ISSN 0021-3640, JETP Letters, 2016, Vol. 104, No. 6, pp. 380–383. © Pleiades Publishing, Inc., 2016.
Original Russian Text © I.V. Timofeev, S.Ya. Vetrov, 2016, published in Pis'ma v Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki, 2016, Vol. 104, No. 6, pp. 393–397.

OPTICS AND LASER PHYSICS

Chiral Optical Tamm States at the Boundary of the Medium with Helical Symmetry of the Dielectric Tensor

I. V. Timofeev[a, b, *] **and S. Ya. Vetrov**[a, c]

[a] *Kirensky Institute of Physics, Siberian Branch, Russian Academy of Sciences, Krasnoyarsk, 660036 Russia*
[b] *Laboratory of Nonlinear Optics and Spectroscopy, Siberian Federal University, Krasnoyarsk, 660041 Russia*
[c] *Institute of Engineering Physics and Radio Electronics, Siberian Federal University, Krasnoyarsk, 660041 Russia*
** e-mail: tiv@iph.krasn.ru*

Received June 28, 2016; in final form, August 1, 2016

A new optical state at the boundary of a chiral medium whose dielectric tensor has a helical symmetry is described analytically and numerically. The case of zero tangential wavenumber is considered. The state localized near the boundary does not transfer energy along this boundary and decreases exponentially with the distance from the boundary. The penetration of the field into the chiral medium is blocked at wavelengths corresponding to the photonic band gap and close to the pitch of the helix. The polarization of light near the boundary has the same sign of chirality as the helical symmetry. It is shown that the homogeneous environment or a substrate should exhibit anisotropic metallic reflection. The spectral manifestation of the state is determined by the angle between the optical axes of the media at the interface. A state at the interface between a cholesteric liquid crystal and an anisotropic metal–dielectric nanocomposite was considered as an example.

DOI: 10.1134/S0021364016180119

A surface state that can be observed at the normal incidence of light on a surface is called the optical Tamm state [1]. Localized light can be treated as being trapped near the interface between two mirrors, which are media at whose interface light is reflected. There are various types of mirrors, including metallic and photonic-crystal, that can reflect normally incident light. Photonic-crystal reflection in media that do not have the mirror symmetry of optical properties but have a continuous helical symmetry of the dielectric tensor is particularly remarkable. We call these media *chiral*. An example of such media is a cholesteric liquid crystal consisting of oriented molecules whose preferable direction is twisted in space as a helix [2]. Another example is a twisted tilted sculptured thin film [3]. Screw periodicity results in the diffraction (bulk reflection) of only light circularly polarized in the direction corresponding to the twisting of the helix. In contrast to nonchiral (mirror symmetric) photonic crystals, light with opposite circular polarization is not diffracted.

Optical Tamm states were detected both at the interface between two nonchiral media [1] and at the interface of two chiral mirrors [4] in the form of defect modes. However, as far as we know, optical Tamm states have not yet been obtained at the interface between chiral and nonchiral mirrors. Difficulty appears because the isotropic mirror changes the polarization of light and a diffracting wave of the chiral photonic crystal is transferred to a nondiffracting wave. As a result, the wave undergoes no more than two cycles of reflections and, then, leaves the boundary of the mirror [5]. In this work, we attempt to avoid the described difficulty by means of an anisotropic substrate. We consider the case of normal incidence where the energy transfer along the surface is absent and the wave vector does not have a tangential component.

We represent the anisotropic substrate as a uniaxial crystal with the optical axis directed along the x axis and the medium itself fills the $z < 0$ half-space, where the field is decomposed into the extraordinary and ordinary waves. The equation for waves propagating in the direction opposite to the direction of the z axis has the form

$$\begin{bmatrix} E_x \\ H_y \\ E_y \\ -H_x \end{bmatrix} = E_x^0 \begin{bmatrix} 1 \\ -n_e^0 \\ 0 \\ 0 \end{bmatrix} \exp\left(-i\kappa n_e^0 z - i\omega t\right) + E_y^0 \begin{bmatrix} 0 \\ 0 \\ 1 \\ -n_o^0 \end{bmatrix} \exp\left(-i\kappa n_o^0 z - i\omega t\right), \tag{1}$$

where $E_{x,y}$ and $H_{x,y}$ are the complex projections of the electric and magnetic field strengths, respectively; $\kappa = \omega/c$ is the wave vector in vacuum; and $n^0_{e,o}$ are the extraordinary and ordinary refractive indices of the substrate, respectively.

