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# Single-layer model of reflective nanostructures for magnetoellipsometry data analysis 

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#### Abstract

In this work we present the method of magneto-ellipsometry data analysis. Magnetoellipsometry measurements are conducted in situ during nanostructures synthesis. Magnetic field is applied in configuration of magneto-optical transverse Kerr effect. Single-layer model of reflective nanostructures is in focus.


## 1. Introduction

Magneto-ellipsometry is considered as one of powerful reliable nondestructive methods for nanostructures synthesis control that is highly important for spintronics, electronics and nanotechnology. This technique combines ellipsometry and magneto-optical Kerr effect measurements. Magneto-ellipsometry has to be developed and in this work we report on magnetoellipsometry measurements analysis for the case of single-layer nanostructures study. We have developed the approach that can be applied to investigation of reflective ferromagnetic/nonferromagnetic nanostructures that are a subject of interest due to observed spin transport phenomena. We offer an algorithm that yields information about dielectric permittivity tensor of ferromagnetic layer [1], where diagonal tensor elements are responsible for refractive index and extinction coefficient, off-diagonal tensor elements are related to magneto-optical effects:

$$
[\varepsilon]=\left[\begin{array}{ccc}
\varepsilon_{11} & \varepsilon_{12} & 0  \tag{1}\\
\varepsilon_{21} & \varepsilon_{22} & 0 \\
0 & 0 & \varepsilon_{33}
\end{array}\right]=\left[\begin{array}{ccc}
\varepsilon_{11}^{\prime}-i \varepsilon_{11}^{\prime \prime} & -i\left(\varepsilon_{12}^{\prime}-i \varepsilon_{12}^{\prime \prime}\right)\left(Q_{1}-i Q_{2}\right) & 0 \\
i\left(\varepsilon_{12}^{\prime}-i \varepsilon_{12}^{\prime \prime}\right)\left(Q_{1}-i Q_{2}\right) & \varepsilon_{11}^{\prime}-i \varepsilon_{11}^{\prime \prime} & 0 \\
0 & 0 & \varepsilon_{11}^{\prime}-i \varepsilon_{11}^{\prime \prime}
\end{array}\right],
$$

where $\varepsilon$ is a complex permittivity of a medium ( $\varepsilon_{11}=\varepsilon_{22} \approx \varepsilon_{33}, \varepsilon_{12}=-\varepsilon_{21}$ ), real parts are marked by ', imaginary by ", $Q=Q_{1}-i Q_{2}$ is a proportional to magnetization magneto-optical parameter. In the nonmagnetic condition $(Q=0)$ the off-diagonal tensor elements vanish.

In the following, we describe the method of interpretation of the ellipsometric and magnetoellipsometric measurements data from the in situ setup of a magneto-optical generalized ellipsometer, which is integrated into an ultra-high vacuum chamber with the electromagnet for magnetization reversal of the sample. The key idea of this approach and the case of the model of a homogeneous semi-infinite medium have been reported in [2] and at the $8^{\text {th }}$ Joint European Magnetic Symposia (JEMS-2016) [3]. Here we repeat some of our basic ideas and present developed expressions for experimental data processing for a single-layer model of reflective nanostructures in order to study their optical and magneto-optical properties. We consider the case of electromagnetic wave incidence from non-magnetic dielectric medium (characterized by the refraction index $N_{0}$ ) onto ferromagnetic metal (the refraction index $N_{I}$ ) on substrate (the refraction index $N_{2}$ ). We set the magnetization vector to be z -axis directed, so that YX plane is a plane of incidence, YZ plane is a boundary plane. The transverse configuration is in focus because of the design features of high-vacuum chamber and


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electromagnet that are commonly used for magnetization reversal. In this paper, we consider the visible light range, as a great amount of modern ellipsometers work in this range.

