

# Coexistence of the Chiral Superconductivity and Noncollinear Magnetic Order in the Ensemble of Hubbard Fermions on a Triangular Lattice

V. V. Val'kov\* and A. O. Zlotnikov

*Kirensky Institute of Physics, Federal Research Center KSC, Siberian Branch, Russian Academy of Sciences, Akademgorodok, Krasnoyarsk, 660036 Russia*

\* e-mail: vvv@iph.krasn.ru

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For the system of strongly correlated electrons on a triangular lattice, the possibility of coexisting superconductivity with the chiral order parameter and the  $120^\circ$ -type noncollinear spin ordering is demonstrated. The integral self-consistency equation for the superconducting order parameter is derived using the diagram technique for Hubbard operators taking into account the spin structure, exchange interaction within two coordination spheres, and intersite Coulomb repulsion.

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## 1. INTRODUCTION

Recently, the studies exploring the possibility for the formation of spiral structures in materials with strongly correlated electrons again became active [1]. The new surge of interest in such studies was initiated by paper [2], which put forward the suggestion on the possible formation of the Majorana mode if the superconducting phase with the chiral order parameter on a triangular lattice coexists with the noncollinear magnetic ordering. However, Lu and Wang [2] did not address the issue whether the suggested magnetic structure indeed allows for the coexistence with the chiral superconductivity. Since the Majorana-related problems are quite topical, such analysis appears to be necessary, the more so that we demonstrate in this work that the integral self-consistency equations for the chiral superconducting order parameter are not satisfied for the magnetic structure suggested in [2]. Therefore, it turns out to be especially important to find such type of noncollinear magnetic ordering for which the chiral superconducting phase satisfies the coexistence conditions, i.e., the chiral order parameter indeed satisfies the coexistence equations. Our work is focused mainly on such problem.

There is a common opinion that the chiral superconducting phase can exist in water-intercalated sodium cobaltites  $\text{Na}_x\text{CoO}_2 \cdot y\text{H}_2\text{O}$  including conducting layers with the triangular lattice. The first theoretical studies based on the  $t-J$  model for the triangular lattice predicted the chiral  $d_{x^2-y^2} + id_{xy}$  symmetry type for the order parameter or  $p$ -wave pairing [3,

4]. For these two symmetry types, the superconducting gap opens over the whole Fermi surface. This contradicts the nuclear quadrupole resonance (NQR) measurements. Therefore, it was suggested that these compounds exhibit superconductivity with other symmetry types [5–7].

The arising contradiction was removed in [8], where it was shown that the inclusion of the pairing interaction in the second coordination sphere of the triangular lattice induces a set of nodal points located within the Brillouin zone even in the case of the  $d_{x^2-y^2} + id_{xy}$  symmetry type of the superconducting order parameter. This eliminates the existing discrepancy between the preferable existence of the chiral superconducting phase for the triangular lattice and the gapless superconductivity in sodium cobaltites observed in experiment. Later on, in the framework of the  $t-J_1-J_2-V$  model on the triangular lattice, it was shown that the exchange interaction within both the first and second coordination spheres in the presence of the nearest-neighbor intersite Coulomb repulsion leads to the formation of a new set of nodal points [9].

In this work, we derive an integral equation determining the Cooper instability in the case of the noncollinear spin ordering in the framework of the  $t-J_1-J_2-V$  model on the triangular lattice using the diagram technique for the Hubbard operators [10–12]. The analysis of this equation demonstrates the possibility of the coexistence of chiral superconductivity and the  $120^\circ$ -type spin structure. In this situation, the nearest-neighbor intersite Coulomb repulsion of

fermions leads to the formation of the chiral superconducting order parameter described by the superposition of two singlet and one triplet invariant. Note that, for the stripe structure of the spin ordering [2], the phase with the coexisting order parameters also appears, but the superconducting order parameter does not correspond to the chiral symmetry of the triangular lattice.

The performed analysis of the Cooper instability in the limit of strong electron correlations has two characteristic features. One of them is related to the use of Hubbard operators, for which the commutation relations are not fermionic. As a result, the system exhibits the kinematic interaction and the diagrams contain the end factors [10–14]. The latter factors lead to an additional contribution to the fermion excitation spectrum in the noncollinear phase, which manifests itself in the calculations of the charge-carrier-density dependence of the temperature corresponding to the transition to the phase with the coexistence of chiral superconductivity and noncollinear spin ordering. The second characteristic feature is related to the Hubbard fermions corresponding to the upper Hubbard subband, which is formed owing to strong correlations. The presented results demonstrate that the conditions underlying the existence of the Majorana modes on the triangular lattice are determined not only by the type of spin ordering but also by strong electron correlations playing an important role.