The eigenwave for a chiral medium can be described as follows [2]:

$$\begin{bmatrix} E_x \\ H_y \\ E_y \\ -H_x \end{bmatrix} = A\begin{bmatrix} 1 \\ (q+\tau)/\kappa \\ -i \\ -i(q+\tau)/\kappa \end{bmatrix}\exp(i(qz+\tilde{\varphi}(z)-\omega t)) + B\begin{bmatrix} 1 \\ -(q-\tau)/\kappa \\ i \\ -i(q-\tau)/\kappa \end{bmatrix}\exp(i(qz-\tilde{\varphi}(z)-\omega t)). \quad (2)$$

Here, A and B are the complex amplitudes of waves that are circularly polarized along the helix and travel forward and backward along the z axis, respectively. The angle of twisting of the optical axis $\tilde{\varphi}(z) = \tau z + \varphi$ is measured from the x axis in the direction of the y axis; $\tau = 2\pi/p$ is the wave vector of twisting of the optical axis; p is the pitch of the helix; and the wave vector q is given by the expression

$$q = \sqrt{\tau^2 + \epsilon\kappa^2 - 2\tau\kappa\sqrt{\epsilon + \delta^2\kappa^2/4\tau^2}}.$$

The components of the dielectric tensor have the form $\epsilon_{\parallel,\perp} = \epsilon \pm \delta$.

The wave vector q of a diffracting wave is imaginary in the band gap given by the inequalities

$$\frac{\tau}{\sqrt{\epsilon+\delta}} < \kappa = \frac{\omega}{c} < \frac{\tau}{\sqrt{\epsilon-\delta}}. \quad (3)$$

The strengths of the waves A and B in the band gap have the same length and their phase difference Φ depends on the frequency and varies from 0 to π:

$$\frac{A}{B} = e^{+i\Phi(\kappa)} = \frac{(q-\tau)^2/\kappa^2 - \epsilon}{\delta},$$

$$\frac{B}{A} = e^{-i\Phi(\kappa)} = \frac{(q+\tau)^2/\kappa^2 - \epsilon}{\delta}.$$

For further consideration, it is convenient to exclude q by representing equations in the form

$$\kappa\sqrt{\varepsilon + \delta\exp(-i\Phi(\kappa))} = \tau - q,$$
$$\kappa\sqrt{\varepsilon + \delta\exp(+i\Phi(\kappa))} = \tau + q,$$

and taking the sum of these two equations:

$$\kappa\mathrm{Re}(\sqrt{\epsilon + \delta\exp(i\Phi(\kappa))}) = \tau. \quad (4)$$

It is convenient to divide all wave vectors and refractive indices by the average refractive index of the chiral medium; i.e., the refractive index of the substrate is

$$n_{e,o} = n^0_{e,o}/\sqrt{\epsilon}. \quad (5)$$

To derive Eq. (4), we use the small anisotropy approximation $\delta \ll \epsilon$. In this approximation, the ratio of the amplitudes of the electric and magnetic field strengths is approximately unity: $(q \pm \tau)/\kappa \approx \pm 1$. The matching conditions for fields at the interface can be obtained by directly equating strengths given by Eqs. (1) and (2) at the $z = 0$ interface at the time $t = 0$:

$$E^{\varphi}_x\begin{bmatrix} 1 \\ -n_e \\ 0 \\ 0 \end{bmatrix} + E^{\varphi}_y\begin{bmatrix} 0 \\ 0 \\ 1 \\ -n_o \end{bmatrix} = r\begin{bmatrix} 1 \\ 1 \\ -i \\ -i \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ i \\ -i \end{bmatrix}, \quad (6)$$

$$r = \exp(i\Phi(\kappa) + 2i\varphi). \quad (7)$$

Here, $E^{\varphi}_{x,y} = E^0_{x,y}/B\exp(i\varphi)$. Four unknowns in the substrate are expressed in terms of r as

$$E^{\varphi}_x = 1 + r; \quad E^{\varphi}_y = i(1-r);$$
$$n_e = \frac{1-r}{1+r}; \quad n_o = \frac{1+r}{1-r}. \quad (8)$$

