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METALS

Specific Features of Quantum Oscillations of Magnetization in Quasi-Two-Dimensional Antiferromagnetic Semimetals

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Abstract—The specific features of quantum oscillations of the magnetization in quasi-two-dimensional wide-band-gap antiferromagnetic semimetals with a low concentration of charge carriers have been considered theoretically. It has been shown that, in these systems, the Fermi energy determined from the analysis of the frequency of the de Haas–van Alphen oscillations according to the standard procedure can differ significantly from the true value. For the correct determination of the Fermi energy in the canted phase, it has been proposed to analyze quantum oscillations of the magnetization M not as a function of the inverse magnetic field $1/H$, but as a function of $1/\cos\gamma$, where the angle γ characterizes the inclination angle of the magnetic field with respect to the plane of the quasi-two-dimensional semimetal.

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1. INTRODUCTION

Compounds in which the dynamics of charge carriers is substantially bounded by the planes of the crystal are of great theoretical and experimental interest. Suffice to say that this class of systems includes high-temperature cuprate superconductors (for example, $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_y$), heavy-fermion compounds of the 1–1–5 type, $CeMIn_5$ (M = Co, Rh, Ir), and relatively recently discovered iron-based superconductors [1–3].

The problem of the experimental determination of the shape of the Fermi surface in copper-based high-temperature superconductors initially was solved mainly using angle-resolved photoelectron spectroscopy (ARPES) [4]. Later, with the improvement of the experimental technology and quality of the samples, it became possible to measure the Fermi surface by the methods based on the analysis of quantum oscillations of the magnetization (de Haas–van Alphen effect [5, 6]) and the electrical conductivity (Shubnikov–de Haas effect [7]). Both these methods (ARPES and de Haas–van Alphen effect) have been actively used in the study of the topology of the Fermi surface in 1–1–5 heavy-fermion systems [8, 9] and iron-based superconductors [10, 11].

When comparing the results obtained by different experimental methods, quite often there arise discrepancies in the evaluations of the size and shape of the Fermi surface. For example, in lightly doped $YBa_2Cu_3O_y$, the hole Fermi surfaces obtained in the ARPES experiments are represented as Fermi arcs [4]. At the same time, from the experiments on quantum oscillations, it follows that Fermi surfaces are closed and their sizes amount to approximately 2% of the area of the Brillouin zone [7, 12]. Moreover, as turned out [13], these surfaces are the electron Fermi ones, which is unusual for p-type high-temperature superconductors. A possible explanation for the appearance of small pockets of electrons is associated with the transformation of the Fermi surface due to the formation of charge- and spin-density waves [14].

However, we cannot ignore the fact that a strong magnetic field in some systems with strong electron correlations by itself can lead to a transformation of the Fermi surface. In the case where the magnetic field strength H is large enough to change the size or topology of the Fermi surface, the frequency of quantum oscillations will no longer correspond to the Fermi surface in the absence of a magnetic field.

In systems with a strong coupling of the charge and spin degrees of freedom, there can exist another mechanism that is responsible for the inconsistency between the frequency of the de Haas–van Alphen oscillations and the size of the Fermi surface in the absence of a magnetic field. It should be noted that this mechanism does not imply a substantial transformation of the Fermi surface. The analysis of the de Haas–van Alphen effect in wide-band-gap antiferro-

magnetic (AFM) semimetals [15] revealed that the canting of the magnetizations of the sublattices in a magnetic field due to the s–d exchange coupling results in the motion of the bands of charge carriers. Under the conditions of the pinning of the chemical potential, this motion causes a significant change in the frequency of quantum oscillations in the canted phase. Although the main emphasis in [15] was made on the justification of the possibility of the experimental observation of an abrupt change in the frequency of the de Haas–van Alphen oscillations at the spin-flip transition point, from the formulas derived in [15] it clearly follows that, in the canted phase, the frequency of the de Haas–van Alphen oscillations will not correspond to the true Fermi surface.

The aforementioned iron-based superconductors are most similar in the electronic structure to the AFM semimetals considered in [15]. Undoped iron-based compounds are compensated semimetals with the hole Fermi surfaces at the Γ point of the Brillouin zone and the electron Fermi surfaces at the M points. At temperatures of $\simeq$100 K, in these systems there is a long-range antiferromagnetic order. For the description of the experimentally observed diversity of magnetic structures, the spin-fermion model was proposed in [16, 17]. The key idea advanced in [16, 17] is the possibility to divide the 3d electronic states of iron ions into two groups: the first group includes the d_{xz} and d_{yz} states, which are considered to be itinerant, and the second group contains all other d states, which are assumed to be well localized. In addition to the kinetic energy of charge carriers, the Hamiltonian of the spin-fermion model takes into account the superexchange interaction between the localized spins and the Hund's ferromagnetic coupling between the localized moments and itinerant electrons and holes. Moreover, the Hund's coupling is the key one in the description of the magnetic structure type.

In this paper, based on the spin-fermion model we will consider the specific features of quantum oscillations of the magnetization of itinerant quasiparticles in quasi-two-dimensional (quasi-2D) compensated AFM semimetals with the model parameters corresponding to iron-based superconductors. In particular, we will show that, in the canted AFM phase, due to the strong Hund's coupling, the frequency of the de Haas–van Alphen oscillations is determined not only by the size of the Fermi surface, but also the dynamics of the canting of the magnetic sublattices. Consequently, the recovery of the size of the Fermi surface in the canted phase directly from the frequency of the de Haas–van Alphen oscillations will lead to a significant error.

Taking into account, however, that the two-dimensional character of the motion of charge carriers makes it possible to observe quantum oscillations of the magnetization with a variation in the inclination angle γ of the magnetic field with respect to the plane of the quasi-two-dimensional semimetal, we will proposed a technique that makes it possible to correctly reproduce the size of the Fermi surface. This method is based on the analysis of the frequency of the magnetization quantum oscillations as a function of the parameter $1/\cos\gamma$.

2. HAMILTONIAN OF A QUASI-TWO-DIMENSIONAL AFM SEMIMETAL

Charge carriers (electrons and holes), as well as the subsystem of localized spin moments in an external magnetic field, will be described on the basis of the Hamiltonian of the spin-fermion model

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_e + \hat{\mathcal{H}}_h + \hat{\mathcal{H}}_m + \hat{\mathcal{H}}_{\mathrm{H}} + \hat{\mathcal{H}}_{\mathrm{Z}}. \quad (1)$$

The first and second terms in expression (1) correspond to the energy operators of non-interacting electrons and holes, respectively. The third term $\hat{\mathcal{H}}_m$ describes the localized spin moments interrelated by the antiferromagnetic exchange coupling. The fourth term accounts for the Hund's coupling of the localized moments with charge carriers: electrons and holes. The last term in expression (1) corresponds to the Zeeman energy operator of the system in a magnetic field.