### 2. GREEN'S FUNCTIONS AND THE EXCITATION SPECTRUM AT THE NONCOLLINEAR SPIN ORDERING

We solve the problem concerning the phase with the coexistence of chiral superconductivity and noncollinear spin ordering using the  $t$ - $J_1$ - $J_2$ - $V$  model, in which the electron states correspond to the upper Hubbard subband

$$\begin{aligned}
 H = & \sum_{f\sigma} (\varepsilon - \mu) X_f^{\sigma\sigma} + \sum_f (2\varepsilon + U - 2\mu) X_f^{22} \\
 & + \sum_{fm\sigma} t_{fm} X_f^{2\bar{\sigma}} X_m^{\bar{\sigma}2} + \sum_{fm} J_{fm} \left( X_f^{\uparrow\downarrow} X_m^{\downarrow\uparrow} - X_f^{\uparrow\uparrow} X_m^{\downarrow\downarrow} \right) \quad (1) \\
 & + \frac{V}{2} \sum_{f\delta} n_f n_{f+\delta}.
 \end{aligned}$$

The first and second terms in the Hamiltonian describe the one- and two-electron states in the atomic representation with the initial energies measured relative to the chemical potential. The third term in the Hamiltonian corresponds to the electron hopping between sites in the triangular lattice. The parameter  $t_{fm}$  is the probability amplitude for such hoppings. The fourth term describes the exchange interaction characterized by the parameter  $J_{fm}$ . The last term in the Hamiltonian is related to the Coulomb

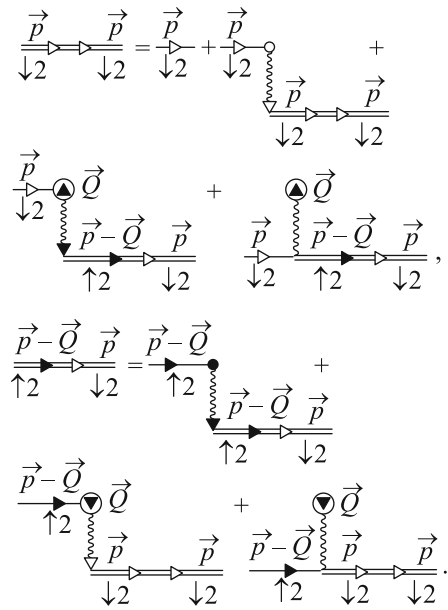


Fig. 1. Set of equations for fermion Green's functions corresponding to the collective excitations at the spiral spin ordering.

interaction  $V$  between electrons located at the nearest-neighbor sites. The electron number operator is  $n_f = X_f^{\uparrow\uparrow} + X_f^{\downarrow\downarrow} + 2X_f^{22}$ .

Further on, we assume the existence of the spin ordering with the wave vector  $\mathbf{Q}$ :  $\langle \mathbf{S}_f \rangle = M_0(\cos(\mathbf{Q}\mathbf{R}_f), -\sin(\mathbf{Q}\mathbf{R}_f), 0)$ .

To analyze the characteristics of the system both in the phase with the spiral spin ordering and in the phase corresponding to the coexistence with superconductivity, we introduce the Green's functions [10, 11]

$$D_{\alpha,\beta}(f\tau; f'\tau') = -\langle T_\tau \tilde{X}_f^\alpha(\tau) \tilde{X}_{f'}^\beta(\tau') \rangle, \quad (2)$$

where  $\tilde{X}_f^\alpha(\tau)$  are the Hubbard operators in the Matsubara representation, and  $\alpha = \alpha(n, m)$  is the root vector [10, 11], determining the transition from the  $m$ th single-site state to the  $n$ th single-site state.

The spectrum of cooperative fermion excitations is determined by the poles of the Green's function  $G_{\downarrow 2, \downarrow 2}^{(0)}(\mathbf{p}, i\omega_n)$ . The application of the diagram technique for Hubbard operators [10–12] allows writing out the set of equations determining this function in the form of diagrams, which are shown in Fig. 1.

