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## Single-layer model of reflective nanostructures for magnetoellipsometry data analysis

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Abstract. In this work we present the method of magneto-ellipsometry data analysis. Magnetoellipsometry measurements are conducted in situ during nanostructures synthesis. Magnetic field is applied in configuration of magneto-optical transverse Kerr effect. Single-layer model of reflective nanostructures is in focus.

#### 1. Introduction

Magneto-ellipsometry is considered as one of powerful reliable nondestructive methods for nanostructures synthesis control that is highly important for spintronics, electronics and nanotechnology. This technique combines ellipsometry and magneto-optical Kerr effect measurements. Magneto-ellipsometry has to be developed and in this work we report on magnetoellipsometry measurements analysis for the case of single-layer nanostructures study. We have developed the approach that can be applied to investigation of reflective ferromagnetic/nonferromagnetic nanostructures that are a subject of interest due to observed spin transport phenomena. We offer an algorithm that yields information about dielectric permittivity tensor of ferromagnetic layer [1], where diagonal tensor elements are responsible for refractive index and extinction coefficient, off-diagonal tensor elements are related to magneto-optical effects:

$$\begin{bmatrix} \varepsilon \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ \varepsilon_{21} & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \varepsilon_{11}' - i\varepsilon_{11}'' & -i(\varepsilon_{12}' - i\varepsilon_{12}'')(Q_1 - iQ_2) & 0 \\ i(\varepsilon_{12}' - i\varepsilon_{12}'')(Q_1 - iQ_2) & \varepsilon_{11}' - i\varepsilon_{11}'' & 0 \\ 0 & 0 & \varepsilon_{11}' - i\varepsilon_{11}'' \end{bmatrix},$$
(1)

where  $\varepsilon$  is a complex permittivity of a medium ( $\varepsilon_{11} = \varepsilon_{22} \approx \varepsilon_{33}$ ,  $\varepsilon_{12} = -\varepsilon_{21}$ ), real parts are marked by ', imaginary by ",  $Q=Q_1-iQ_2$  is a proportional to magnetization magneto-optical parameter. In the nonmagnetic condition (Q=0) the off-diagonal tensor elements vanish.

In the following, we describe the method of interpretation of the ellipsometric and magnetoellipsometric measurements data from the in situ setup of a magneto-optical generalized ellipsometer, which is integrated into an ultra-high vacuum chamber with the electromagnet for magnetization reversal of the sample. The key idea of this approach and the case of the model of a homogeneous semi-infinite medium have been reported in [2] and at the 8<sup>th</sup> Joint European Magnetic Symposia (JEMS-2016) [3]. Here we repeat some of our basic ideas and present developed expressions for experimental data processing for a single-layer model of reflective nanostructures in order to study their optical and magneto-optical properties. We consider the case of electromagnetic wave incidence from non-magnetic dielectric medium (characterized by the refraction index  $N_0$ ) onto ferromagnetic metal (the refraction index  $N_1$ ) on substrate (the refraction index  $N_2$ ). We set the magnetization vector to be z-axis directed, so that YX plane is a plane of incidence, YZ plane is a boundary plane. The transverse configuration is in focus because of the design features of high-vacuum chamber and

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electromagnet that are commonly used for magnetization reversal. In this paper, we consider the visible light range, as a great amount of modern ellipsometers work in this range.

#### 2. Ellipsometry and magneto-ellipsometry measurements data

Let us denote the ellipsometric parameters in the non-magnetic condition (Q=0) as  $\psi_0$  and  $\Delta_0$  [1]. In the case of magneto-ellipsometric characterization of the sample  $(Q=Q_1-iQ_2\neq 0)$  the surface transverse magneto-optical Kerr effect results in the ellipsometric angles corrections  $\delta\psi$  and  $\delta\Delta$ . Thus, the ellipsometric parameters become  $\psi_0 + \delta\psi$ ,  $\Delta_0 + \delta\Delta$ . It means that four independent real-valued quantities  $(\psi_0, \delta\psi, \Delta_0, \delta\Delta)$  are measured and, as a result, four real-valued quantities  $(\varepsilon'_{11}, \varepsilon''_{11}, \varepsilon'_{12}, \varepsilon''_{12})$  can be derived.

