Optics Letters

Design of bandpass filters composed of dielectric layers separated by gratings of strip conductors

B. A. Belyaev[1,2,3,*] and V. V. Tyurnev[1,2]

[1]L. V. Kirensky Institute of Physics SB RAS, 50/38 Akademgorodok Str., Krasnoyarsk, 660036, Russia
[2]Siberian Federal University, 79 Svobodny Ave., Krasnoyarsk, 660041, Russia
[3]Siberian State Aerospace University, 31 Krasnoyarsky Rabochy Ave., Krasnoyarsk, 660014, Russia
*Corresponding author: belyaev@iph.krasn.ru

Received 21 September 2015; revised 23 November 2015; accepted 18 December 2015; posted 21 December 2015 (Doc. ID 250419); published 29 January 2016

We derive the design formulas for novel multilayer bandpass filters in which every dielectric layer (resonator) is separated from the adjacent layer or external medium by a grating of strip conductors. Every grating acts as a semi-reflecting mirror. Such novel filters have wide stop bands compared to conventional filters with multilayer dielectric mirrors between resonators at the same passband width. The parameters of the lowpass, lumped-element prototype filter, as well as the theory of resonator-coupling coefficients, are considered in our approach. The computed frequency response of the fifth-order Chebyshev filter that was synthesized using the proposed formulas is also presented.

OCIS codes: (230.7408) Wavelength filtering devices; (310.6805) Theory and design; (230.4555) Coupled resonators; (310.4165) Multilayer design; (050.2770) Gratings.

http://dx.doi.org/10.1364/OL.41.000536

A novel multilayer bandpass filter containing interlayer gratings of strip conductors was proposed in [1]; the cross section of its structure is shown in Fig. 1. The filter works similarly to the conventional dielectric multilayer bandpass filter for which multilayer dielectric mirrors separating half-wavelength resonant layers are composed of quarter-wavelength layers with alternating high- and low-refractive indices [2–5]. The salient feature of the novel filter is its stop bands are extended manyfold. This filter may operate in microwave, infrared, and optical ranges.

Every dielectric layer in the filter under consideration is a half-wavelength resonator. It is separated from the adjacent resonator or free space by semi-reflecting mirrors located on both of its sides. Each mirror is a planar grating of strip conductors, and the conductors in all the gratings are parallel to each other. Every period T_i for the ith grating in the filter must be less than the thickness h_i of the adjacent dielectric layers in order to suppress the contribution of evanescent modes generated by the grating. The third set of structure parameters of the filter is the spacing s_i between strip conductors. All three sets of parameters have to have their optimal values corresponding to index i.

The behavior of the reflection properties of the single grating with the ideal strip conductors situated on a boundary between two media when its structure parameters vary was studied in [6]. In that paper, one can also find the derived formulas for the reflection and transmission coefficients, which we use during filter design.

In this Letter, we derive formulas for computing the optimal values of s_i and h_i at fixed values of T_i for every layer of the structure in order to ensure the desired Chebyshev frequency response of the filter. We restrict ourselves to the case in which the wave vector of the incident wave is orthogonal to the filter plane and the electric field $\mathbf{E}$ is parallel to the strip conductors.

We shall characterize the frequency response near the passband by the frequencies f_l and f_h, which are the low and high edges of the passband as defined by the minimum return loss level L_r (i.e., the reflection loss measured in decibels). The order n_f of the filter defines the number of reflection zeroes within the passband and is equal to the number of layers.

In accordance with the theory of filters, the only physical values that are to be adjusted in order to obtain the desired passband parameters (L_r, f_l, and f_h) are the resonant frequencies of all the resonators, f_i; the coupling coefficients of all the resonator pairs, $k_{i,i+1}$; and the external quality factor of the input and output resonators, Q_e. In the case of a narrow passband, all the resonant frequencies f_i must be equal to the center frequency $f_0 = (f_l f_h)^{1/2}$. Moreover, the coefficients $k_{i,i+1}$, together with the factor Q_e, must satisfy the following equations [7,8]:

$$Q_e^{-1} = w/g_1, \tag{1}$$

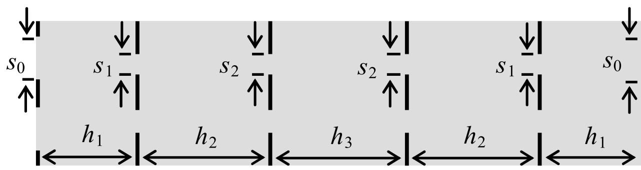

Fig. 1. Cross section of the fifth-order bandpass filter.

0146-9592/16/030536-04$15/0$15.00

$$|k_{i,i+1}| = w/\sqrt{g_i g_{i+1}}, \tag{2}$$

where w is the fractional bandwidth defined by the equation

$$w \equiv (f_h - f_l)/f_0, \tag{3}$$

and g_i are the normalized lowpass prototype parameters that can be computed using the following equations:

$$g_1 = 2a_1/\gamma,$$
$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}, \qquad k = 2, 3, \ldots, n_f. \tag{4}$$

Here we use the following notations:

$$\gamma = \sinh\left(\frac{\operatorname{atanh}\sqrt{1 - 10^{-L_r/10}}}{2n_f}\right),$$
$$g_1 = 2a_1/\gamma,$$
$$g_k = \frac{4a_{k-1}a_k}{b_{k-1}g_{k-1}}, \qquad k = 2, 3, \ldots, n_f,$$
$$a_k = \sin\frac{(2k-1)\pi}{2n_f},$$
$$b_k = \gamma^2 + \sin^2\frac{k\pi}{n_f}, \qquad k = 1, 2, \ldots, n_f. \tag{5}$$

We should note that the coefficients k_i are symmetrical relative to the filter center—i.e., $k_1 = k_{n-1}$, $k_2 = k_{n-2}$ etc.