In the diagrams shown in Fig. 1, the double thin line with two open arrows corresponds to the function  $G_{\downarrow 2, \downarrow 2}^{(0)}(\mathbf{p}, i\omega_n)$  describing the propagation of the Hubbard fermion with spin up and thin lines with open (spin up) and closed (spin down) arrows describe the bare propagator  $G(i\omega_n) = (i\omega_n - \xi_0)^{-1}$ ,  $\xi_0 = U - \mu +$

$(1 - n/2)J_0 + nV_0$ . The wavy lines with arrows denote the Fourier transforms of the hopping integrals  $t_{fm}$ , whereas the wavy lines in the last term on the right-hand side of the equations correspond to the Fourier transforms of the exchange interaction  $J_{fm}$ . The existence of the noncollinear spin ordering in the system manifests itself in the presence of the function  $G_{\uparrow 2, \downarrow 2}^{(0)}(\mathbf{p} - \mathbf{Q}, \mathbf{p}; i\omega_n)$  describing the propagation of a fermion associated with the change in the spin projection. This function corresponds to the double thin line with the open and closed arrows. Here, the incoming and outgoing momenta differ by the spin structure vector  $\mathbf{Q}$ . The large closed triangle put into the circle is associated with the Fourier component corresponding to the structure of spin ordering. The direction of such arrow determines the addition or subtraction of the  $\mathbf{Q}$  vector characterizing the structure. The other notation is conventional [13, 14].

The system of diagrams corresponds to the equations

$$\begin{aligned} (i\omega_n - \xi_{\mathbf{p}})G_{\downarrow 2, \downarrow 2}^{(0)} &= 1 + R_{\mathbf{p}-\mathbf{Q}}G_{\uparrow 2, \downarrow 2}^{(0)}, \\ (i\omega_n - \xi_{\mathbf{p}-\mathbf{Q}})G_{\uparrow 2, \downarrow 2}^{(0)} &= R_{\mathbf{p}}G_{\downarrow 2, \downarrow 2}^{(0)}, \end{aligned} \quad (3)$$

where  $R_{\mathbf{p}} = M_0(t_{\mathbf{p}} - J_{\mathcal{Q}})$ . By solving this set of equations, we find

$$G_{\downarrow 2, \downarrow 2}^{(0)}(\mathbf{p}; i\omega_n) = \frac{i\omega_n - \xi_{\mathbf{p}-\mathbf{Q}}}{(i\omega_n - \varepsilon_{1\mathbf{p}})(i\omega_n - \varepsilon_{2\mathbf{p}})}, \quad (4)$$

$$G_{\uparrow 2, \downarrow 2}^{(0)}(\mathbf{p} - \mathbf{Q}, \mathbf{p}; i\omega_n) = \frac{R_{\mathbf{p}}}{(i\omega_n - \varepsilon_{1\mathbf{p}})(i\omega_n - \varepsilon_{2\mathbf{p}})}, \quad (5)$$

where  $\xi_{\mathbf{p}} = \xi_0 + (n/2)t_{\mathbf{p}}$ . Two branches of the fermion excitation spectrum are given by the expressions

$$\varepsilon_{1,2\mathbf{p}} = \frac{\xi_{\mathbf{p}} + \xi_{\mathbf{p}-\mathbf{Q}}}{2} \mp \sqrt{\left(\frac{\xi_{\mathbf{p}} - \xi_{\mathbf{p}-\mathbf{Q}}}{2}\right)^2 + R_{\mathbf{p}}R_{\mathbf{p}-\mathbf{Q}}}. \quad (6)$$

### 3. COOPER INSTABILITY IN THE PHASE WITH THE NONCOLLINEAR SPIN ORDERING

For finding the superconducting transition temperature and determining the symmetry of the order parameter, it is sufficient to write in the linear approximation the relation between the anomalous Gorkov Green's function  $F_{2\uparrow, \downarrow 2}(\mathbf{p}; i\omega_n)$  and the anomalous components of the mass operator. The graphical representation of this relation is shown in Fig. 2.

The encircled parameters  $\Delta_1$  and  $\Delta_2$  correspond to the anomalous components of the mass operator. The explicit expression for the function  $G_{2\uparrow, 2\uparrow}^{(0)}(\mathbf{p}; i\omega_n)$  (with two closed arrows) involved in this equation can be found from the expression for  $G_{\downarrow 2, \downarrow 2}^{(0)}(\mathbf{p}; i\omega_n)$  based on

Fig. 2. Linearized representation for the Gorkov Green's function.