## 2. Ellipsometry and magneto-ellipsometry measurements data

Let us denote the ellipsometric parameters in the non-magnetic condition $(Q=0)$ as $\psi_{0}$ and $\Delta_{0}$ [1]. In the case of magneto-ellipsomertic characterization of the sample ( $Q=Q_{1}-i Q_{2} \neq 0$ ) the surface transverse magneto-optical Kerr effect results in the ellipsometric angles corrections $\delta \psi$ and $\delta \Delta$. Thus, the ellipsometric parameters become $\psi_{0}+\delta \psi, \Delta_{0}+\delta \Delta$. It means that four independent real-valued quantities $\left(\psi_{0}, \delta \psi, \Delta_{0}, \delta \Delta\right)$ are measured and, as a result, four real-valued quantities $\left(\varepsilon_{11}^{\prime}, \varepsilon^{\prime \prime}{ }_{11}, \varepsilon_{12}^{\prime}, \varepsilon^{\prime \prime}{ }_{12}\right)$ can be derived.

To start analysis of magneto-ellipsometry experimental data ( $\psi$ and $\Delta$ ) we have to write the real and imaginary parts of complex reflection coefficients in the basic equation of ellipsometry [4, 5]:

$$
\begin{equation*}
\rho=\operatorname{tg}\left(\psi_{0}+\delta \psi\right) \exp \left(i\left(4_{0}+\delta \Delta\right)\right)=R_{p} R_{S}^{-1}=\left(R_{p}^{\prime}-i R_{P}^{\prime \prime}\right)\left(R_{S}^{\prime}-i R_{S}^{\prime \prime}\right)^{-1}, \tag{2}
\end{equation*}
$$

where $\rho$ is the complex ellipsometric parameter, $R_{p}$ and $R_{s}$ are complex reflection coefficients corresponding to in-plane p-polarization and out-of-plane s-polarization respectively, real parts again are marked by ', imaginary by ". According to mode conversion from the p to the s polarized channel we can write that

$$
\begin{align*}
& R_{p}=R_{p p}+R_{p s}=R_{p 0}^{\prime}+R_{p 1}^{\prime}-i\left(R_{p 0}^{\prime \prime}+R_{p 1}^{\prime \prime}\right),  \tag{3}\\
& R_{S}=R_{S S}+R_{S p}=R_{S 0}=R_{S 0}^{\prime}-i R_{S 0}^{\prime \prime}, \tag{4}
\end{align*}
$$

where we have distinguished the magnetic field contribution and marked it by subscript 1 , nonmagnetic summands - by subscript 0 . One can see that transverse Kerr effect yields to $R^{\prime \prime}{ }_{s l}=0, R_{s l}^{\prime}=0$.

By substituting equations (3-4) into (2) we obtain for non-magnetic condition:

$$
\begin{align*}
& \operatorname{tg} \psi_{0}=\sqrt{\frac{\left(R_{p 0}^{\prime} R_{S 0}^{\prime}+R_{S 0}^{\prime \prime} R_{p 0}^{\prime \prime}\right)^{2}+\left(R_{s 0}^{\prime \prime} R_{p 0}^{\prime}-R_{p 0}^{\prime \prime} R_{S 0}^{\prime}\right)^{2}}{R_{s 0}^{\prime 2}+R_{s 0}^{\prime \prime 2}}},  \tag{5}\\
& \Delta_{0}=\operatorname{arctg} \frac{R_{S 0}^{\prime \prime} R_{p 0}^{\prime}-R_{p 0}^{\prime \prime} R_{S 0}^{\prime}}{R_{p 0}^{\prime} R_{S 0}^{\prime}+R_{p 0}^{\prime \prime} R_{S 0}^{\prime \prime}}, \tag{6}
\end{align*}
$$

while the influence of an external magnetic field leads to ellipsometric parameters $\delta \psi$ and $\delta \Delta$ :

$$
\begin{gather*}
\delta \psi=\psi-\psi_{0}=\operatorname{arctg}\left(F \operatorname{tg}\left(\psi_{0}\right)\right)-\psi_{0},  \tag{7}\\
\delta \Delta=\Delta-\psi_{0}=\operatorname{arctg} \frac{R_{s 0}^{\prime \prime}\left(R_{p 0}^{\prime}+R_{p 1}^{\prime}\right)-\left(R_{p 0}^{\prime \prime}+R_{p 1}^{\prime \prime}\right) R_{s 0}^{\prime}}{\left(R_{p 0}^{\prime}+R_{p 1}^{\prime}\right) R_{S 0}^{\prime}+\left(R_{p 0}^{\prime \prime}+R_{p 1}^{\prime \prime}\right) R_{s 0}^{\prime \prime}}-\operatorname{arctg} \frac{R_{s 0}^{\prime \prime} R_{p 0}^{\prime}-R_{p 0}^{\prime \prime} R_{s 0}^{\prime}}{R_{p 0}^{\prime} R_{S 0}^{\prime}+R_{p 0}^{\prime \prime} R_{s 0}^{\prime \prime}}, \tag{8}
\end{gather*}
$$