In the second-quantization representation, the sum of the operators $\hat{\mathcal{H}}_e$ and $\hat{\mathcal{H}}_h$ has the form

$$\hat{\mathcal{H}}_e + \hat{\mathcal{H}}_h = \sum_{\lambda k\sigma} (\varepsilon_{\lambda k} - \mu_\lambda) c^+_{\lambda k\sigma} c_{\lambda k\sigma}, \quad (2)$$

where $\varepsilon_{\lambda k}$ is the energy of quasiparticles of the type $\lambda(\{e, h\})$ in the state with quasi-momentum k. The chemical potentials for electrons and holes are $\mu_e = \mu$ and $\mu_h = -\mu$, respectively. The operator $c^+_{\lambda k\sigma}(c_{\lambda k\sigma})$ creates (annihilates) a quasiparticle of the type λ in the kth state with the projection of the spin moment σ, which takes on two values $\pm 1/2$.

The antiferromagnetic state of a localized spin subsystem, as is known, can be conveniently described in the two-sublattice representation. Owing to the Hund's coupling, the doubling of the lattice period in the AFM phase causes a modification of the spectrum of charge carriers [18]. In order to describe the aforementioned effect, we change over in Hamiltonian (2) to the Wannier representation and introduce the two sublattices. As a result, we obtain

$$\begin{aligned}\hat{\mathcal{H}}_e + \hat{\mathcal{H}}_h = &\sum_{\lambda ff'\sigma} t^\lambda(f, f') c^+_{\lambda f\sigma} c_{\lambda f'\sigma} \\ &+ \sum_{\lambda gg'\sigma} t^\lambda(g, g') d^+_{\lambda g\sigma} d_{\lambda g'\sigma} \\ &+ \sum_{\lambda fg\sigma} t^\lambda_{fg} (c^+_{\lambda f\sigma} d_{\lambda g\sigma} + d^+_{\lambda g\sigma} c_{\alpha f\sigma}).\end{aligned} \quad (3)$$

In expression (3), the sites related to the F sublattice are denoted by the subscripts f and f', and the sites

belonging to the G sublattice—by the subscripts g and g'. The operators of creation (annihilation) of a quasiparticles of the type λ with spin σ at the site g belonging to the G sublattice are denoted by $d^+_{\lambda g\sigma}$ ($d_{\lambda g\sigma}$). For the operators creating and annihilating the corresponding quasiparticles at the f site in the F sublattice, we use the previous notation $c^+_{\lambda f\sigma}$ ($c_{\lambda f\sigma}$). The first two sums in formula (3) describe the quasiparticle transitions within the same sublattice F or G with the tunneling integrals $t^{\lambda}_{ff'}$ and $t^{\lambda}_{gg'}$. The third sum corresponds to transitions from different sublattices with the hopping parameter t^{λ}_{fg}. In this case, we have

$$t^{\lambda}(f,f') = t^{\lambda}_{ff'} + \delta_{ff'}(\varepsilon_{\lambda} - \mu_{\lambda}), \tag{4}$$

where $\delta_{ff'}$ is the Kronecker symbol and the parameters ε_e and ε_h represent the binding energies of electrons and holes at the f site, respectively. The determination of the function $t^{\lambda}(g,g')$ follows from formula (4) after the change $f \to g$.

The third term in Hamiltonian (1), which takes into account the exchange interaction in the localized spin subsystem, is describe in the framework of the Heisenberg model

$$\hat{\mathcal{H}}_m = -\frac{1}{2}\sum_{ff'} I_{ff'}(\mathbf{S}_f\mathbf{S}_{f'}) - \frac{1}{2}\sum_{gg'} I_{gg'}(\mathbf{S}_g\mathbf{S}_{g'}) + \sum_{fg} K_{fg}(\mathbf{S}_f\mathbf{S}_g). \tag{5}$$

Here, $\mathbf{S}_{f(g)}$ is the vector operator of the spin moment at the $f(g)$ site in the $F(G)$ sublattice, $I_{ff'}$ and $I_{gg'}$ are the exchange interaction integrals for the spin moments from the same sublattice, and K_{fg} is the parameter corresponding to the exchange interaction energy of the spins from different sublattices. In order to implement the long-range AFM order, all the aforementioned exchange parameters should be positive. Note that, with respect to the magnetic properties, the systems under consideration are three-dimensional. The term quasi-two-dimensionality in this paper is used only to characterize the transport properties of AFM semimetals.

The operator of the Hund's coupling between the localized magnetic moments and spin moments of charge carriers (the fourth term in formula (1)) can be written as

$$\hat{\mathcal{H}}_H = -\sum_{f\lambda} J_{\lambda}(\mathbf{S}_f\boldsymbol{\sigma}_{\lambda f}) - \sum_{g\lambda} J_{\lambda}(\mathbf{S}_g\boldsymbol{\sigma}_{\lambda g}), \tag{6}$$

where the intensity of Hund's coupling of the localized spin with quasiparticles of the type λ is determined by the parameter J_{λ}, and the components of the spin moment operators of charge carriers $\boldsymbol{\sigma}_{\lambda f(g)}$ have the following representation in terms of the Fermi operators:

$$\begin{aligned}
\sigma^+_{\lambda f} &= c^+_{\lambda f\uparrow}c_{\lambda f\downarrow}, \quad \sigma^+_{\lambda f} = c^+_{\lambda f\downarrow}c_{\lambda f\uparrow},\\
\sigma^z_{\lambda f} &= \sum_{\sigma}\sigma c^+_{\lambda f\sigma}c_{\lambda f\sigma},\\
\sigma^+_{\lambda g} &= d^+_{\lambda g\uparrow}d_{\lambda g\downarrow}, \quad \sigma^-_{\lambda g} = d^+_{\lambda g\downarrow}d_{\lambda g\uparrow},\\
\sigma^z_{\lambda g} &= \sum_{\sigma}\sigma d^+_{\lambda g\sigma}d_{\lambda f\sigma}.
\end{aligned} \tag{7}$$

Taking into account representations (7), the last term in Hamiltonian (1), which corresponds to the Zeeman energy, can be written in the form

$$\hat{\mathcal{H}}_Z = -2\mu_B\left(\sum_{f\lambda}\boldsymbol{\sigma}_{\lambda f} + \sum_{g\lambda}\boldsymbol{\sigma}_{\lambda g}\right)\mathbf{H} - g_L\mu_B\left(\sum_f \mathbf{S}_f + \sum_g \mathbf{S}_g\right)\mathbf{H}, \tag{8}$$

where g_L is the Landé factor, μ_B is the Bohr magneton, and $\mathbf{H}$ is the vector of the external magnetic field. The first term in expression (8) describes the interaction with the magnetic field of itinerant electrons and holes, and the second term—the localized spin moments.