To start analysis of magneto-ellipsometry experimental data ( $\psi$  and  $\Delta$ ) we have to write the real and imaginary parts of complex reflection coefficients in the basic equation of ellipsometry [4, 5]:

$$\rho = tg(\psi_0 + \delta\psi) \exp(i(\varDelta_0 + \delta\varDelta)) = R_p R_S^{-1} = (R'_p - iR''_p)(R'_S - iR''_S)^{-1},$$
(2)

where  $\rho$  is the complex ellipsometric parameter,  $R_p$  and  $R_s$  are complex reflection coefficients corresponding to in-plane p-polarization and out-of-plane s-polarization respectively, real parts again are marked by ', imaginary by ". According to mode conversion from the p to the s polarized channel we can write that

$$R_{p} = R_{pp} + R_{ps} = R'_{p0} + R'_{p1} - i(R''_{p0} + R''_{p1}), \qquad (3)$$

$$R_{S} = R_{SS} + R_{SP} = R_{S0} = R_{S0}' - i R_{S0}'',$$
(4)

where we have distinguished the magnetic field contribution and marked it by subscript 1, nonmagnetic summands – by subscript 0. One can see that transverse Kerr effect yields to  $R''_{sl}=0$ ,  $R'_{sl}=0$ .

By substituting equations (3-4) into (2) we obtain for non-magnetic condition:

$$tg\psi_0 = \sqrt{\frac{\left(R'_{p0}R'_{s0} + R''_{s0}R''_{p0}\right)^2 + \left(R''_{s0}R'_{p0} - R''_{p0}R'_{s0}\right)^2}{R'_{s0}^2 + R''_{s0}^2}} ,$$
(5)

$$\Delta_0 = \operatorname{arctg} \frac{R_{s0}^{\prime\prime} R_{p0}^{\prime} - R_{p0}^{\prime\prime} R_{s0}^{\prime}}{R_{p0}^{\prime} R_{s0}^{\prime} + R_{p0}^{\prime\prime} R_{s0}^{\prime\prime}},\tag{6}$$

while the influence of an external magnetic field leads to ellipsometric parameters  $\delta \psi$  and  $\delta \Delta$ :

$$\delta \psi = \psi - \psi_0 = \operatorname{arc} tg \left( F tg(\psi_0) \right) - \psi_0, \tag{7}$$

$$\delta\Delta = \Delta - \Delta_0 = \operatorname{arctg} \frac{R_{s0}^{\prime\prime}(R_{p0}^{\prime} + R_{p1}^{\prime}) - (R_{p0}^{\prime\prime} + R_{p1}^{\prime\prime})R_{s0}^{\prime}}{(R_{p0}^{\prime} + R_{p1}^{\prime})R_{s0}^{\prime} + (R_{p0}^{\prime\prime} + R_{p1}^{\prime\prime})R_{s0}^{\prime\prime}} - \operatorname{arctg} \frac{R_{s0}^{\prime\prime}R_{p0}^{\prime} - R_{p0}^{\prime\prime}R_{s0}^{\prime}}{R_{p0}^{\prime}R_{s0}^{\prime} + R_{p0}^{\prime\prime}R_{s0}^{\prime\prime}},$$
(8)

where *F* is a helpful notation:

$$tg\left(\psi_{0}+\delta\psi\right) = Ftg\left(\psi_{0}\right) = tg\left(\psi_{0}\right) \times \\ \times \sqrt{1 + \frac{\left(R_{s0}^{\prime\prime}R_{p1}^{\prime\prime}\right)^{2} + \left(R_{p1}^{\prime\prime}R_{s0}^{\prime\prime}\right)^{2} + 2R_{p0}^{\prime\prime}R_{p1}^{\prime\prime}\left(R_{s0}^{\prime\,2} + R_{s0}^{\prime\prime\prime}\right)^{2}}{\left(R_{p0}^{\prime}R_{s0}^{\prime} + R_{p0}^{\prime\prime}R_{s0}^{\prime\prime}\right)^{2} + \left(R_{s0}^{\prime\prime}R_{p0}^{\prime\prime} - R_{p0}^{\prime\prime}R_{s0}^{\prime\prime}\right)^{2}} + \frac{\left(R_{p1}^{\prime}R_{s0}^{\prime\prime}\right)^{2} + \left(R_{p1}^{\prime\prime}R_{s0}^{\prime\prime}\right)^{2} + 2R_{p0}^{\prime}R_{p1}^{\prime}\left(R_{s0}^{\prime\,2} + R_{s0}^{\prime\prime\prime}\right)^{2}}{\left(R_{p0}^{\prime}R_{s0}^{\prime} + R_{p0}^{\prime\prime}R_{s0}^{\prime\prime}\right)^{2} + \left(R_{s0}^{\prime\prime}R_{p0}^{\prime\prime} - R_{p0}^{\prime\prime}R_{s0}^{\prime\prime}\right)^{2}}$$
(9)

#### 3. Data analysis

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In Figure 1 one can see a diagram of single-layer model, where 0 – ambient medium, 1 – ferromagnetic metal (d – thickness), 2 – substrate,  $\varphi_0$ ,  $\varphi_1$  and  $\varphi_2$  are the angles of incidence and refraction and related to each other by Snell's law.



Figure 1. Single-layer model of reflective nanostructures

For a single-layer model complex refractive indices of the material under study  $(N_1=n_1-ik_1)$  are calculated from  $\psi_0$ ,  $\Delta_0$  measurements by the Nelder-Mead method. The values of  $N_0=n_0-ik_0$ ,  $N_1=n_1-ik_1$ ,  $N_2=n_2-ik_2$  are necessary for the following ellipsometric angles calculation.

Analytical expressions for the Fresnel coefficients that take into account the magneto-optical parameter in the off-diagonal permittivity tensor elements were presented in [2, 4]. It was shown that the following expressions should be used for a single-layer model:

$$R_{p} = r_{01p} + \frac{t_{01p} t_{10p} r_{12p} \exp(-i2\beta)}{1 - r_{10p} r_{12p} \exp(-i2\beta)},$$
(10)

$$R_{s} = \frac{r_{01s} + r_{12s} \exp(-i2\beta)}{1 + r_{01s} r_{12s} \exp(-i2\beta)},$$
(11)

$$C_{01p} = \frac{N_1 \cos \varphi_0 - N_0 \cos \varphi_1}{N_1 \cos \varphi_0 + N_0 \cos \varphi_1} - i \frac{2QN_0^2 \sin \varphi_0 \cos \varphi_0}{(N_1 \cos \varphi_0 + N_0 \cos \varphi_1)^2},$$
(12)

$$r_{12p} = \frac{N_2 \cos \varphi_1 - N_1 \cos \varphi_2}{N_2 \cos \varphi_1 + N_1 \cos \varphi_2} - i \frac{2QN_1^2 \sin \varphi_1 \cos \varphi_1}{(N_2 \cos \varphi_1 + N_1 \cos \varphi_2)^2},$$
(13)

$$r_{10p} = \frac{N_0 \cos \varphi_1 - N_1 \cos \varphi_0}{N_0 \cos \varphi_1 + N_1 \cos \varphi_0} + i \frac{2QN_1^2 \sin \varphi_1 \cos \varphi_1}{(N_0 \cos \varphi_1 + N_1 \cos \varphi_0)^2},$$
(14)