where $F$ is a helpful notation:

$$
\times \sqrt{\operatorname{tg}\left(\psi_{0}+\delta \psi\right)=F \operatorname{tg}\left(\psi_{0}\right)=\operatorname{tg}\left(\psi_{0}\right) \times} \begin{align*}
& 1+\frac{\left(R_{s 0}^{\prime \prime} R_{p 1}^{\prime \prime}\right)^{2}+\left(R_{p 1}^{\prime \prime} R_{s 0}^{\prime}\right)^{2}+2 R_{p 0}^{\prime \prime} R_{p 1}^{\prime \prime}\left(R_{s 0}^{\prime 2}+R_{s 0}^{\prime \prime 2}\right)}{\left(R_{p 0}^{\prime} R_{s 0}^{\prime}+R_{p 0}^{\prime \prime} R_{s 0}^{\prime \prime}\right)^{2}+\left(R_{s 0}^{\prime \prime} R_{p 0}^{\prime}-R_{p 0}^{\prime} R_{s 0}^{\prime} R_{s 0}^{\prime}\right)^{2}+\left(R_{p 1}^{\prime} R_{s 0}^{\prime \prime}\right)^{2}+2 R_{p 0}^{\prime} R_{p 1}^{\prime}\left(R_{s 0}^{\prime}{ }^{2}+R_{s 0}^{\prime \prime 2}\right)}  \tag{9}\\
& \left(R_{p 0}^{\prime} R_{s 0}^{\prime}+R_{p 0}^{\prime \prime} R_{s 0}^{\prime \prime}\right)^{2}+\left(R_{s 0}^{\prime \prime} R_{p 0}^{\prime}-R_{p 0}^{\prime \prime} R_{s 0}^{\prime}\right)^{2}
\end{align*} .
$$

## 3. Data analysis

In Figure 1 one can see a diagram of single-layer model, where $0-$ ambient medium, $1-$ ferromagnetic metal ( d - thickness), $2-\operatorname{substrate}, \varphi_{0}, \varphi_{1}$ and $\varphi_{2}$ are the angles of incidence and refraction and related to each other by Snell's law.


Figure 1. Single-layer model of reflective nanostructures
For a single-layer model complex refractive indices of the material under study ( $N_{l}=n_{l}-i k_{l}$ ) are calculated from $\psi_{0}, \Delta_{0}$ measurements by the Nelder-Mead method. The values of $N_{0}=n_{0}-i k_{0}, N_{1}=n_{1}-i k_{1}$, $N_{2}=n_{2}-i k_{2}$ are necessary for the following ellipsometric angles calculation.

Analytical expressions for the Fresnel coefficients that take into account the magneto-optical parameter in the off-diagonal permittivity tensor elements were presented in [2, 4]. It was shown that the following expressions should be used for a single-layer model:

$$
\begin{align*}
& R_{p}=r_{01 p}+\frac{t_{01 p} t_{10 p} r_{12 p} \exp (-i 2 \beta)}{1-r_{10 p} r_{12 p} \exp (-i 2 \beta)},  \tag{10}\\
& R_{S}=\frac{r_{01 S}+r_{12 S} \exp (-i 2 \beta)}{1+r_{01 S} r_{12 S} \exp (-i 2 \beta)},  \tag{11}\\
& r_{01 p}=\frac{N_{1} \cos \varphi_{0}-N_{0} \cos \varphi_{1}}{N_{1} \cos \varphi_{0}+N_{0} \cos \varphi_{1}}-i \frac{2 Q N_{0}^{2} \sin \varphi_{0} \cos \varphi_{0}}{\left(N_{1} \cos \varphi_{0}+N_{0} \cos \varphi_{1}\right)^{2}},  \tag{12}\\
& r_{12 p}=\frac{N_{2} \cos \varphi_{1}-N_{1} \cos \varphi_{2}}{N_{2} \cos \varphi_{1}+N_{1} \cos \varphi_{2}}-i \frac{2 Q N_{1}^{2} \sin \varphi_{1} \cos \varphi_{1}}{\left(N_{2} \cos \varphi_{1}+N_{1} \cos \varphi_{2}\right)^{2}},  \tag{13}\\
& r_{10 p}=\frac{N_{0} \cos \varphi_{1}-N_{1} \cos \varphi_{0}}{N_{0} \cos \varphi_{1}+N_{1} \cos \varphi_{0}}+i \frac{2 Q N_{1}^{2} \sin \varphi_{1} \cos \varphi_{1}}{\left(N_{0} \cos \varphi_{1}+N_{1} \cos \varphi_{0}\right)^{2}},  \tag{14}\\
& r_{01 S}=\frac{N_{0} \cos \varphi_{0}-N_{1} \cos \varphi_{1}}{N_{0} \cos \varphi_{0}+N_{1} \cos \varphi_{1}},  \tag{15}\\
& r_{12 S}=\frac{N_{1} \cos \varphi_{1}-N_{2} \cos \varphi_{2}}{N_{1} \cos \varphi_{1}+N_{2} \cos \varphi_{2}},  \tag{16}\\
& t_{01 p}=\frac{2 N_{0} \cos \varphi_{0}}{N_{1} \cos \varphi_{0}+N_{0} \cos \varphi_{1}}+i \frac{2 Q N_{0}^{3} \sin \varphi_{0} \cos \varphi_{0}}{N_{1}\left(N_{1} \cos \varphi_{0}+N_{0} \cos \varphi_{1}\right)^{2}},  \tag{17}\\
& t_{10 p}=\frac{2 N_{1} \cos \varphi_{1}}{N_{1} \cos \varphi_{0}+N_{0} \cos \varphi_{1}}-i \frac{2 Q N_{1}^{3} \sin \varphi_{1} \cos \varphi_{1}}{N_{0}\left(N_{1} \cos \varphi_{0}+N_{0} \cos \varphi_{1}\right)^{2}}, \tag{18}
\end{align*}
$$

$$
\begin{equation*}
\beta_{1}=\frac{2 \pi}{\lambda} N_{1} \cos \varphi_{1} d_{1}, \tag{19}
\end{equation*}
$$

where $\beta_{I}$ is phase thickness of the film. Indices $r_{o l p}, r_{0 I s}$ and $r_{12 p}, r_{12 s}$ in expressions $(12,15)$ and (13, 16) are the refractive indices for interfaces $0-1$ and $1-2$, respectively. Indices $t_{0 l p}$ and $t_{l o_{p}}$ in expressions $(17,18)$ are transmission coefficients. Indices $r_{o l p}$ and $t_{0 l_{p}}$ correspond to the wave propagation from medium 0 to medium 1, while $r_{10_{p}}$ and $t_{I \theta_{p}}$ - to the backward propagation. Taking into account expressions (3,4) let us write $r_{0 l p}, r_{I 2 p}, r_{0 l s}, r_{12 s}, r_{I p_{p},}, t_{0 l p}, t_{l 0_{p}}$ in the same manner as the refractive indices and transmission coefficients for the model of a homogeneous semi-infinite medium:

$$
\begin{gather*}
r_{01 S}=\left(R_{S 0}^{\prime}\right)_{01}-i\left(R_{s 0}^{\prime \prime}\right)_{01} .  \tag{20}\\
r_{12 S}=\left(R_{S 0}^{\prime}\right)_{12}-i\left(R_{S 0}^{\prime \prime}\right)_{12} .  \tag{21}\\
r_{01 p}=\left(R_{p 0}^{\prime}\right)_{01}+\left(R_{p 1}^{\prime}\right)_{01}-i\left(\left(R_{p 0}^{\prime \prime}\right)_{01}+\left(R_{p 1}^{\prime \prime}\right)_{01}\right)=r r_{01}-i r i_{01},  \tag{22}\\
r_{12 p},\left(R_{p 0}^{\prime}\right)_{12}+\left(R_{p 1}^{\prime}\right)_{12}-i\left(\left(R_{p 0}^{\prime \prime}\right)_{12}+\left(R_{p 1}^{\prime \prime}\right)_{12}\right)=r r_{12}-i r i_{12},  \tag{23}\\
r_{10 p}=\left(R_{p 0}^{\prime}\right)_{10}+\left(R_{p 1}^{\prime}\right)_{10}-i\left(\left(R_{p 0}^{\prime \prime}\right)_{10}+\left(R_{p 1}^{\prime \prime}\right)_{10}\right)=r r_{10}-i r i_{10},  \tag{24}\\
t_{01 p}=\left(T_{p 0}^{\prime}\right)_{01}+\left(T_{p 1}^{\prime}\right)_{01}-i\left(\left(T_{p 0}^{\prime \prime}\right)_{01}+\left(T_{p 1}^{\prime \prime}\right)_{01}\right)=t r_{01}-i t i_{01},  \tag{25}\\
t_{10 p}=\left(T_{p 0}^{\prime}\right)_{10}+\left(T_{p 1}^{\prime}\right)_{10}-i\left(\left(T_{p 0}^{\prime \prime}\right)_{10}+\left(T_{p 1}^{\prime \prime}\right)_{10}\right)=t r_{10}-i t i_{10}, \tag{26}
\end{gather*}
$$