In addition, we assume that, in the subsystem of localized spin moments, there is a strong easy-plane anisotropy, and this easy plane coincides with the range of motion of charge carriers.

3. UNITARY TRANSFORMATIONS OF THE HAMILTONIAN

The specificity of this problem is determined not only by the fact that the motion of charge carriers is bounded to the two-dimensional plane, but also by the fact that the magnetic field is directed at an angle to this plane. The geometry of the problem under consideration is shown in Fig. 1. The y axis is directed perpendicular to the plane within which the charges move. Accordingly, the x and z axes lie in this plane. The angle between the direction of the external magnetic field $\mathbf{H}$ and the normal to the plane is denoted by γ, and the angle between the projection of the magnetic field onto the zx plane and the x axis is designated by α.

Owing to the strong easy-plane anisotropy, the localized magnetic moments lie in the zx plane. Therefore, the canting of the magnetization vectors of the sublattices [19] also occurs within the zx plane. In this case, the canting is caused exclusively by the component of the magnetic field $\mathbf{H}_{\parallel}$, which is parallel to the zx plane.

For identical sublattices, with which we are dealing here, the angle between the equilibrium magnetization vector $\mathbf{R}_F$ for the F sublattice and the direction of the magnetic field projection vector $\mathbf{H}_{\parallel}$ coincides with the angle between the vector $\mathbf{R}_G$ from the G sublattice and the magnetic field $\mathbf{H}_{\parallel}$. It should be noted, however,

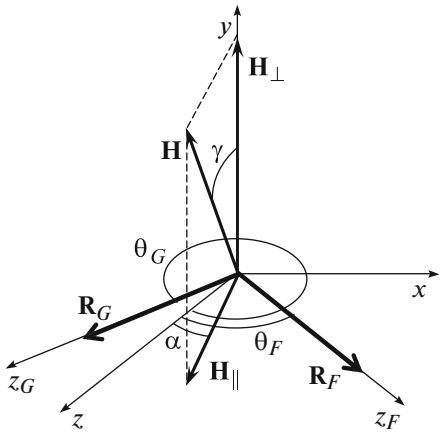

Fig. 1. Relative orientations of the external magnetic field **H** and the equilibrium magnetization vectors $\mathbf{R}_F$ and $\mathbf{R}_G$ in the F and G sublattices. The motion of charge carriers is bounded by the zx plane. The magnetic field **H** is directed at an angle γ with respect to the y axis. The angle between the z axis and the direction of projection of the magnetic field $\mathbf{H}_\parallel$ onto the zx plane is denoted by α. The vectors $\mathbf{R}_F$ and $\mathbf{R}_G$ making the angles θ_F and θ_G with the z axis lie in the zx plane due to the strong anisotropy. The new axes of the local coordinate systems z_F and z_G after unitary transformations (10) are directed along the vectors $\mathbf{R}_F$ and $\mathbf{R}_G$, respectively.

that, if in the easy plane of magnetization, there is an additional single-ion anisotropy, for example, tetragonal symmetry, these angles are not necessarily equal to each other. In this paper, we do not consider the effect of an additional anisotropy on the quantum oscillations. Nonetheless, we calculate the spectrum of Fermi excitations for this more general case. In order to describe an asymmetric canting of the magnetization vectors of the sublattices, we introduce the angles θ_F and θ_G, which determine the directions of the vectors $\mathbf{R}_F$ and $\mathbf{R}_G$ with respect to the z axis.

The state of the localized subsystem due to the Hund's coupling, generally speaking, should be determined taking into account itinerant electrons and holes. However, in the compounds with low concentrations of charge carriers, their influence on the subsystem of localized spins can be ignored. This means that the equilibrium configuration of localized spin moments is determined in the main approximation by interactions (5) and (8). At the same time, the spectral properties of electrons and holes, due to Hund's coupling (6), are determined, to a large extent, by the state of the subsystem of localized spins. We note in this regard that the concentration of charge carriers in iron-based superconductors is of the order of ~0.06 per unit cell [2]. Therefore, in this paper, we will restrict ourselves to the case of a low concentration of band quasiparticles.

In order to find the spectrum of quasiparticles of an AFM semimetal in a magnetic field, for each of the magnetic sublattices we consider a particular local coordinate system. The new coordinate system is chosen so that the equilibrium magnetization vector in the $F(G)$ sublattice is directed along the new axis $z_F(z_G)$ (see Fig. 1). For this purpose, we perform the unitary transformation of Hamiltonian (1):

$$\hat{\mathcal{H}} \to \hat{\mathcal{H}}' = \hat{U}\hat{\mathcal{H}}\hat{U}^+, \tag{9}$$

where the unitary operator $\hat{U}$ is chosen in the form

$$\hat{U} = \prod_{fg} \exp\{i\theta_F(S_f^y + \sigma_{ef}^y + \sigma_{hf}^y)\} \times \exp\{i\theta_G(S_g^y + \sigma_{eg}^y + \sigma_{hg}^y)\}. \tag{10}$$

The choice of the y-components of the spin moment operators as generators of the transformations means that the unitary transformation (10) describes the rotation of the localized and itinerant subsystems in the spin space around the y axis through the angle θ_F for the F sublattice and the angle θ_G for the G sublattice.

Using expression (10), we obtain the transformation laws for the electron and hole operators

$$c_{\lambda f\sigma}(\theta_F) = \hat{U}c_{\lambda f\sigma}\hat{U}^+ = c_{\lambda f\sigma}\cos\frac{\theta_F}{2} + 2\bar{\sigma}c_{\lambda f\bar{\sigma}}\sin\frac{\theta_F}{2}, \tag{11}$$

as well as for the spin operators

$$\begin{aligned} S_f^x(\theta_F) &= \hat{U}S_f^x\hat{U}^+ = S_f^x\cos\theta_F + S_f^z\sin\theta_F, \\ S_f^y(\theta_F) &= \hat{U}S_f^y\hat{U}^+ = S_f^y, \\ S_f^z(\theta_F) &= \hat{U}S_f^z\hat{U}^+ = S_f^z\cos\theta_F - S_f^x\sin\theta_F, \end{aligned} \tag{12}$$

where $\bar{\sigma} = -\sigma$. The transformation laws for the operators $d_{\lambda g\sigma}$ and $S_g^{x(y,z)}$ related to the G sublattice follow from formulas (11) and (12) after the change $\theta_F \to \theta_G$.