$$r_{018} = \frac{N_0 \cos \varphi_0 - N_1 \cos \varphi_1}{N_1 - N_1 -$$

$$N_0 \cos \varphi_0 + N_1 \cos \varphi_1$$

$$r_{12S} = \frac{N_1 \cos \varphi_1 - N_2 \cos \varphi_2}{N_1 \cos \varphi_1 + N_2 \cos \varphi_2},$$
(16)

$$t_{01p} = \frac{2N_0 \cos \varphi_0}{N_1 \cos \varphi_0 + N_0 \cos \varphi_1} + i \frac{2QN_0^3 \sin \varphi_0 \cos \varphi_0}{N_1 (N_1 \cos \varphi_0 + N_0 \cos \varphi_1)^2},$$
(17)

$$t_{10p} = \frac{2N_1 \cos \varphi_1}{N_1 \cos \varphi_0 + N_0 \cos \varphi_1} - i \frac{2QN_1^3 \sin \varphi_1 \cos \varphi_1}{N_0 (N_1 \cos \varphi_0 + N_0 \cos \varphi_1)^2},$$
(18)

$$\beta_1 = \frac{2\pi}{\lambda} N_1 \cos \varphi_1 d_1, \qquad (19)$$

where  $\beta_1$  is phase thickness of the film. Indices  $r_{01p}$ ,  $r_{01s}$  and  $r_{12p}$ ,  $r_{12s}$  in expressions (12, 15) and (13, 16) are the refractive indices for interfaces 0-1 and 1-2, respectively. Indices  $t_{01p}$  and  $t_{10p}$  in expressions (17, 18) are transmission coefficients. Indices  $r_{01p}$  and  $t_{01p}$  correspond to the wave propagation from medium 0 to medium 1, while  $r_{10p}$  and  $t_{10p}$  – to the backward propagation. Taking into account expressions (3, 4) let us write  $r_{01p}$ ,  $r_{12p}$ ,  $r_{01s}$ ,  $r_{12s}$ ,  $r_{10p}$ ,  $t_{01p}$ ,  $t_{10p}$  in the same manner as the refractive indices and transmission coefficients for the model of a homogeneous semi-infinite medium:

$$r_{01S} = (R'_{S0})_{01} - i(R''_{S0})_{01}.$$
<sup>(20)</sup>

$$r_{12S} = (R'_{S0})_{12} - i(R''_{S0})_{12}.$$
<sup>(21)</sup>

$$r_{01p} = (R'_{p0})_{01} + (R'_{p1})_{01} - i((R''_{p0})_{01} + (R''_{p1})_{01}) = rr_{01} - i ri_{01},$$
(22)

$$r_{12p} = (R'_{p0})_{12} + (R'_{p1})_{12} - i((R''_{p0})_{12} + (R''_{p1})_{12}) = rr_{12} - i ri_{12},$$
(23)

$$r_{10p} = (R'_{p0})_{10} + (R'_{p1})_{10} - i((R''_{p0})_{10} + (R''_{p1})_{10}) = rr_{10} - i ri_{10},$$
(24)

$$t_{01p} = (T'_{p0})_{01} + (T'_{p1})_{01} - i((T''_{p0})_{01} + (T''_{p1})_{01}) = tr_{01} - i ti_{01},$$
(25)

$$t_{10p} = (T'_{p0})_{10} + (T'_{p1})_{10} - i((T''_{p0})_{10} + (T''_{p1})_{10}) = tr_{10} - i ti_{10},$$
(26)