Fig. 3. Diagrams for the anomalous component  $\Delta_1(\mathbf{p})$  of the mass operator.

the symmetry considerations. Indeed, the change in the spin projection in the expressions for the diagonal with respect to spin normal Green's functions is accompanied by the inversion of the  $\mathbf{Q}$  vector. Moreover, the definitions of the Green's functions in the atomic representation and of their Fourier transforms directly imply the relation  $G_{2\downarrow, 2\downarrow}^{(0)}(\mathbf{p}; i\omega_n) = -G_{2\downarrow, 2\downarrow}^{(0)}(-\mathbf{p}; -i\omega_n)$ . Having in mind everything mentioned above, we find

$$G_{2\uparrow, 2\uparrow}^{(0)}(\mathbf{p}; i\omega_n) = \frac{i\omega_n + \xi_{\mathbf{p}-\mathbf{Q}}}{(i\omega_n + \varepsilon_{1\mathbf{p}})(i\omega_n + \varepsilon_{2\mathbf{p}})}, \quad (7)$$

$$G_{2\uparrow, 2\downarrow}^{(0)}(\mathbf{p}, \mathbf{p} - \mathbf{Q}; i\omega_n) = -\frac{R_{\mathbf{p}-\mathbf{Q}}}{(i\omega_n + \varepsilon_{1\mathbf{p}})(i\omega_n + \varepsilon_{2\mathbf{p}})}. \quad (8)$$

Using the obtained expressions, we find the linearized relation between the anomalous Green's function and the anomalous components of the mass operator ( $F = F_{2\uparrow, \downarrow 2}(\mathbf{p}; i\omega_n)$ )

$$F = \frac{[(i\omega_n)^2 - (\xi_{\mathbf{p}-\mathbf{Q}})^2] \Delta_1(\mathbf{p}) - R_{\mathbf{p}}R_{\mathbf{p}-\mathbf{Q}} \Delta_2(\mathbf{p} - \mathbf{Q})}{[(i\omega_n)^2 - (\varepsilon_{1\mathbf{p}})^2][(i\omega_n)^2 - (\varepsilon_{2\mathbf{p}})^2]}. \quad (9)$$

The graphical representation for  $\Delta_1(\mathbf{p})$  is shown in Fig. 3. We can see that the expression includes not only the anomalous function  $F_{2\uparrow, \downarrow 2}(\mathbf{p}; i\omega_n)$  considered above but also the anomalous function  $F_{2\downarrow, \uparrow 2}(\mathbf{p}; i\omega_n)$ . Its explicit form can be found using the aforementioned symmetry. As a result, we obtain  $F_{2\downarrow, \uparrow 2}(\mathbf{p}; i\omega_n) = -F_{2\uparrow, \downarrow 2}(-\mathbf{p}; -i\omega_n)$ . Writing out the graphical representation for  $\Delta_2(\mathbf{p})$  and taking into account the symmetry properties, we find that  $\Delta_2(\mathbf{p}) = -\Delta_1(-\mathbf{p})$ . Using these properties and the explicit expressions for the normal Green's functions,

we can obtain after the summation over the Matsubara frequencies the integral equation for the superconducting order parameter

$$\Delta_1(\mathbf{p}) = \frac{1}{N} \sum_{\mathbf{q}} \Phi(\mathbf{p}, \mathbf{q}) \Delta_1(\mathbf{q}), \quad (10)$$

where the kernel is given by the expression

$$\begin{aligned} \Phi(\mathbf{p}, \mathbf{q}) = & \sum_{\lambda=1}^2 \left\{ \left[ (J_{\mathbf{p}+\mathbf{q}} + J_{\mathbf{p}-\mathbf{q}} - V_{\mathbf{p}-\mathbf{q}}) (\varepsilon_{\lambda\mathbf{q}}^2 - \xi_{\mathbf{q}-\mathbf{Q}}^2) \right. \right. \\ & \left. \left. + (J_{\mathbf{p}+\mathbf{q}-\mathbf{Q}} + J_{\mathbf{p}-\mathbf{q}+\mathbf{Q}} - V_{\mathbf{p}+\mathbf{q}-\mathbf{Q}}) R_{\mathbf{q}} R_{\mathbf{q}-\mathbf{Q}} \right] \right. \\ & \left. \times \frac{(-1)^\lambda \tanh(\varepsilon_{\lambda\mathbf{q}}/2T)}{2\varepsilon_{\lambda\mathbf{q}} (\varepsilon_{2\mathbf{q}}^2 - \varepsilon_{1\mathbf{q}}^2)} \right\}. \end{aligned}$$

The existence of the noncollinear spin ordering manifests itself in the term containing the product  $R_{\mathbf{q}} R_{\mathbf{q}-\mathbf{Q}}$ , which appears in the integral equation for the superconducting order parameter.