Physically, r is the amplitude reflection coefficient of the substrate. Expressing r in terms of each of the refractive indices, we arrive at the known Fresnel equations for the ordinary and extraordinary waves:

$$r_e = r = \frac{1-n_e}{1+n_e}; \quad r_o = -r = \frac{1-n_o}{1+n_o}. \quad (9)$$

One of these waves is reflected in antiphase. Consequently, a diffracting circular wave is reflected to a diffracting wave. The substrate satisfying the condition of inversion of the refractive index $n_e = 1/n_o$ is a polarization-conserving anisotropic mirror [6].

The found solution given by Eq. (8) ensures phase matching. After the reflection from two mirrors, the wave should return to the initial state in the same phase, ensuring constructive interference and resonance. The phase shift is compensated by the angle φ between the optical axes at the interface because the angle of spatial rotation about the direction of propagation for circularly polarized light is equal to the phase of the wave. A smooth variation of this angle by π at double reflection changes the phase by 2π and makes it possible to ensure phase matching at the interface between the mirrors.

The frequency of the optical Tamm state can be found from the second matching condition given by

Eq. (7) ensuring phase matching, which can be represented in the form

$$\Phi(\kappa) = \rho - 2\varphi,$$

where ρ is the complex phase of the amplitude reflection coefficient $r = |r|\exp(i\rho)$, which is defined up to 2π. Substituting this expression for $\Phi(\kappa)$ in the cholesteric liquid crystal into Eq. (4), we obtain the frequency of the optical Tamm state as a function of the angle φ:

$$\kappa = \frac{\omega}{c} = \frac{\tau}{\mathrm{Re}(\sqrt{\epsilon + \delta\exp(i\rho - 2i\varphi)})}. \quad (10)$$

Expression (10) determines the spectral manifestation of the optical Tamm state inside band gap (3) of the chiral medium. It is a dispersion relation because the tangential wave vector of the described optical Tamm state is zero and cannot determine the frequency of the state. Solution in the case of a nonzero tangential wave vector is beyond the scope of this work.

Another condition of the existence of the optical Tamm state is *the localization of the field* near the interface. Localization on the side of the chiral medium is ensured by the imaginary wave vector q in the band gap. Localization on the side of the substrate requires that both refractive indices have a positive imaginary part, which corresponds to damping at chosen signs of the complex factor $\exp(ikz - i\omega t)$. However, in the approximation under consideration, the condition of inversion of the refractive indices $n_e = 1/n_o$ provides the condition $\mathrm{Im}(n_e)\cdot\mathrm{Im}(n_o) < 0$. For this reason, for Eqs. (8) to be consistent with localization, only the limiting case of a small imaginary part where the Q factor of the state and the localization length simultaneously tend to infinity is correct. A finite imaginary part of the refractive index of the substrate leads to losses through nondiffracting polarization and through metallic-type absorption.

We construct a localized state by the Berreman numerical method. A chiral medium is a right-handed cholesteric liquid crystal with the anisotropy $\delta = 0.2$ and the normalized pitch of the helix $p\sqrt{\epsilon} = 500$ nm; the thickness of the layer is five pitches of the helix. A substrate is a nanocomposite of silver spheroids that are oblate in the x direction and are placed in a matrix with the refractive index equal to the average refractive index of the chiral medium. The Maxwell–Garnett formula makes it possible to select the parameters of the nanocomposite such that $n_o = (1+i)n_m$ and $n_e = (1+i)/2n_m$ at a certain frequency in the visible range [6]. Let $n_m = 10$. The dispersion of the nanocomposite is disregarded. The condition $n_e = 1/n_o^*$ is valid, which substitutes the condition of inversion of the refractive index following from the analytical solution given by Eq. (8). The asterisk stands for complex conjugation. The proposed substitution ensures the relations $r_e = -r_o^*$ and $\mathrm{Re}(r_e) = -\mathrm{Re}(r_o)$ between the amplitude reflection coefficients of the substrate. As a result, reflection to diffracting polarization is consistent with conditions (7) and (10). The reflection phase is $\rho = 0$ because the amplitude of reflection is a positive real quantity. At $\sin(\varphi) < 0$, an optical Tamm state does not appear.