By performing the unitary transformations (10) for the terms of Hamiltonian (1), after changing over to the representation of the quasi-momentum by the formulas

$$t_{ff'}^\lambda = \frac{1}{N}\sum_k e^{ik(f-f')}t_k^\lambda, \quad t_{gg'}^\lambda = \frac{1}{N}\sum_k e^{ik(g-g')}t_k^\lambda,$$
$$t_{fg}^\lambda = \frac{1}{N}\sum_k e^{ik(f-g)}\Gamma_k^\lambda, \tag{13}$$
$$c_{\lambda f\sigma} = \frac{1}{\sqrt{N}}\sum_k e^{ik\lambda}c_{\lambda k\sigma}, \quad d_{\lambda g\sigma} = \frac{1}{\sqrt{N}}\sum_k e^{ikg}d_{\lambda k\sigma}$$

we obtain the electron–hole Hamiltonian

$$\begin{aligned} \hat{\mathcal{H}}_{eh} = \sum_{\lambda k\sigma}\Bigg[& \varepsilon_{\lambda k\sigma}^F c_{\lambda k\sigma}^+ c_{\lambda k\sigma} + \varepsilon_{\lambda k\sigma}^G d_{\lambda k\sigma}^+ d_{\lambda k\sigma} \\ & + \Gamma_k^\lambda\cos\left(\frac{\theta_F - \theta_G}{2}\right)(c_{\lambda k\sigma}^+ d_{\lambda k\sigma} + d_{\lambda k\sigma}^+ c_{\lambda k\sigma}) \\ & + 2\sigma\Gamma_k^\lambda\sin\left(\frac{\theta_F - \theta_G}{2}\right)(c_{\lambda k\sigma}^+ d_{\lambda k\bar{\sigma}} + d_{\lambda k\bar{\sigma}}^+ c_{\lambda k\sigma}) \\ & + h_\sigma(\theta_F)c_{\lambda k\bar{\sigma}}^+ c_{\lambda k\sigma} + h_\sigma(\theta_G)d_{\lambda k\bar{\sigma}}^+ d_{\lambda k\sigma}\Bigg]. \end{aligned} \tag{14}$$

When writing expressions (14), we introduced the following notation:

$$\varepsilon_{\lambda k\sigma}^{F(G)} = \varepsilon_\lambda + t_k^\lambda - \mu_\lambda - 2\sigma\mu_B H_\parallel \cos(\alpha - \theta_{F(G)}) - \sigma J_\lambda R_{F(G)}, \quad (15)$$

$$h_\sigma(\theta) = -\mu_B H_\parallel \sin(\alpha - \theta) - 2\sigma i \mu_B H_\perp,$$

where R_F and R_G are the absolute values of the magnetization vectors in the F and G sublattices, respectively.

In the derivation of the expression for $\hat{\mathcal{H}}_{eh}$, in the Hamiltonian $\hat{\mathcal{H}}_H$ after the unitary transformation, we carried out the conventional separation of the terms corresponding to the inclusion of the mean field. These terms are taken into account in the expressions for $\varepsilon_{\lambda k\sigma}^{F(G)}$. The possibility of using the mean-field approximation is determined by the low concentration of charge carriers and low temperatures ($T \ll T_N$).

4. ENERGY SPECTRUM OF ELECTRONS AND HOLES IN AN ANTIFERROMAGNETIC SEMIMETAL

In order to find the energy spectrum of charge carriers in an antiferromagnetic semimetal in the canted AFM phase, we use the method of equations of motion for two-time temperature Green's functions [20]. We define two types of Green's functions

$$G_{\lambda k}^{\sigma'\sigma}(t-t') = -i\Theta(t-t')\langle\{c_{\lambda k\sigma'}(t), c_{\lambda k\sigma}^+(t')\}\rangle,$$
$$F_{\lambda k}^{\sigma'\sigma}(t-t') = -i\Theta(t-t')\langle\{d_{\lambda k\sigma'}(t), c_{\lambda k\sigma}^+(t')\}\rangle. \quad (16)$$

Here, $\Theta(t-t')$ is the Heaviside step function equal to unity for $t > t'$ and zero for $t < t'$. The angle brackets denote the averaging over the grand canonical ensemble with Hamiltonian (14). The curly brackets denote the anticommutator. All operators in formulas (16) are taken in the Heisenberg representation.

The system of equations of motion for the Fourier transforms of the Green's functions $G_{\lambda k}^{\sigma'\sigma}(\omega)$ and $F_{\lambda k}^{\sigma'\sigma}(\omega)$ is conveniently written in the matrix form

$$\hat{M}\mathbf{G} = \mathbf{B}, \quad (17)$$

where $\mathbf{G}$ is the column vector, the elements of which are four Green's functions $G_{\lambda k}^{\sigma\sigma}(\omega)$, $G_{\lambda k}^{\bar{\sigma}\sigma}(\omega)$, $F_{\lambda k}^{\sigma\sigma}(\omega)$, and $F_{\lambda k}^{\bar{\sigma}\sigma}(\omega)$; the vector $\mathbf{B}$ is equal to $(1, 0, 0, 0)^T$; and the matrix $\hat{M}$ has the form

$$\begin{pmatrix} \omega - \varepsilon_{\lambda k\sigma}^F & -h_\sigma(\theta_F) & -\Gamma_k^\lambda \cos\left(\frac{\theta_F - \theta_G}{2}\right) & -2\sigma\Gamma_k^\lambda \sin\left(\frac{\theta_F - \theta_G}{2}\right) \\ -h_\sigma(\theta_F) & \omega - \varepsilon_{\lambda k\bar{\sigma}}^F & 2\sigma\Gamma_k^\lambda \sin\left(\frac{\theta_F - \theta_G}{2}\right) & -\Gamma_k^\lambda \cos\left(\frac{\theta_F - \theta_G}{2}\right) \\ -\Gamma_k^\lambda \cos\left(\frac{\theta_F - \theta_G}{2}\right) & 2\sigma\Gamma_k^\lambda \sin\left(\frac{\theta_F - \theta_G}{2}\right) & \omega - \varepsilon_{\lambda k\sigma}^G & -h_{\bar{\sigma}}(\theta_G) \\ -2\sigma\Gamma_k^\lambda \sin\left(\frac{\theta_F - \theta_G}{2}\right) & -\Gamma_k^\lambda \cos\left(\frac{\theta_F - \theta_G}{2}\right) & -h_\sigma(\theta_G) & \omega - \varepsilon_{\lambda k\bar{\sigma}}^G \end{pmatrix}. \quad (18)$$