#### 4. EFFECT OF THE NONCOLLINEAR MAGNETIC ORDERING ON THE FORMATION OF CHIRAL SUPERCONDUCTIVITY

In the absence of noncollinear spin ordering, the exact solution of integral equation (10) corresponds to the  $d_{x^2-y^2} + id_{xy}$  symmetry type and can be written as a superposition of two chiral invariants [9]:

$$\Delta_{2\mathbf{p}} = 2\Delta_{21}\varphi_{21}(\mathbf{p}) + 2\Delta_{22}\varphi_{22}(\mathbf{p}), \quad (11)$$

where

$$\begin{aligned} \varphi_{21}(\mathbf{p}) = & \cos(p_2) + e^{i2\pi/3} \cos(p_1) \\ & + e^{i4\pi/3} \cos(p_1 + p_2), \end{aligned} \quad (12)$$

$$\begin{aligned} \varphi_{22}(\mathbf{p}) = & \cos(2p_1 + p_2) + e^{i2\pi/3} \cos(2p_2 + p_1) \\ & + e^{i4\pi/3} \cos(p_1 - p_2). \end{aligned} \quad (13)$$

It is well known [15] that triplet anomalous amplitudes in the presence of a spin density wave in superconductors arise along with singlet pairings. This stems from the broken time-reversal symmetry in a magnetically ordered state and manifests itself via the admixture of triplet invariants.

According to these facts, the solution of the integral self-consistency equations (10) for the 120°-type magnetic ordering with  $\mathbf{Q} = (Q, Q)$ ,  $Q = 2\pi/3$  can be represented as a linear superposition of three chiral invariants

$$\Delta_1(\mathbf{p}) = 2\Delta_{21}\varphi_{21}(\mathbf{p}) + 2\Delta_{22}\varphi_{22}(\mathbf{p}) + 2\Delta_{11}\varphi_{11}(\mathbf{p}), \quad (14)$$

where the basis function

$$\begin{aligned} \varphi_{11}(\mathbf{p}) = & \sin(p_2) + e^{i\pi/3} \sin(p_1 + p_2) \\ & + e^{i2\pi/3} \sin(p_1) \end{aligned} \quad (15)$$

corresponds to the chiral  $p$ -wave symmetry for the triangular lattice. Let us emphasize that the triplet invariant is induced only when the Coulomb intersite interactions turn out to be involved in the case of noncollinear magnetic ordering. Without such interactions,  $\Delta_1(\mathbf{p})$  in the phase with the coexistence of superconductivity and the 120°-type magnetic ordering is determined by Eq. (11).

The splitting in the kernel of the integral equation allows obtaining the set of three algebraic equations determining the relations between the amplitudes in  $\Delta_1(\mathbf{p})$ :

$$\begin{aligned} (1 - A_{21})\Delta_{21} - A_{22}\Delta_{22} - A_{11}\Delta_{11} &= 0, \\ -B_{21}\Delta_{21} + (1 - B_{22})\Delta_{22} - B_{11}\Delta_{11} &= 0, \\ C_{21}\Delta_{21} + C_{22}\Delta_{22} + (1 + C_{11})\Delta_{11} &= 0. \end{aligned} \quad (16)$$

The coefficients in this set of equations are given by the expressions

$$\begin{aligned} A_{ij} = & \left( J_1 - \frac{V}{2} \right) \frac{2}{N} \sum_{\mathbf{q}\lambda} \frac{\varphi_{ij}(\mathbf{q}) \tanh(\varepsilon_{\lambda\mathbf{q}}/2T)}{\varepsilon_{\lambda\mathbf{q}} [\varepsilon_{\lambda\mathbf{q}}^2 - \varepsilon_{\lambda'\mathbf{q}}^2]} \\ & \times \left\{ \cos(q_2) [\varepsilon_{\lambda\mathbf{q}}^2 - \xi_{\mathbf{q}-\mathbf{Q}}^2] + \cos(q_2 - Q) R_{\mathbf{q}} R_{\mathbf{q}-\mathbf{Q}} \right\}, \end{aligned} \quad (17)$$