where $\left(R_{s 0}^{\prime}\right)_{01},\left(R_{s 0}^{\prime \prime}\right)_{01},\left(R_{p 0}^{\prime}\right)_{01},\left(R_{p 0}^{\prime \prime}\right)_{01},\left(R_{p 1}^{\prime}\right)_{01},\left(R_{p l}^{\prime \prime}\right)_{01}$ are $R_{s i}^{\prime}, R_{s o}^{\prime \prime}, R_{p 0}^{\prime}, R_{p 0}^{\prime \prime}, R_{p l}^{\prime}, R_{p 1}^{\prime \prime}$ in the model of a homogeneous semi-infinite medium, respectively. Subscript 01 denotes the electromagnetic wave incidence from ambient medium 0 onto layer 1. Indices $\left(R_{s 0}^{\prime}\right)_{12},\left(R_{s 0}^{\prime \prime}\right)_{12},\left(R_{p 0}^{\prime}\right)_{12}$, $\left(R_{p 0}^{\prime \prime}\right)_{12},\left(R_{p l}^{\prime}\right)_{12},\left(R_{p l}^{\prime \prime}\right)_{12}$ are also calculated by formulae for the model of a homogeneous semi-infinite medium, the only difference is that subscript 12 denotes the electromagnetic wave incidence from layer 1 onto substrate 2 that leads to the following changes: $\cos \varphi_{0} \rightarrow \cos \varphi_{1}, \cos \varphi_{1} \rightarrow \cos \varphi_{2}, \sin$ $\varphi_{0} \rightarrow \sin \varphi_{1}, n_{l} \rightarrow n_{2}, n_{0} \rightarrow n_{1}, k_{l} \rightarrow k_{2}, k_{0} \rightarrow k_{1}$. Likewise, indices $\left(R_{p 0}^{\prime}\right)_{10},\left(R_{p 0}^{\prime \prime}\right)_{10},\left(R_{p l}^{\prime}\right)_{10},\left(R_{p l}^{\prime \prime}\right)_{10}$ describe the electromagnetic wave propagation from layer 1 to medium $0: \cos \varphi_{0} \leftrightarrow \cos \varphi_{1}, \sin \varphi_{0} \leftrightarrow \sin \varphi_{1}$, $n_{0} \leftrightarrow n_{l}, k_{0} \leftrightarrow k_{1}$.

Transmission coefficients were not involved into algorithm of data processing for the model of a homogeneous semi-infinite medium. Therefore, we report on $\left(T_{p 0}^{\prime}\right)_{01},\left(T^{\prime \prime}{ }_{p 0}\right)_{01},\left(T_{p 1}^{\prime}\right)_{01},\left(T_{p l}^{\prime \prime}\right)_{01}$ here:

$$
\begin{gather*}
\left(T_{p 0}^{\prime}\right)_{01}=2 \frac{\left(n_{0} n_{1}+k_{0} k_{1}\right)\left(a^{2}+c^{2}\right)+\left(n_{0}{ }^{2}+k_{0}{ }^{2}\right)(a b+c d)}{A_{3}{ }^{2}+B_{3}{ }^{2}},  \tag{27}\\
\left(T_{p 0}^{\prime \prime}\right)_{01}=2 \frac{\left(n_{0}{ }^{2}+k_{0}{ }^{2}\right)(a d-b c)+\left(n_{1} k_{0}-n_{0} k_{1}\right)\left(a^{2}+c^{2}\right)}{A_{3}{ }^{2}+B_{3}{ }^{2}},  \tag{28}\\
\left(T_{p 1}^{\prime}\right)_{01}=2 \frac{Q_{1}(p q+r s)-Q_{2}(p r-s q)}{\left(n_{1}{ }^{2}+k_{1}{ }^{2}\right)\left(A_{3}{ }^{2}+B_{3}{ }^{2}\right)^{2}},  \tag{29}\\
\left(T_{p 1}^{\prime \prime}\right)_{01}=2 \frac{Q_{1}(p r-s q)+Q_{2}(p q+r s)}{\left(n_{1}{ }^{2}+k_{1}{ }^{2}\right)\left(A_{3}{ }^{2}+B_{3}^{2}\right)^{2}}, \tag{30}
\end{gather*}
$$

where

$$
\begin{align*}
& A_{3}=n_{1} a+k_{1} c+n_{0} b+k_{0} d,  \tag{31}\\
& B_{3}=k_{1} a-n_{1} c+k_{0} b-n_{0} d,  \tag{32}\\
& p=N\left(3 n_{0}^{2} k_{0}-k_{0}^{3}\right)+P\left(n_{0}^{3}-3 n_{0} k_{0}^{2}\right), \\
& q=n_{1}\left(A_{3}^{2}-B_{3}^{2}\right)-2 A_{3} B_{3} k_{1},  \tag{34}\\
& r=k_{1}\left(B_{3}{ }^{2}-A_{3}^{2}\right)-2 A_{3} B_{3} n_{1}, \tag{35}
\end{align*}
$$

$$
\begin{gather*}
s=N\left(n_{0}^{3}-3 n_{0} k_{0}^{2}\right)-P\left(3 n_{0}^{2} k_{0}-k_{0}^{3}\right),  \tag{36}\\
a=\operatorname{Re}\left(\cos \varphi_{0}\right),  \tag{37}\\
b=\operatorname{Re}\left(\cos \varphi_{1}\right),  \tag{38}\\
c=\operatorname{Im}\left(\cos \varphi_{0}\right),  \tag{39}\\
d=\operatorname{Im}\left(\cos \varphi_{1}\right),  \tag{40}\\
N=\operatorname{Re}\left(\sin \varphi_{0}\right) a-\operatorname{Im}\left(\sin \varphi_{0}\right) \mathrm{c}  \tag{41}\\
P=-\operatorname{Re}\left(\sin \varphi_{0}\right) c-\operatorname{Im}\left(\sin \varphi_{0}\right) a \tag{42}
\end{gather*}
$$

Coefficients $\left(T_{p 0}^{\prime}\right)_{10},\left(T^{\prime \prime}{ }_{p 0}\right)_{10},\left(T_{p l}^{\prime}\right)_{10},\left(T^{\prime \prime}{ }_{p l}\right)_{10}$ correspond to the electromagnetic wave propagation from layer 1 to medium 0, that leads to the changes: $\cos \varphi_{0} \leftrightarrow \cos \varphi_{l}, \sin \varphi_{0} \leftrightarrow \sin \varphi_{1}, n_{0} \leftrightarrow n_{l}, k_{0} \leftrightarrow k_{1}$.

Let us take into account $N_{0}=n_{0}-i k_{0}, N_{l}=n_{1}-i k_{1}, N_{2}=n_{2}-i k_{2}, Q=Q_{1}-i Q_{2}$ and compare expressions (10, 11 ) with (2, 3). Thus we obtain expressions for $R_{p 0}^{\prime}, R_{p 0}^{\prime \prime}, R_{p l}^{\prime}, R_{p l}^{\prime \prime}, R_{s 0}^{\prime}$ and $R_{s o}^{\prime \prime}$.