The electron–hole spectrum is found from the solution to the fourth-degree equation: $\det|\hat{M}| = 0$. The calculation of the determinant $\det|\hat{M}|$ is substantially simplified after the preliminary unitary transformation of the matrix $\hat{M}$: $\hat{M}' = \hat{T}^+\hat{M}\hat{T}$, where the unitary operator $\hat{T}$ is determined using the second-order matrices $\hat{u} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$ by the expression $\hat{T} = \begin{pmatrix} \hat{u} & 0 \\ 0 & \hat{u} \end{pmatrix}$. As a result, for the determinant of the matrix $\hat{M}'$, we find

$$\begin{vmatrix} \tilde{\omega} - 2\sigma\mu_B H_\perp & \sigma\tilde{H}_F & -\Gamma_k^\lambda e^{-i2\sigma\frac{\theta_F - \theta_G}{2}} & 0 \\ \sigma\tilde{H}_F^\dagger & \tilde{\omega} + 2\sigma\mu_B H_\perp & 0 & -\Gamma_k^\lambda e^{i2\sigma\frac{\theta_F - \theta_G}{2}} \\ -\Gamma_k^\lambda e^{i2\sigma\frac{\theta_F - \theta_G}{2}} & 0 & \tilde{\omega} - 2\sigma\mu_B H_\perp & \sigma\tilde{H}_G \\ 0 & -\Gamma_k^\lambda e^{-i2\sigma\frac{\theta_F - \theta_G}{2}} & \sigma\tilde{H}_G^\dagger & \tilde{\omega} + 2\sigma\mu_B H_\perp \end{vmatrix}, \quad (19)$$

where $\tilde{\omega} = \omega - \varepsilon_\lambda - t_k^\lambda + \mu_\lambda$ and the effective field is given by the formula $\tilde{H}_{F(G)} = 2\mu_B H_\parallel e^{i2\sigma(\alpha-\theta_{F(G)})} + J_\lambda R_{F(G)}$.

Expanding determinant (19) and equating it to zero, we obtain the biquadratic equation, which determines the energy spectrum of electrons and holes. Four solutions to the biquadratic equation can be written as

$$E^{\pm}_{\lambda k\sigma} = \varepsilon_\lambda + t_k^\lambda - \mu_\lambda \pm \sqrt{-C_2 + 2\sigma\sqrt{C_2^2 - C_0}}, \quad (20)$$

where

$$C_2 = -\Gamma_k^{\lambda^2} - \mu_B^2\mathbf{H}^2 - \frac{J_\lambda^2}{8}(R_F^2 + R_G^2) - \frac{1}{2}J_\lambda\mu_B H_\parallel \times [R_F\cos(\alpha-\theta_F) + R_G\cos(\alpha-\theta_G)],$$

$$C_0 = \left(\frac{J_\lambda}{2}\right)^4 R_F^2R_G^2 + (\Gamma_k^{\lambda^2} - (\mu_B\mathbf{H})^2)^2 + \left(\frac{J_\lambda}{2}\right)^2(\mu_B\mathbf{H})^2(R_F^2+R_G^2) + (\Gamma_k^{\lambda^2} - (\mu_B\mathbf{H})^2)J_\lambda\mu_B H_\parallel \times [R_F\cos(\alpha-\theta_F) + R_G\cos(\alpha-\theta_G)] + \frac{J_\lambda^3}{4}\mu_B H_\parallel R_F R_G[R_F\cos(\alpha-\theta_G) + R_G\cos(\alpha-\theta_F)] - \frac{J_\lambda^2}{2}R_F R_G[((\mu_B H_\parallel)^2 - \Gamma_k^{\lambda^2})\cos(\theta_F-\theta_G) + (\mu_B H_\parallel)^2\cos(2\alpha-\theta_F-\theta_G)]. \quad (21)$$

Expressions (20) and (21) give the solution to the problem on the spectrum of electrons and holes of an antiferromagnetic semimetal in the canted AFM phase in the most general case with arbitrary directions of the magnetic field vector and equilibrium magnetization vectors of the sublattices (see Fig. 1).

Taking into account the symmetry of the initially formulated problem, we direct the z axis along the projection of the magnetic field $\mathbf{H}_\parallel$ (i.e., we set $\alpha = 0$). In this case, there is a relation between the angles and magnetizations from different sublattices

$$\theta_F = -\theta_G = \theta, \quad R_F = R_G = R. \quad (22)$$

Considering these equalities, the expression for spectrum (20) can be substantially simplified and represented in the form

$$E^{\pm}_{\lambda k\sigma} = \varepsilon_\lambda + t_k^\lambda - \mu_\lambda \pm \left\{\left[\Gamma_k^\lambda + 2\sigma\sqrt{(\mu_B H_\perp)^2 + \left(\mu_B H_\parallel + \frac{J_\lambda R}{2}\cos\theta\right)^2}\right]^2 + \left(\frac{J_\lambda R}{2}\right)^2\sin^2\theta\right\}^{1/2}. \quad (23)$$

It should be noted that, under the additional conditions $\mathbf{H}_\perp = 0$ and $\mathbf{H}_\parallel = \mathbf{H}$, expression (23) coincides with the electron–hole energy spectrum obtained previously in [15].

Let us assume that the motion of quasiparticles over the square lattice is caused by their hopping only between the nearest and next-nearest neighbors. Then, for the functions Γ_k^λ and t_k^λ, we can write

$$\Gamma_k^\lambda = 4t_1^\lambda\cos(k_x b/2)\cos(k_y b/2), \quad t_k^\lambda = 2t_2^\lambda(\cos(k_x b) + \cos(k_y b)), \quad (24)$$

where t_1^λ and t_2^λ are the hopping integrals for quasiparticles of the type λ between the nearest and next-nearest neighbors, respectively, and b is the square-lattice parameter in the AFM phase.

If the band width W determined by the tunneling integrals t_1^λ and t_2^λ is significantly less than the Hund's coupling constant J_λ, then the model considered here corresponds to the double exchange model used, for example, to describe the phase diagram of manganites. In iron-based superconductors, the situation is reversed: $W \gg J_\lambda$. For example, in [21], the authors obtained the estimate for the Hund's exchange: $J_\lambda \sim 0.35$ eV. In our study, we will use the following relationships between the model parameters:

$$W \gg J_\lambda \sim \mu_\lambda \gg K_0 \sim T_N \sim \mu_B H \gg T. \quad (25)$$

Taking into account relationships (25) and the fact that, in semimetals with a low concentration of charge carriers, the thermodynamic properties of the electron–hole subsystem are determined by small values of the quasi-momentum $\mathbf{k}$, the energy spectrum (23) corresponding to the lower band can be written as

$$E^{-}_{\lambda k\sigma} = -\Delta_\lambda + \frac{\hbar^2\mathbf{k}^2}{2m_\lambda} - \mu_\lambda - 2\sigma\bar{H}_\lambda \quad (|\mathbf{k}|b \ll 1). \quad (26)$$

In this formula, the parameter $-\Delta_\lambda = \varepsilon_\lambda + 4(t_2^\lambda - |t_1^\lambda|)$ sets the origin of the quasiparticle energy. For electrons, the value of ε_e is selected from the condition $\Delta_e = 0$, whereas for holes, the value of Δ_h is taken to be positive. The effective masses of holes m_h and electrons m_e in formula (26) are expressed in terms of the tunneling integrals

$$\frac{\hbar^2}{2m_\lambda} = \left(\frac{|t_1^\lambda|}{2} - t_2^\lambda\right)b^2, \quad (27)$$

and the effective field $\bar{H}_\lambda$ is determined by the expression

$$\bar{H}_\lambda^2 = (\mu_B H_\perp)^2 + \left(\mu_B H_\parallel + \frac{1}{2}J_\lambda R\cos\theta\right)^2. \quad (28)$$