$$\begin{aligned} B_{ij} = & J_2 \frac{2}{N} \sum_{\mathbf{q}\lambda} \frac{\varphi_{ij}(\mathbf{q}) \cos(2q_1 + q_2) \tanh(\varepsilon_{\lambda\mathbf{q}}/2T)}{\varepsilon_{\lambda\mathbf{q}} [\varepsilon_{\lambda\mathbf{q}}^2 - \varepsilon_{\lambda'\mathbf{q}}^2]} \\ & \times \left\{ [\varepsilon_{\lambda\mathbf{q}}^2 - \xi_{\mathbf{q}-\mathbf{Q}}^2] + R_{\mathbf{q}} R_{\mathbf{q}-\mathbf{Q}} \right\}, \end{aligned} \quad (18)$$

$$\begin{aligned} C_{ij} = & V \frac{1}{N} \sum_{\mathbf{q}\lambda} \frac{\varphi_{ij}(\mathbf{q}) \tanh(\varepsilon_{\lambda\mathbf{q}}/2T)}{\varepsilon_{\lambda\mathbf{q}} [\varepsilon_{\lambda\mathbf{q}}^2 - \varepsilon_{\lambda'\mathbf{q}}^2]} \\ & \times \left\{ \sin(q_2) [\varepsilon_{\lambda\mathbf{q}}^2 - \xi_{\mathbf{q}-\mathbf{Q}}^2] - \sin(q_2 - Q) R_{\mathbf{q}} R_{\mathbf{q}-\mathbf{Q}} \right\}. \end{aligned} \quad (19)$$

Here,  $\lambda' \neq \lambda$ .

If the spin structure is characterized by the vector  $\mathbf{Q} = (Q, 0)$ , the performed analysis demonstrates that the Cooper instability also takes place, but  $\Delta_1(\mathbf{p})$  is determined by a more complicated expression

$$\begin{aligned} \tilde{\Delta}_1(\mathbf{p}) &= 2\tilde{\Delta}_{21} [\cos(p_2) + a_1 \cos(p_1) + a_2 \cos(p_1 + p_2)] \\ &+ 2\tilde{\Delta}_{22} [\cos(2p_1 + p_2) + b_1 \cos(2p_2 + p_1) \\ &+ b_2 \cos(p_1 - p_2)] \\ &+ 2\tilde{\Delta}_{11} [\sin(p_2) + c_1 \sin(p_1 + p_2) + c_2 \sin(p_1)]. \end{aligned}$$

Here, the coefficients  $a_i$ ,  $b_i$ , and  $c_i$  are determined not only by the symmetry of the triangular lattice but also by the parameters of the model.

These features of the superconducting order parameter in the phase with the coexistence of superconductivity and the noncollinear magnetic order should significantly affect the conditions favoring the formation of Majorana modes.

It is well known that the  $120^\circ$ -type noncollinear magnetic ordering becomes favorable for the triangular lattice in the Heisenberg regime at  $J_2/J_1 < 1/8$  [16, 17]. Bearing this in mind, below we consider the phase corresponding to the coexistence with the  $\mathbf{Q} = (Q, Q)$  noncollinear magnetic ordering.

### 5. DEPENDENCE OF THE TEMPERATURE CORRESPONDING TO THE TRANSITION TO THE COEXISTENCE PHASE ON THE CHARGE CARRIER DENSITY

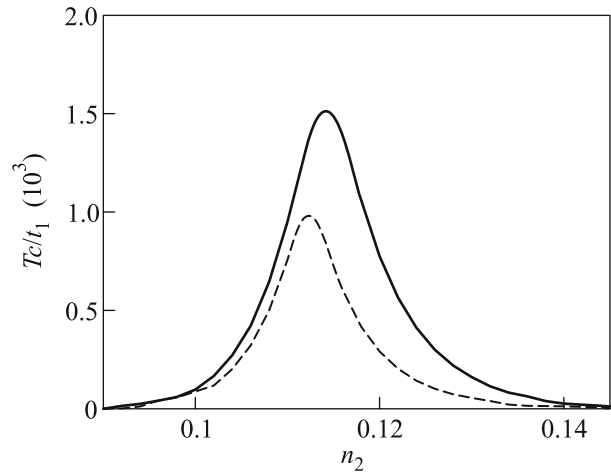
The equation determining the temperature of the transition to the phase with the coexistence of the  $120^\circ$ -type magnetic order and superconductivity follows from the compatibility condition for Eqs. (16)

$$\begin{aligned} & [(1 - A_{21})(1 - B_{22}) - A_{22}B_{21}](1 + C_{11}) \\ & + A_{11}[B_{21}C_{22} + (1 - B_{22})C_{21}] \\ & + B_{11}[A_{22}C_{21} + (1 - A_{21})C_{22}] = 0. \end{aligned} \quad (20)$$