$$
\begin{align*}
& R_{p 0}^{\prime}=\left(\left(\left(R_{p 0}^{\prime}\right)_{01}+\xi_{1}\left(R_{p 0}^{\prime}\right)_{12}-\eta_{1}\left(R_{p 0}^{\prime \prime}\right)_{12}\right)\left(1+\xi_{1} L_{0112}-\eta_{1} M_{0112}\right)+\right. \\
& \left.+\left(\left(R_{p 0}^{\prime \prime}\right)_{01}+\eta_{1}\left(R_{p 0}^{\prime}\right)_{12}+\xi_{1}\left(R_{p 0}^{\prime \prime}\right)_{12}\right)\left(\xi_{1} M_{0112}+\eta_{1} L_{0112}\right)\right)\left(\left(1+\xi_{1} L_{0112}-\eta_{1} M_{0112}\right)^{2}+\left(\xi_{1} M_{0112}+\eta_{1} L_{0112}\right)^{2}\right)^{-1},  \tag{43}\\
& R_{p 0}^{\prime \prime}=\left(\left(\left(R_{p 0}^{\prime \prime}\right)_{01}+\eta_{1}\left(R_{p 0}^{\prime}\right)_{12}+\xi_{1}\left(R_{p 0}^{\prime \prime}\right)_{12}\right)\left(1+\xi_{1} L_{0112}-\eta_{1} M_{0112}\right)-\right. \\
& \left.-\left(\left(R_{p 0}^{\prime}\right)_{01}+\xi_{1}\left(R_{p 0}^{\prime}\right)_{12}-\eta_{1}\left(R_{p 0}^{\prime \prime}\right)_{12}\right)\left(\xi_{1} M_{0112}+\eta_{1} L_{0112}\right)\right)\left(\left(1+\xi_{1} L_{0112}-\eta_{1} M_{0112}\right)^{2}+\left(\xi_{1} M_{0112}+\eta_{1} L_{0112}\right)^{2}\right)^{-1},  \tag{44}\\
& R_{p 1}^{\prime}=\frac{\Omega \chi-\Gamma \varpi}{\Omega^{2}+\Gamma^{2}}-R_{p 0}^{\prime},  \tag{45}\\
& R_{p 1}^{\prime \prime}=\frac{\Omega \varpi+\Gamma \chi}{\Omega^{2}+\Gamma^{2}}-R_{p 0}^{\prime \prime},  \tag{46}\\
& R_{s 0}^{\prime}=\left(\left(\left(R_{s 0}^{\prime}\right)_{01}+\xi_{1}\left(R_{s 0}^{\prime}\right)_{12}-\eta_{1}\left(R_{s 0}^{\prime \prime}\right)_{12}\right)\left(1+\xi_{1} H_{0112}-\eta_{1} J_{0112}\right)+\right. \\
& \left.+\left(\left(R_{s 0}^{\prime \prime}\right)_{01}+\eta_{1}\left(R_{s 0}^{\prime}\right)_{12}+\xi_{1}\left(R_{s 0}^{\prime \prime}\right)_{12}\right)\left(\xi_{1} J_{0112}+\eta_{1} H_{0112}\right)\right)\left(\left(1+\xi_{1} H_{0112}-\eta_{1} J_{0112}\right)^{2}+\left(\xi_{1} J_{0112}+\eta_{1} H_{0112}\right)^{2}\right)^{-1},  \tag{47}\\
& R_{s 0}^{\prime \prime}=\left(\left(\left(R_{s 0}^{\prime \prime}\right)_{01}+\eta_{1}\left(R_{s 0}^{\prime}\right)_{12}+\xi_{1}\left(R_{s 0}^{\prime \prime}\right)_{12}\right)\left(1+\xi_{1} H_{0112}-\eta_{1} J_{0112}\right)-\right. \\
& \left.-\left(\left(R_{s 0}^{\prime}\right)_{01}+\xi_{1}\left(R_{s 0}^{\prime}\right)_{12}-\eta_{1}\left(R_{s 0}^{\prime \prime}\right)_{12}\right)\left(\xi_{1} J_{0112}+\eta_{1} H_{0112}\right)\right)\left(\left(1+\xi_{1} H_{0112}-\eta_{1} J_{0112}\right)^{2}+\left(\xi_{1} J_{0112}+\eta_{1} H_{0112}\right)^{2}\right)^{-1}, \tag{48}
\end{align*}
$$

where the following notations are used:

$$
\begin{gather*}
L_{0112}=\left(R_{p 0}^{\prime}\right)_{12}\left(R_{p 0}^{\prime}\right)_{01}-\left(R_{p 0}^{\prime \prime}\right)_{12}\left(R_{p 0}^{\prime \prime}\right)_{01},  \tag{49}\\
M_{0112}=\left(R_{p 0}^{\prime}\right)_{01}\left(R_{p 0}^{\prime \prime}\right)_{12}+\left(R_{p 0}^{\prime}\right)_{12}\left(R_{p 0}^{\prime \prime}\right)_{01},  \tag{50}\\
\xi_{1}=\operatorname{Re}\left(\exp \left(-i 2 \beta_{1}\right)\right),  \tag{51}\\
\eta_{1}=-\operatorname{Im}\left(\exp \left(-i 2 \beta_{1}\right)\right),  \tag{52}\\
J_{0112}=\left(R_{S 0}^{\prime}\right)_{01}\left(R_{S 0}^{\prime \prime}\right)_{12}+\left(R_{S 0}^{\prime}\right)_{12}\left(R_{S 0}^{\prime \prime}\right)_{01},  \tag{53}\\
H_{012}=\left(R_{S 0}^{\prime}\right)_{01}\left(R_{S 0}^{\prime}\right)_{12}-\left(R_{S 0}^{\prime \prime}\right)_{12}\left(R_{S 0}^{\prime \prime}\right)_{01},  \tag{54}\\
\Omega=1-\xi_{1}\left(r r_{10} r r_{12}-r i_{10} r i_{12}\right)+\eta_{1}\left(r i_{10} r r_{12}+r r_{10} r i_{12}\right),  \tag{55}\\
\sigma=r i_{01}-\left(\xi_{1} r r_{12}-\eta_{1} i_{12}\right)\left(r i_{01} r r_{10}+r i_{10} r r_{01}-\tau\right)-\left(\xi_{1} r i_{12}+\eta_{1} r r_{12}\right)\left(r r_{01} r r_{10}-r i_{01} r i_{10}-\theta\right),  \tag{56}\\
\Gamma=\xi_{1}\left(r i_{10} r r_{12}+r r_{10} r 1_{12}\right)+\eta_{1}\left(r r_{10} r r_{12}-r i_{10} r i_{12}\right),  \tag{57}\\
\chi=r r_{01}-\left(\xi_{1} r r_{12}-\eta_{1} r i_{12}\right)\left(r r_{01} r r_{10}-r i_{01} r i_{10}-\theta\right)+\left(\xi_{1} r i_{12}+\eta_{1} r r_{12}\right)\left(r i_{01} r r_{10}+r i_{10} r r_{01}-\tau\right),  \tag{58}\\
\theta=t r_{01} t r_{10}-t i_{01} t i_{10},  \tag{59}\\
\tau=t i_{01} t r_{10}+t i_{10} t r_{01} . \tag{60}
\end{gather*}
$$

Thus, we have all formulae that are necessary for theoretical calculation of the ellipsometric angles (2-9) in case of a single-layer model. Final step is giving the best fit to the experimental data by the use of the wavelength-to-wavelength Nelder-Mead minimization of the ellipsometric angles. It yields

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the spectral dependences of real $\left(Q_{1}\right)$ and imaginary parts $\left(Q_{2}\right)$ of magneto-optical parameter $Q$. So, we have information about all elements of the dielectric permittivity tensor.

## 4. Conclusion

To conclude, we have proposed an approach to studying single-layer nanomaterials by means of magneto-ellipsometry. The algorithm of experimental data analysis ( $\psi_{0}, \Delta_{0}, \psi_{0}+\delta \psi, \Delta_{0}+\delta \Delta$ ) is presented. As a result, optical and magneto-optical properties can be easily and reliably characterized during synthesis.

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## References

[1] Sokolov A V 1961 Optical Properties of Metals (Moscow: GIFML) p. 322
[2] Maksimova O A et al. 2014 Journal Of Structural Chemistry 55. No. 6 pp 1134-41
[3] Maximova O, Kosyrev N, Varnakov S et al. Proc of JEMS-2016 (Glasgow: IOP) p 117-118
[4] Maximova O A, Ovchinnikov S G, Hartmann U et al. 2013 J. SibSAU Vestnik (Krasnoyarsk) 49. No. 3 pp121-127
[5] Azzam R M A and Bashara N M 1977 Ellipsometry and Polarized Light (Amsterdam: NorthHolland) chapter 4