As was noted above, in systems with a low concentration of charge carriers, the equilibrium configuration of localized spin moments is determined primar-

ily by the exchange interaction (5) and the Zeeman interaction (8). By performing the unitary transformation (9) of interaction operators (5) and (8) and the further disengagement in the mean-field approximation, we obtain the condition for the equilibrium angle θ:

$$\cos\theta = g_L\mu_B H_{\parallel}/2RK_0. \quad (29)$$

Here, $K_0 = zK_1$, where K_1 is the exchange integral between the nearest spin moments, and z is the number of nearest neighbors, which for the square lattice is equal to four. Equation (29) allows us to find the critical field $H_c = 2RK_0/g_L\mu_B \sin\gamma$, at which the collapse of the sublattice magnetizations occurs and the angle θ becomes zero.

From expressions (26), (28), and (29), it follows that, in the canted AFM phase, a change in the external magnetic field (both in the absolute value of the vector **H** and its direction) leads to a shift of the reference point of the dispersion relations without changing the effective mass of the quasiparticles. Moreover, if in the collinear phase ($H > H_c$), this shift occurs with the rate $dE^-_{\lambda k\sigma}/dH \sim \mu_B$, then, in the canted phase ($H < H_c$), the rate of motion of the spectrum significantly increases by a factor of approximately J_λ/K_0.

5. SPECIFIC FEATURES OF QUANTUM OSCILLATIONS IN QUASI-TWO-DIMENSIONAL AFM SEMIMETALS

For the study of quantum oscillations of the magnetization in quasi-two-dimensional antiferromagnetic semimetals, we use the formula (see, for example, [22])

$$M_{\sim} = -\sum_{\lambda\sigma} M_0^{\lambda} \sum_{n=1}^{+\infty} \frac{(-1)^n \sin\left(\frac{2\pi\tilde{\mu}_{\lambda\sigma}}{\hbar\omega_{c\lambda}} n\right)}{\sinh\left(\frac{2\pi^2 T}{\hbar\omega_{c\lambda}} n\right)}. \quad (30)$$

In the above expression, $M_{\sim}$ is the oscillating part of the magnetization; $\omega_{c\lambda} = |e|H_\perp/m_\lambda c$ is the cyclotron frequency; T is the temperature expressed in energy units; and $\tilde{\mu}_{\lambda\sigma}$ is the renormalized chemical potential, which at low temperatures coincides with the Fermi energy

$$\tilde{\mu}_{\lambda\sigma} = \Delta_\lambda + \mu_\lambda + 2\sigma\bar{H}_\lambda. \quad (31)$$

The amplitude of quantum oscillations of the magnetization M_0^λ in formula (30) can be written as

$$M_0^\lambda = \frac{T}{(|t_1^\lambda| - 2t_2^\lambda)\mu_B H_\perp} \frac{\tilde{\mu}_{\lambda\sigma}}{\mu_B} \frac{\mu_B}{b^3}, \quad (32)$$

if the thickness of the quasi-two-dimensional layer is taken to be equal to the lattice parameter b.

In order to reveal specific features of the de Haas–van Alphen effect in AFM semimetals, we note that, under the conditions of canting the sublattice magnetizations ($\theta < \pi/2$) and with due regard for relationships (25) for the effective field (28), we can write: $\bar{H}_\lambda \approx J_\lambda R\cos\theta/2$. It can be seen that the Hund's coupling J_λ ($\sim\mu_\lambda$) makes a significant contribution to the energy of quasiparticles, which for $H < H_c$ depends linearly on the magnetic field. Then, taking into account expressions (29) and (31), we find that the phase of the oscillating terms (i.e., the argument of the function sin with $n = 1$ in formula (30)) for $H < H_c$ has the form

$$\phi^<_{\lambda\sigma} = \frac{\pi m_\lambda}{m_0}\left(\frac{\Delta_\lambda + \mu_\lambda}{\mu_B H\cos\gamma} + \frac{\sigma J_\lambda g_L}{2K_0}\tan\gamma\right), \quad (33)$$

where m_0 is the mass of a free electron. If $H > H_c$, then the phase is given by the expression

$$\phi^>_{\lambda\sigma} = \frac{\pi m_\lambda}{m_0}\frac{\Delta_\lambda + \mu_\lambda + \sigma J_\lambda R}{\mu_B H\cos\gamma}. \quad (34)$$

When writing expressions (33) and (34), we took into account that $H_\parallel = H\sin\gamma$ and $H_\perp = H\cos\gamma$.

Equation (33) shows that, due to the linear dependence of the Fermi energy (31) on the magnetic field H in the canted AFM phase, the frequency of the de Haas–van Alphen oscillations is determined not by the renormalized chemical potential $\tilde{\mu}_{\lambda\sigma}$, which is directly responsible for the number of charge carriers and, therefore, for the size of the Fermi surface, but by the parameter $\Delta_\lambda + \mu_\lambda$. This parameter differs from the Fermi energy by the value of $\bar{H}_\lambda$, which in the canted phase has the same order of magnitude as the chemical potential $\tilde{\mu}_{\lambda\sigma}$.

An increase in the magnetic field strength to a level equal to the critical value leads to a spin-flip transition, and the angle θ becomes zero. In the ferromagnetic phase (at $H > H_c$), the contribution to the energy of quasiparticles due to the Hund's coupling almost does not depend on the magnetic field. As follows from formula (34), in this case, the frequency of the de Haas–van Alphen oscillations is proportional to the Fermi energy $\tilde{\mu}_{\lambda\sigma}$.

Thus, when passing through the spin-flip transition point, we should observe an abrupt change in the frequency of magnetization oscillations. This effect was predicted for the first time in [15] in the study of three-dimensional AFM semimetals.