where  $(R'_{s0})_{01}$ ,  $(R''_{s0})_{01}$ ,  $(R''_{p0})_{01}$ ,  $(R''_{p1})_{01}$ ,  $(R''_{p1})_{01}$  are  $R'_{s0}$ ,  $R''_{s0}$ ,  $R''_{p0}$ ,  $R''_{p0}$ ,  $R''_{p1}$ ,  $R''_{p1}$  in the model of a homogeneous semi-infinite medium, respectively. Subscript 01 denotes the electromagnetic wave incidence from ambient medium 0 onto layer 1. Indices  $(R'_{s0})_{12}$ ,  $(R''_{s0})_{12}$ ,  $(R''_{p0})_{12}$ ,  $(R''_{p1})_{12}$  are also calculated by formulae for the model of a homogeneous semi-infinite medium, the only difference is that subscript 12 denotes the electromagnetic wave incidence from layer 1 onto substrate 2 that leads to the following changes:  $\cos \varphi_0 \rightarrow \cos \varphi_1$ ,  $\cos \varphi_1 \rightarrow \cos \varphi_2$ ,  $\sin \varphi_0 \rightarrow \sin \varphi_1$ ,  $n_1 \rightarrow n_2$ ,  $n_0 \rightarrow n_1$ ,  $k_1 \rightarrow k_2$ ,  $k_0 \rightarrow k_1$ . Likewise, indices  $(R'_{p0})_{10}$ ,  $(R''_{p0})_{10}$ ,  $(R''_{p1})_{10}$  describe the electromagnetic wave propagation from layer 1 to medium 0:  $\cos \varphi_0 \leftrightarrow \cos \varphi_1$ ,  $\sin \varphi_0 \leftrightarrow \sin \varphi_1$ ,  $n_0 \leftrightarrow n_1$ ,  $k_0 \leftrightarrow k_1$ .

Transmission coefficients were not involved into algorithm of data processing for the model of a homogeneous semi-infinite medium. Therefore, we report on  $(T'_{p0})_{01}$ ,  $(T''_{p1})_{01}$ ,  $(T''_{p1})_{01}$ ,  $(T''_{p1})_{01}$  here:

$$(T'_{p0})_{01} = 2 \frac{(n_0 n_1 + k_0 k_1)(a^2 + c^2) + (n_0^2 + k_0^2)(ab + cd)}{A_0^2 + B_0^2},$$
(27)

$$(T_{p0}^{\prime\prime})_{01} = 2 \frac{(n_0^2 + k_0^2)(ad - bc) + (n_1k_0 - n_0k_1)(a^2 + c^2)}{A_0^2 + B_0^2},$$
(28)

$$(T'_{p1})_{01} = 2 \frac{Q_1(pq+rs) - Q_2(pr-sq)}{(n^2 + k^2)(A_2^2 + B_2^2)^2},$$
(29)

$$(T_{p1}'')_{01} = 2 \frac{Q_1(pr - sq) + Q_2(pq + rs)}{(n_1^2 + k_1^2)(A_3^2 + B_3^2)^2},$$
(30)

where

$$A_3 = n_1 a + k_1 c + n_0 b + k_0 d , \qquad (31)$$

$$B_3 = k_1 a - n_1 c + k_0 b - n_0 d , \qquad (32)$$

$$p = N(3n_0^2k_0 - k_0^3) + P(n_0^3 - 3n_0k_0^2), \qquad (33)$$

$$q = n_1 (A_3^2 - B_3^2) - 2A_3 B_3 k_1, \qquad (34)$$

$$r = k_1 (B_3^2 - A_3^2) - 2A_3 B_3 n_1,$$
(35)

 $\langle \mathbf{a} \mathbf{a} \rangle$ 

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 $+((R_{s_0}^{\prime\prime})_{01}+\eta_1(R_{s_0}^{\prime})_{12})$ 

$$s = N(n_0^3 - 3n_0k_0^2) - P(3n_0^2k_0 - k_0^3), \qquad (36)$$

$$a = \operatorname{Re}(\cos\varphi_0), \qquad (37)$$

$$b = \operatorname{Re}(\cos \varphi_1), \qquad (38)$$

$$c = \operatorname{Im}(\cos \varphi_1) \qquad (39)$$

$$c = \operatorname{Im}(\cos\varphi_0), \qquad (39)$$

$$d = \operatorname{Im}(\cos\varphi_1), \qquad (40)$$

$$N = \operatorname{Re}(\sin\varphi_0)a - \operatorname{Im}(\sin\varphi_0)c \tag{41}$$

$$P = -\operatorname{Re}(\sin\varphi_0)c - \operatorname{Im}(\sin\varphi_0)a \tag{42}$$

