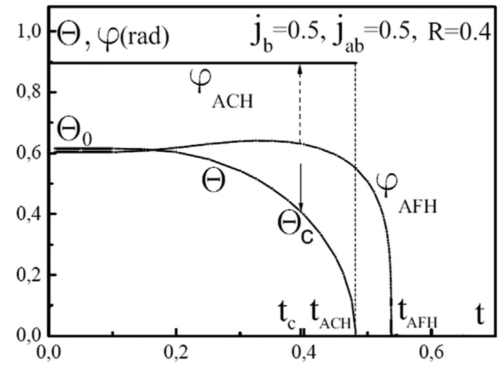

Fig. 5. Temperature dependences of the cone angle θ and pitch of the helixes φ at the succession of the phase transitions $AF \rightarrow ACH \rightarrow AFH$. The change of the cone angle from $\pi/2$ up to θ_c and the pitch jump at temperature t_c are shown by the solid and dashed arrows, respectively.

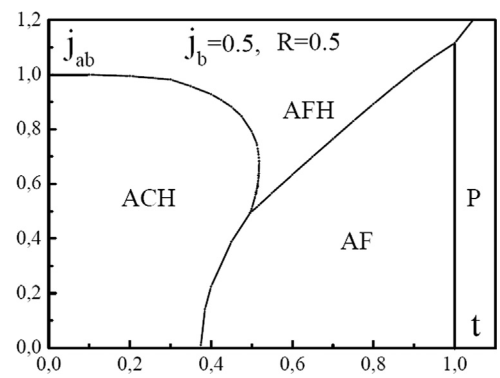

Fig. 6. Phase diagram of temperature vs intersubsystem exchange at the fixed exchanges in the B-subsystem. The area P corresponds to the paramagnetic phase with the average spins being absent in both subsystem.

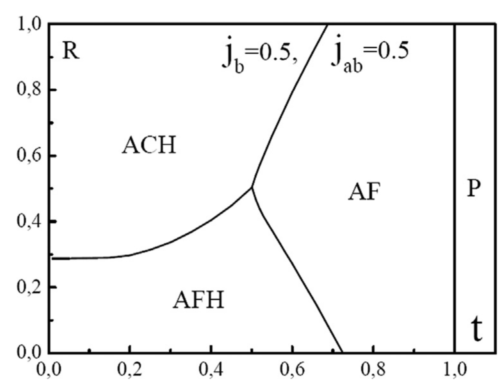

Fig. 7. Phase diagram of temperature vs ratio of the competing exchanges in the B-subsystem at the fixed intersubsystem exchange j_{ab} and exchange j_b.

6. Summary and conclusion

In the considered model of the ferrimagnet with the dominant nonfrustrated exchange in one magnetic subsystem the frustration of the intersubsystem exchange and the competition of the exchanges in the second subsystem lead to different incommensurate structures – antiferromagnetic flat and antiferromagnetic conical helixes, respectively. The helix arises at temperature lowering from the AF phase where only the subsystem with the dominant exchange is ordered with the magnetization appearance in the second subsystem and its type depends on the relation between the values of the intersubsystem and competing exchanges. In the flat helix phase the canting of the antiferromagnetic sublattices in the A-subsystem reduces the effective field on the spins $\mathbf{S}_{a,i}$. The free energy of the total system is decreased by ordering in the B-subsystem. At the fulfillment of the threshold condition on the competing exchanges further temperature lowering leads to the first order phase transition from the flat helix phase into the conical helix one. Here, the helix pitch is increased by jump. Nevertheless, the A-spins reorientation in the symmetrical cones decreases the canting angle between the spins and the effective field is restored up to the field of the collinear A-sublattices. Despite the reduction of the exchange field on the B-spins (and, consequently, the average value of S_b) the system free energy is lowered due to faster decrease of the A-spins energy. Upon further lowering the temperature the magnetic structure wave vector remains constant but the cone angle increases monotonically remaining intermediate even at zero temperature $0 < \theta_0 < \pi/2$.

The considered model (1) is an isotropic one. Depending on the relation between the frustrated exchanges (Fig. 3) the transition into the conical helix phase can occur either from the AF phase by the continuous increase of the cone angle from the zero value (by the second order phase transition) or through the intermediate flat helix phase with the common spin polarization plane. In the second case the cone angle appears by jump from $\pi/2$ down to the finite value θ_c (the first order phase transition). The exchange energy lowering in the subsystem with competing interactions plays the role of effective easy-plane anisotropy in this subsystem [18]. An equilibrium cone angle of the A-spins is determined by the relation between the fields of intersubsystem (A–B) and the dominant (A–A) exchanges. The latter promotes collinear ordering of A-spins and leads to an effective anisotropy with an easy axis which is normal to a helix plane. The temperature variation of the cone angle is determined by the temperature dependence of the subsystem magnetizations. In the considered model the main contribution in this variation is made by the change of B-magnetization as contrasted to the hexagonal ferrimagnet $DyMn_6Ge_6$ [20], where the easy-axis R-subsystem moments provide such a contribution. As a result, in our case the cone angle has the inverse temperature dependence as compared with the similar one in $DyMn_6Ge_6$.

Taking into consideration of the single ion (or exchange) easy-plane anisotropy of the A-spins can change the phase space considerably up to the appearance of the partially disordered states instead of the conical phase in the limit case of XY-model [21]. The finite anisotropy of this type changes the character of the AF–ACH phase transition. The anisotropy field would act on the spins $\mathbf{S}_a$ similarly to the field from the B-spins retaining the spins within the easy plane and stabilizing the flat phase. The transition into the conical phase would occur at the finite exchange energy of the interaction between B-spins (and, consequently, the finite value of S_b), which is comparable with the anisotropy energy. As a result, the transition from the AF phase becomes the first order phase transition similarly to the transition from the AFH phase.

In the framework of the isotropic model the external magnetic field applied in any direction should rotate the polarization plane of the incommensurate structures and orientate it orthogonally relative to the field direction. The symmetry relative to the polarization plane of the B-spins is broken and the projection of the A-spins from the upper and lower cones on the plane becomes different. It leads to the appearance of the alternate helix pitch similarly to the case of the triangular helix [3]. As in the case with easy-plane anisotropy of the A-spins the phase transition into the ACH phase becomes the first order one.

In conclusion, comparison is made on the result of the numerical free energy minimization taking into account the collinearity condition for the average spins and effective fields (the solid line in Fig. 4) and without the constraint (the dashed line). For the selected values of the exchange parameters the noncollinear solution has a lower energy of the conical state in comparison with the flat helix energy at any temperatures. So, ignoring the collinearity condition leads to a significant distortion of the phase space. The mean field approach allows one to clearly consider different magnetic states from the physical viewpoint and to describe the phase transitions both of the first and second order from the unified position. Within this approach using the collinearity condition in the explicit form reduces the number of variables and excludes nonphysical states arising in the numerical calculations.

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