Figure 1a shows the dependence of the local intensity (square of the amplitude of the electric field strength) on the distance to the boundary for the optical Tamm state. The local intensity is measured in units of intensity of the wave that excites the optical Tamm state, is incident from the right from the chiral medium, and has left circular polarization. The result of the direct numerical calculation by the Berreman method is presented. Wave B propagating to the left includes a wave exciting the optical Tamm state and, thereby, has a higher intensity than wave A that is reflected from the substrate and propagates to the right. In order to avoid encumbering of Fig. 1a, the total local intensity $|A+B|^2$ is not shown, which near the boundary is almost seven times higher than the local intensity of the wave exciting the optical Tamm state. The contribution from the ordinary wave in the substrate is also unseen on the chosen scale because this wave is rapidly damped with depth of the substrate and has a small amplitude of the electric field strength. To obtain a smooth exponentially decreasing envelope consistent with Eqs. (1) and (2), the wave exciting the optical Tamm state should be subtracted from the solution.

Figure 1b illustrates the condition of matching of the fields at the interface. The polarization ellipse of the resulting field is elongated in the x direction for both the electric and magnetic field strengths. Its major and minor semiaxes in the chiral medium are proportional to the sum $|A| + |B|$ and difference $|A| - |B|$ of the amplitudes, respectively. The electric field strength for the extraordinary wave, as well as the magnetic field strength for the ordinary wave, is elongated in the substrate. The equality of the ratios of the major and minor semiaxes of the ellipse in the chiral medium and substrate gives

$$\frac{|A| + |B|}{|A| - |B|} = \frac{1}{n_e} n_o \gg 1. \quad (11)$$

This relation is consistent with analytical solution (8).

Figure 2 shows the reflectance from the interface for light with right circular polarization incident from the right-handed chiral medium normally to the interface. The dependence (10) of the frequency of the optical Tamm state on the angle φ qualitatively corresponds to a similar dependence on the modulation phase in a nonchiral photonic crystal (see [7] and references therein).

The smooth rotation of the mirrors results in the shift of the frequency of the optical Tamm state to the edges of band gap (3). When the optical axes coincide with each other, the optical Tamm state is at the high-

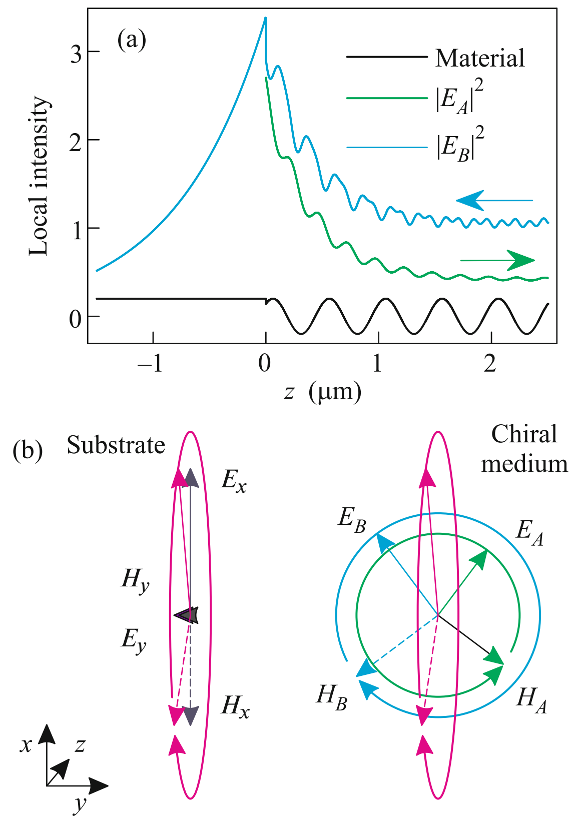

Fig. 1. (Color online) (a) Local intensity of the field versus the distance from the boundary. Line Material corresponds to the orientations of the optical axis in the substrate (straight segment) and in the chiral medium (sinusoidal projection on the x axis). (b) Matching of fields at the interface. The solid and dashed arrows mark the electric and magnetic field strengths, respectively. The field in the chiral medium is represented in the form of the circular waves incident on the interface (B) and reflected from it (A). The field in the substrate is decomposed into the extraordinary (x) and ordinary (y) waves.