Coefficients  $(T'_{p0})_{10}$ ,  $(T''_{p0})_{10}$ ,  $(T'_{p1})_{10}$ ,  $(T''_{p1})_{10}$  correspond to the electromagnetic wave propagation from layer 1 to medium 0, that leads to the changes:  $\cos \varphi_0 \leftrightarrow \cos \varphi_1$ ,  $\sin \varphi_0 \leftrightarrow \sin \varphi_1$ ,  $n_0 \leftrightarrow n_1$ ,  $k_0 \leftrightarrow k_1$ .

Let us take into account  $N_0 = n_0 - ik_0$ ,  $N_1 = n_1 - ik_1$ ,  $N_2 = n_2 - ik_2$ ,  $Q = Q_1 - iQ_2$  and compare expressions (10, 11) with (2, 3). Thus we obtain expressions for  $R'_{p0}$ ,  $R''_{p0}$ ,  $R''_{p1}$ ,  $R''_{p1}$ ,  $R''_{s0}$  and  $R''_{s0}$ .

$$R'_{p0} = (((R'_{p0})_{01} + \xi_1 (R'_{p0})_{12} - \eta_1 (R''_{p0})_{12})(1 + \xi_1 L_{0112} - \eta_1 M_{0112}) + ((R''_{p0})_{01} + \eta_1 (R'_{p0})_{12} + \xi_1 (R''_{p0})_{12})(\xi_1 M_{0112} + \eta_1 L_{0112}))((1 + \xi_1 L_{0112} - \eta_1 M_{0112})^2 + (\xi_1 M_{0112} + \eta_1 L_{0112})^2)^{-1},$$

$$R''_{p0} = (((R''_{p0})_{01} + \eta_1 (R'_{p0})_{12} + \xi_1 (R''_{p0})_{12})(1 + \xi_1 L_{0112} - \eta_1 M_{0112}) - (43)$$

$$-((R'_{p0})_{01} + \xi_1(R'_{p0})_{12} - \eta_1(R''_{p0})_{12})(\xi_1M_{0112} + \eta_1L_{0112}))((1 + \xi_1L_{0112} - \eta_1M_{0112})^2 + (\xi_1M_{0112} + \eta_1L_{0112})^2)^{-1},$$
(44)

$$R'_{p1} = \frac{\Omega \chi - \Gamma \varpi}{\Omega^2 + \Gamma^2} - R'_{p0}, \qquad (45)$$

$$R_{p1}^{\prime\prime} = \frac{\Omega \overline{\omega} + \Gamma \chi}{\Omega^2 + \Gamma^2} - R_{p0}^{\prime\prime}, \qquad (46)$$

$$R'_{50} = (((R'_{50})_{01} + \xi_1 (R'_{50})_{12} - \eta_1 (R''_{50})_{12})(1 + \xi_1 H_{0112} - \eta_1 J_{0112}) + \xi_1 (R''_{50})_{12})(\xi_1 J_{0112} + \eta_1 H_{0112}))((1 + \xi_1 H_{0112} - \eta_1 J_{0112})^2 + (\xi_1 J_{0112} + \eta_1 H_{0112})^2)^{-1},$$
(47)

$$R_{s0}^{\prime\prime} = (((R_{s0}^{\prime\prime})_{01} + \eta_1 (R_{s0}^{\prime})_{12} + \xi_1 (R_{s0}^{\prime\prime})_{12})(1 + \xi_1 H_{0112} - \eta_1 J_{0112}) -$$