Knowing the change in the frequency of the de Haas–van Alphen oscillations, we can determine, in particular, the ratio of the Hund's coupling J_λ to the Fermi energy $\tilde{\mu}_{\lambda\sigma}$. Indeed, taking into account the relation between the phase change $\Delta\phi$ and the frequency of oscillations $F_{\lambda\sigma}$, i.e., $\Delta\phi = 2\pi F_{\lambda\sigma}\Delta(1/H)$, and using expressions (33) and (34), we obtain

$$\frac{F^>_{\lambda\sigma} - F^<_{\lambda\sigma}}{F^>_{\lambda\sigma}} = \frac{J_\lambda}{\tilde{\mu}_{\lambda\sigma}}\sigma R, \quad (35)$$

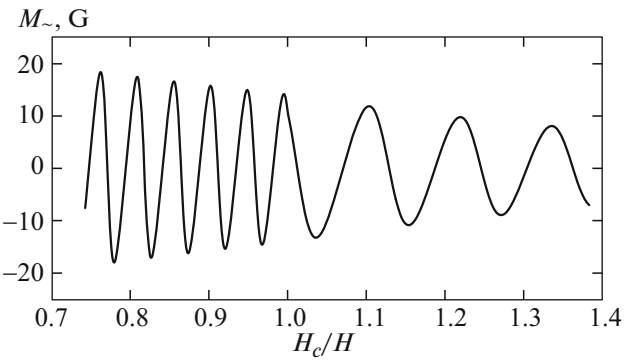

Fig. 2. Abrupt change in the de Haas–van Alphen oscillation frequency upon the transition of the spin subsystem from the canted AFM phase to the collinear ferromagnetic phase. The model parameters are chosen as follows: $t_1^e = t_1^h = -1$ eV, $t_2^e = t_2^h = 0$, $J_e = J_h = 0.15$ eV, $\Delta = 0.15$ eV, $R = 3/2$, $g_L = 2$, and $K_0 = 5.5 \times 10^{-3}$ eV. In the calculation, $\gamma = 45°$ is the angle between the magnetic field and the normal to the plane. $T = 10$ K. The lattice parameter is $b = 2 \times 10^{-8}$ cm.

where $F_{\lambda\sigma}^{<}$ ($F_{\lambda\sigma}^{>}$) is the frequency of the de Haas–van Alphen oscillations for $H < H_c$ ($H > H_c$).

The above feature of the de Haas–van Alphen effect is demonstrated in Fig. 2. For simplicity, the parameters determining the energy spectrum of electrons and holes are chosen to be identical. The constant K_0 is related to the Néel temperature T_N according to the approximate formula $T_N \approx S(S + 1)K_0/3$, where the temperature T_N is assumed to be 80 K. Since the calculation of $M_\sim$ was performed at a temperature $T \ll T_N$, the average magnetization was taken as $R \approx S$. The spin S was assumed to be 3/2. The chemical potential μ_λ was determined self-consistently for each value of the magnetic field H from the condition that the numbers of holes and electrons are equal to each other. For the chosen values of the model parameters, only the subbands of quasiparticles with $\sigma = +1/2$ proved to be occupied (the subbands with $\sigma = -1/2$ were unoccupied). Therefore, on each side of the spin-flip transition point ($H = H_c$), there is only one frequency of oscillations. The magnetization curves shown in Fig. 2 demonstrate a twofold increase in the oscillation frequency upon the transition from the canted phase to the collinear phase. The evaluation of the Hund's coupling constant J_λ from formula (35) gives two-thirds of the Fermi energy $\tilde{\mu}_{\lambda\sigma}$.

It should be noted that, in order to observe an abrupt change in the frequency of the de Haas–van Alphen oscillations in quasi-two-dimensional AFM semimetals, it is necessary that both components of the magnetic field, $H_\parallel$ and $H_\perp$, should not be equal to zero. In contrast to three-dimensional AFM semimetals, in this case, the roles of the components $H_\parallel$ and $H_\perp$ differ from each other both by the object of the impact and by the result of this impact. The magnetic field component $H_\parallel$ parallel to the plane acts only on the localized spin subsystem and causes a canting of the magnetization of the sublattices. The perpendicular magnetic field component $H_\perp$ acting only on charge carriers leads to a quantization of their energy spectrum and, consequently, to quantum oscillations of the magnetization. This is the first feature of the discussed effect, which is characteristic of quasi-two-dimensional AFM semimetals.

The second feature is associated with the dependence of the phase of oscillating terms in formula (30) on the inclination angle of the magnetic field **H** with respect to the plane of a quasi-two-dimensional semimetal. If the magnetic field strength is fixed and sufficient to provide the occurrence of the spin-flip transition at a certain inclination angle γ_c, then, in the experiment, an abrupt change should also be observed in the frequency of magnetization oscillations as a function of the angle γ at the point $\gamma = \gamma_c$. The analysis of formulas (33) and (34) demonstrates that, as before, the change in the frequency of magnetization oscillations is associated with the dependence of the Fermi energy $\tilde{\mu}_{\lambda\sigma}$ on the Hund's coupling. Moreover, if the spin-flip transition occurs at angles γ_c close to $\pi/2$ (at which $\sin\gamma_c$ is close to unity), the change in the frequency of magnetization oscillations $M_\sim(\gamma)$ is almost negligible. If the angle γ_c is small, the phase shift $\phi_{\lambda\sigma}^{<}$ due to the second term in formula (33) is insignificant in comparison with the corresponding term in formula (34) for the phase $\phi_{\lambda\sigma}^{>}$. In this case, the change in the frequency of magnetization oscillations $M_\sim$ as a function of the inclination angle γ should be well pronounced. The above conclusions are confirmed by the results of the numerical calculations of $M_\sim(\gamma)$ at the angle $\gamma_c = 30°$ (Fig. 3).

It should also be noted that the experimental observation of the effects associated with the spin-flip transition in AFM semimetals with the Néel temperature $T_N \sim 100$ K requires magnetic fields with a strength of ~180 T. At present, the creation of such strong magnetic fields is a complex problem.

From this point of view, the following circumstance, which is also related to the angular dependence of the phase of the oscillating terms in expression (30), but takes place in significantly weaker magnetic fields (less than H_c), has become important. As was shown above, in the canted phase, the Fermi energy in the AFM semimetals, which is recovered from the de Haas–van Alphen effect (i.e., from the analysis of the quantum oscillations of the magnetization $M_\sim$ as a function of $1/H$), can differ from the true value by a factor of two. This is associated with the fact that the sufficiently large contribution to the Fermi energy due

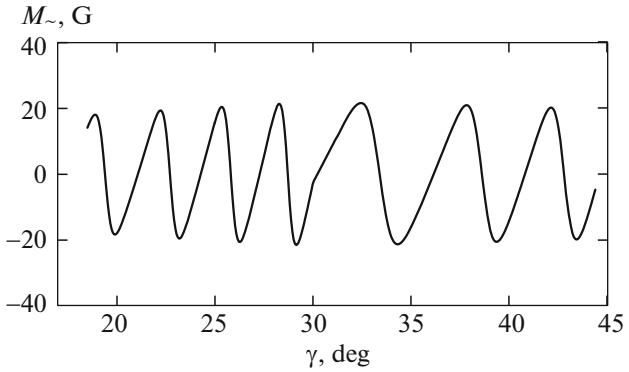

Fig. 3. Abrupt change in the frequency of magnetization oscillations $M_\sim$ as a function of the angle γ upon the spin-flip transition. The model parameters are chosen the same as in Fig. 2. The magnetic field strength H was chosen from the condition $\gamma_c = 30°$.