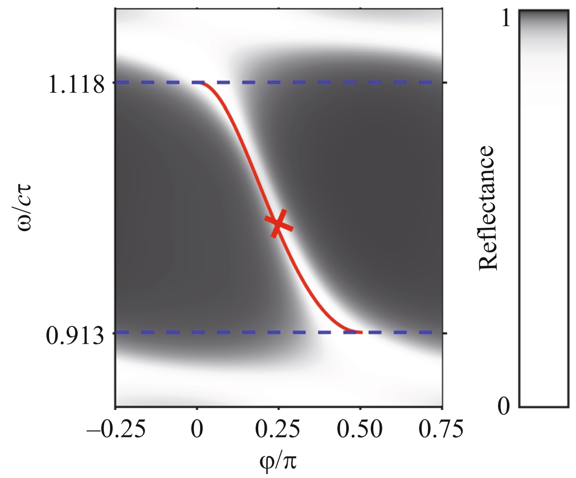

Fig. 2. (Color online) Reflectance from the interface versus the angle φ between the optical axes at the interface. The blue dashed straight segments indicate the edges of the photonic band gap. The red solid line shows analytical dependence (10). The red cross at $\varphi = \pi/4$ and $\kappa = \tau$ corresponds to the parameters in Fig. 1a.

frequency edge of the band. The strongest dip in the reflectance at the angle $\varphi = \pi/4$ corresponds to the middle of the band. In the cases of excitation by light with right and left circular polarizations, the reflectance in the dip is 90% (red cross in Fig. 2) and 45% (Fig. 1a), respectively. This angle is $\varphi = \pi/4$ because the gradient of the refractive index of the chiral medium is maximal at an angle of $\pi/4$ to the optical axis and the electric field oriented in this direction undergoes strong bulk reflection. The optical Tamm state is shifted to the low-frequency edge of the band at perpendicular optical axes. For angles larger than $\pi/2$, the dip of the reflectance is absent because condition (10) is not satisfied.

To summarize, the problem of the existence of an optical Tamm state at the interface between chiral and nonchiral mirrors at zero tangential wave vector has been analytically solved. The solution imposes a strong condition on the parameters of the homogeneous substrate at the interface with the chiral mirror where the chiral optical Tamm state can exist. The Q factor of the state tends to infinity only if the localization length tends to infinity. The found optical Tamm state is localized near the interface and decreases exponentially on both sides of it. At the passage of the interface, the phase is controlled by the rotation of the mirrors in the plane of interface, which is described by condition (10) serving as the dispersion relation. The analytical dependence is in agreement with the direct numerical calculation.

This work was performed within the framework of the state assignment from the Ministry of Education and Science of the Russian Federation for R&D at the Siberian Federal University (assignment no. 3.1276.2014/K).

REFERENCES

1. A. P. Vinogradov, A. V. Dorofeenko, A. M. Merzlikin, and A. A. Lisyanskii, Phys. Usp. **53**, 243 (2010).
2. V. A. Belyakov, *Complex-Structured Periodic Media* (Springer, New York, 1992), p. 352.
3. A. Lakhtakia and M. Russell, *Sculptured Thin Films: Nanoengineered Morphology and Optics,* SPIE Press Monograph, Vol. PM143 (SPIE, Bellingham, WA, 2005).
4. J. Schmidtke and W. Stille, Eur. Phys. J. E **12**, 553 (2003).
5. I. V. Timofeev, V. G. Arkhipkin, S. Ya. Vetrov, V. Ya. Zyryanov, and W. Lee, Opt. Mater. Express **3**, 496 (2013).
6. N. V. Rudakova, I. V. Timofeev, P. S. Pankin, and S. Ya. Vetrov, Izv. Akad. Nauk, Ser. Fiz. (2016, in press).
7. V. A. Belyakov, V. P. Orlov, and G. I. Shilina, Sov. Phys. JETP **75**, 189 (1992).

Translated by R. Tyapaev