 $-((R_{s0}')_{01}+\xi_1(R_{s0}')_{12}-\eta_1(R_{s0}'')_{12})(\xi_1J_{0112}+\eta_1H_{0112}))((1+\xi_1H_{0112}-\eta_1J_{0112})^2+(\xi_1J_{0112}+\eta_1H_{0112})^2)^{-1},$ (48)where the following notations are used:

$$L_{0112} = (R'_{p0})_{12} (R'_{p0})_{01} - (R''_{p0})_{12} (R''_{p0})_{01},$$
(49)

$$M_{0112} = (R'_{p0})_{01} (R''_{p0})_{12} + (R'_{p0})_{12} (R''_{p0})_{01},$$
(50)

$$\xi_1 = \operatorname{Re}(\exp(-i2\beta_1)), \tag{51}$$

$$\eta_1 = -\operatorname{Im}(\exp(-i2\beta_1)), \qquad (52)$$

$$J_{0112} = (R'_{S0})_{01} (R''_{S0})_{12} + (R'_{S0})_{12} (R''_{S0})_{01},$$
(53)

$$H_{0112} = (R'_{S0})_{01} (R'_{S0})_{12} - (R''_{S0})_{12} (R''_{S0})_{01},$$
(54)

$$\Omega = 1 - \xi_1 (rr_{10}rr_{12} - r\dot{i}_{10}r\dot{i}_{12}) + \eta_1 (r\dot{i}_{10}rr_{12} + rr_{10}r\dot{i}_{12}),$$
(55)

$$\varpi = ri_{01} - (\xi_1 rr_{12} - \eta_1 ri_{12})(ri_{01} rr_{10} + ri_{10} rr_{01} - \tau) - (\xi_1 ri_{12} + \eta_1 rr_{12})(rr_{01} rr_{10} - ri_{01} ri_{10} - \theta),$$

$$\Gamma = \xi_1 (ri_{10} rr_{12} + rr_{10} ri_{12}) + \eta_1 (rr_{10} rr_{12} - ri_{10} ri_{12}),$$
(56)
(57)

$$f = \xi_1 (r_{i_10} r_{i_12} + r_{i_10} r_{i_{12}}) + \eta_1 (r_{i_10} r_{i_{12}} - r_{i_{10}} r_{i_{12}}),$$
(57)

$$\chi = rr_{01} - (\xi_1 rr_{12} - \eta_1 ri_{12})(rr_{01} rr_{10} - ri_{01} ri_{10} - \theta) + (\xi_1 ri_{12} + \eta_1 rr_{12})(ri_{01} rr_{10} + ri_{10} rr_{01} - \tau),$$
(58)

$$\theta = tr_{01}tr_{10} - t\dot{t}_{01}t\dot{t}_{10}, \qquad (59)$$

$$\tau = t \dot{i}_{01} t r_{10} + t \dot{i}_{10} t r_{01} \,. \tag{60}$$

Thus, we have all formulae that are necessary for theoretical calculation of the ellipsometric angles (2-9) in case of a single-layer model. Final step is giving the best fit to the experimental data by the use of the wavelength-to-wavelength Nelder-Mead minimization of the ellipsometric angles. It yields

the spectral dependences of real  $(Q_1)$  and imaginary parts  $(Q_2)$  of magneto-optical parameter Q. So, we have information about all elements of the dielectric permittivity tensor.

#### 4. Conclusion

To conclude, we have proposed an approach to studying single-layer nanomaterials by means of magneto-ellipsometry. The algorithm of experimental data analysis ( $\psi_0$ ,  $\Delta_0$ ,  $\psi_0+\delta\psi$ ,  $\Delta_0+\delta\Delta$ ) is presented. As a result, optical and magneto-optical properties can be easily and reliably characterized during synthesis.

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