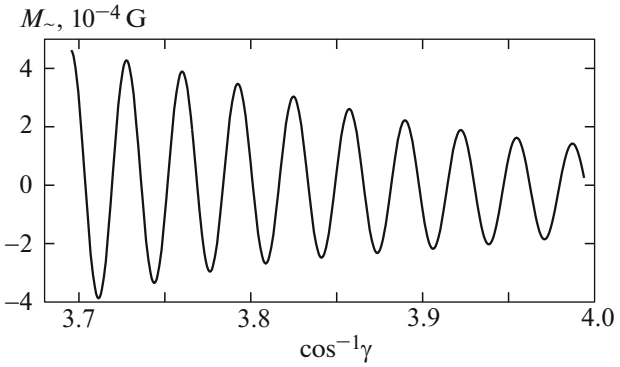

Fig. 4. Magnetization oscillations $M_\sim$ as a function of $1/\cos\gamma$. The model parameters are chosen the same as in Fig. 2. The magnetic field strength is $H = 74$ T.

to the Hund's coupling (i.e., due to the term $2\sigma\bar{H}_\lambda$), in the main approximation, is linear in H. Upon substituting into formula (30), this contribution (after the division by $\hbar\omega_{c\lambda}$) leads only to a phase shift of the function $M_\sim(1/H)$ and, therefore, in the experiments on the de Haas–van Alphen effect, does not manifest itself.

However, if the quantum oscillations of the magnetization $M_\sim$ are analyzed not as a function of the inverse magnetic field, but as a function of $\cos^{-1}\gamma$, then, in the range of angles not too far from $\pi/2$, the frequency of magnetization oscillations $M_\sim(\cos^{-1}\gamma)$ will be proportional to the true chemical potential $\tilde{\mu}_{\lambda\sigma}$. Indeed, the correction to the Fermi energy at $H < H_c$ is given by the expression $\bar{H}_\lambda \approx \frac{J_\lambda R}{2}\cos\theta = \frac{J_\lambda}{4K_0} g_L \mu_B H \sin\gamma$ and, at the angles γ close to $\pi/2$, depends weakly on γ. In this case, the dependence of the phase $\phi^<_{\lambda\sigma}$ on the angle γ in the main approximation is determined by the ratio $\tilde{\mu}_{\lambda\sigma}/\cos\gamma$, and the frequency of magnetization oscillations $M_\sim(\cos^{-1}\gamma)$ is proportional to $\tilde{\mu}_{\lambda\sigma}$.

The dependence of the magnetization oscillations $M_\sim$ on the inverse cosine of the inclination angle of the magnetic field is shown in Fig. 4. It can be seen that the oscillation period remains constant almost over the entire range of variations in the angle γ. Of fundamental importance in this case is the fact that the Fermi energy recovered from the Fourier analysis of the frequency of oscillations of the function $M_\sim(\cos^{-1}\gamma)$ differs by only 2% from the value of $\tilde{\mu}_{\lambda\sigma}$ obtained by solving the equation for the chemical potential.

6. CONCLUSIONS

The main result of the performed investigations is the proposed method for the correct determination of the Fermi energy of a two-dimensional wide-band-gap antiferromagnetic semimetal with a low concentration of charge carriers in the canted AFM phase. The method consists in the Fourier analysis of the frequency of magnetization oscillations $M_\sim$ as a function of $1/\cos\gamma$, where γ is the inclination angle of the magnetic field with respect to the plane of the quasi-two-dimensional semimetal. In conventional experiments on measurements of the de Haas–van Alphen effect, the frequency of magnetization oscillations $M_\sim$ is determined as a function of the inverse magnetic field $1/H$. For quasi-two-dimensional AFM semimetals, this approach is acceptable only when $H_\parallel = 0$ or $H > H_c$. For the canted phase, as was shown in this study, the analysis of the frequency of oscillations of the function $M_\sim(1/H)$ can lead to incorrect values of the Fermi energy. At the same time, the investigation of the magnetization $M_\sim$ as a function of $1/\cos\gamma$ makes it possible to obtain values of the Fermi energy that almost completely coincide with the true values.

It should be noted that a large error in the determination of the Fermi energy from the frequency analysis of the function $M_\sim(1/H)$ arises in the case where the Hund's coupling energy is comparable in magnitude to the Fermi energy. This leads to the necessity to search for the effect under consideration in the systems with a low concentration of charge carriers. Moreover, the experimental observation of a significant difference between the Fermi energies obtained in the canted phase from the analysis of the frequency dependence of the magnetization $M_\sim$ as a function of $1/H$ and as a function of $1/\cos\gamma$ could serve as indirect evidence for the presence of Hund's coupling in these systems.

An abrupt change in the frequency of the de Haas–van Alphen oscillations at the spin-flip transition point was predicted for the first time for three-dimen-

sional AFM semimetals in [15]. In the present study, it was shown that, for the observation of this effect in quasi-two-dimensional wide-band-gap AFM semimetals, the magnetic field should be directed at an angle to the plane: $0 < \gamma < \pi/2$. Furthermore, in this paper, we presented formula (35), according to which the ratio of the Hund's coupling constants to the Fermi energy can be determined from the experimentally measured frequencies of the de Haas–van Alphen oscillations in the left and right neighborhoods of the spin-flip transition point. It was also noted that an abrupt change in the frequency of quantum oscillations of the magnetization in quasi-two-dimensional AFM semimetals can be observed with a variation in the angle γ.

Another result of the investigations should be considered to be expression (20) for the energy spectrum of electrons and holes in two-dimensional wide-band-gap AFM semimetals with the Hund's coupling between the localized spin moments and itinerant quasiparticles. The novelty of the obtained expression lies in the fact that it describes the electron–hole spectrum with an arbitrary relative orientation of the equilibrium magnetization vectors of the sublattices and the external magnetic field and, therefore, has a wider range of applications as compared to that considered in this study. For example, in many two-dimensional systems, apart from the easy-plane anisotropy, there are additional interactions removing the degeneracy in the direction of the in-plane magnetic moment. These interactions lead, in particular, to the preferred orientation of the equilibrium magnetization vectors $\mathbf{R}_F$ and $\mathbf{R}_G$ along the high-symmetry axes of the crystal lattice, which lie in the plane under investigation. Therefore, if there is the magnetic field component $\mathbf{H}_\parallel$ parallel to the plane, the vector $\mathbf{R}_F$ and $\mathbf{R}_G$ already do not necessarily form identical angles with the vector $\mathbf{H}_\parallel$. Expression (20) for the energy spectrum of charge carriers in quasi-two-dimensional AFM semimetals is also applicable in this more general case. The necessity to use expression (20) for the spectrum of charge carriers can also arise in the study of ferrimagnetic semimetals in an external magnetic field in which, for definition, the absolute values of the magnetization vectors $\mathbf{R}_F$ and $\mathbf{R}_G$ differ from each other.

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