In the analysis of this equation, it is important to take into account the intersite Coulomb interaction. Because of strong suppression of the pairing interaction [18, 19] between the nearest-neighbor sites owing to the Coulomb repulsion, the main contribution to the superconducting channel comes from the exchange interaction in the second coordination sphere. Here, we assume that it is possible to neglect the Coulomb interaction between the next nearest neighbors owing to the screening effects. Therefore, the temperature corresponding to the onset of the noncollinear magnetic ordering determined mostly by parameter  $J_1$  is much higher than the superconducting transition temperature related to the  $J_2$  exchange.

The results of the calculation of the superconducting transition temperature to the phase with the chiral order parameter as a function of the density  $n_2$  of two-electron states in the regime under discussion are illustrated in Fig. 4. These results were obtained with the parameters  $J_1 = 0.5t_1$ ,  $J_2 = 0.06t_1$ ,  $V = 0.96t_1$ ,  $t_2 = -0.17t_1$ , and  $t_3 = -0.14t_1$  (parameters characterizing the hoppings to the second and third coordination spheres). All parameters related to the energy are measured in units of the nearest-neighbor hopping parameter  $t_1$ . Earlier, the importance of the distant hoppings for the description of electron bands in cobaltites was demonstrated in the framework of the tight-binding approximation with  $t_1 \approx 0.2$  eV [8].

In Fig. 4, the solid line demonstrates the behavior of the temperature of the transition to the phase with the coexistence of superconductivity and the  $120^\circ$ -type magnetic order. The similar plot in the absence of the noncollinear magnetic ordering is shown by the dashed line. We can see that the induced magnetic order does not suppress the Cooper instability and



**Fig. 4.** Onset temperature for the chiral superconductivity versus the density of two-electron states for the triangular lattice. The solid line demonstrates the behavior of the temperature of the transition to the phase with the coexistence of superconductivity and the  $120^\circ$ -type magnetic order and the dashed line corresponds to the superconducting transition temperature in the absence of the noncollinear magnetic order.

even increases the superconducting transition temperature.

It is well known that the long-range antiferromagnetic order in strongly correlated electron systems often leads to the suppression of the superconducting pairing [20, 21]. In the case of the noncollinear  $120^\circ$ -type magnetic structure, such effect is absent because the solution of the integral equation for the superconducting order parameter exists even when the critical temperature of the magnetic ordering far exceeds the superconducting transition temperature. Moreover, the phase with the coexistence of chiral superconductivity and the  $120^\circ$ -type magnetic structure occurs in the range of relatively low temperatures.

### 6. CONCLUSIONS

The study of Hubbard fermions on the triangular lattice at the noncollinear spin ordering demonstrates that the Cooper instability in such system is appreciably affected by the combined action of the  $120^\circ$ -type spin structure and intersite Coulomb interactions. At the same structure but without the Coulomb interaction or with this interaction but without spin ordering, the superconducting order parameter  $\Delta_p$  can be represented as a sum of two chiral invariants. Under the combined effect of these two factors, the Cooper instability takes place in respect to the phase in which  $\Delta_p$  is described by the superposition of three chiral invariants. Note in this connection that a significant effect of the intersite Coulomb interaction on the symmetry type of the superconducting phase manifests

itself, for example, in the Kohn–Luttinger mechanism of superconductivity [22–24].

The results obtained in this work are quite important for the search for systems exhibiting the Majorana modes. Until recently, it was commonly accepted that the Majorana modes can arise in the systems with the singlet type of superconductivity only if the spin–orbit coupling is included in the consideration. In [2], it was emphasized that the noncollinear magnetic order on the triangular lattice can favor the formation of edge states with zero energy. The performed analysis of the Cooper instability in the case of the noncollinear spin ordering demonstrates that the chiral structure of  $\Delta_p$  remains only for the  $120^\circ$ -type spin ordering. Hence, the Majorana states should be sought just for such spin ordering. The corresponding results will be presented elsewhere.

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