In order to determine the factor Q_e of the input resonator ($i = 1$), including the first dielectric layer together with the external grating ($i = 0$) that separates the layer from free space, we should isolate this resonator from the neighboring resonator. This may be theoretically achieved by applying the boundary condition $\mathbf{E} = 0$ for the electric field on the isolated surface of the first layer. At this stage, we can solve the boundary-value problem and find the complex frequency f_c for free oscillation in the first resonator.

We shall characterize the reflective properties of every grating on the boundary between media with refractive indices n_1 and n_2 by the scattering matrix $\mathbf{S}(n_1, n_2, f, s, T)$. This matrix relates the normalized electric-strength amplitudes of outgoing waves (b_i), with the analogous amplitudes of the incoming waves (a_i) on both sides (port 1 and port 2), according to the following equation [9]:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \mathbf{S}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}. \tag{6}$$

Here the amplitudes a_i and b_i are normalized in such a way that the total power P_i coming into port i per unit area can be expressed by the equation

$$P_i = \frac{1}{2}|a_i|^2 - \frac{1}{2}|b_i|^2. \tag{7}$$

Using the formulas for reflection and transmission coefficients that were derived in [6], we obtain the scattering matrix for the grating

$$\mathbf{S}(n_1, n_2, f, s, T) = \begin{pmatrix} \frac{n_1 - n_2 - i\Lambda}{n_1 + n_2 + i\Lambda} & \frac{2\sqrt{n_1 n_2}}{n_1 + n_2 + i\Lambda} \\ \frac{2\sqrt{n_1 n_2}}{n_1 + n_2 + i\Lambda} & \frac{-n_1 + n_2 + i\Lambda}{n_1 + n_2 + i\Lambda} \end{pmatrix}$$
$$\Lambda \equiv c/\left[fT \ln \sec\left(\frac{\pi s}{2T}\right)\right], \tag{8}$$

where i is the imaginary unit that is present in the time-dependent factor $\exp(-i2\pi f t)$.

Solving the eigenvalue problem concerning the free oscillation in the first resonator, we obtain the following equation:

$$\theta_c = \pi + \frac{i}{2}\ln(-S_{11}(n, 1, f_c, s_0, T_0)), \tag{9}$$

where n is the refractive index of all the layers, and θ_c is phase thickness of the first layer as defined by the following equation:

$$\theta_c \equiv \frac{2\pi f_c}{c} n h_1. \tag{10}$$

The frequency f_c together with thickness h_1 may be easily found by iteratively solving Eq. (10) under the condition $\operatorname{Re} f_c = f_0$. At this stage, the factor Q_e may be computed using the following equation [6,10]:

$$Q_e = \operatorname{Re} f_c/(-2\operatorname{Im} f_c). \tag{11}$$

We can simultaneously estimate the relative thinning of the first layer, affected by the external grating, using the following equation:

$$\delta l_0 = 1 - \operatorname{Re}\theta_c/\pi. \tag{12}$$

Now the optimal value of s_0 in the external grating may be found for a fixed value of T_0 by solving Eq. (1).

In order to determine the coupling coefficients $k_{i,i+1}$ for the ith and $(i+1)$th resonators, we should isolate this pair from all other parts of the filter. This can be theoretically achieved by applying the boundary condition $\mathbf{E} = 0$ for the electric field on both external surfaces of this resonator pair.

Next, we compute the resonant frequencies f_e and f_o for the even- and odd-coupled oscillations for the case of identical layers. Solving this eigenvalue problem, we obtain the following equations:

$$\theta_e = \pi + i\frac{1}{2}\arg(-S_{21} - S_{11}), \tag{13}$$

$$\theta_o = \pi + i\frac{1}{2}\arg(S_{21} - S_{11}). \tag{14}$$

For phase thicknesses, we have

$$\theta_e \equiv \frac{2\pi f_e}{c} n h_i,$$
$$\theta_o \equiv \frac{2\pi f_o}{c} n h_i. \tag{15}$$

Taking Eq. (8) into account, we can rewrite Eq. (14) for the case of $n_1 = n_2 = n$ in the following form:

$$\theta_o = \pi. \tag{16}$$

The frequencies f_e and f_o together with the thickness h_i may be found by iteratively solving Eqs. (13), (15), and (16) under the condition that $(f_e + f_o)/2 = f_0$. Next, the coefficients $k_{i,i+1}$ may be computed using the following equation [11,12]:

$$k_{i,i+1} = (f_o^2 - f_e^2)/(f_o^2 + f_e^2). \tag{17}$$

We can simultaneously estimate the relative thinning of the ith layer, affected by ith grating, as follows:

$$\delta l_i = 1 - (\theta_e + \theta_o)/(2\pi). \tag{18}$$

Now the optimal value of s_i in the ith grating may be found for a fixed value of T_i by solving Eq. (2).

At this stage, the final thicknesses of the dielectric layers in accordance with Eqs. (10), (12), (15), and (18) may be computed as follows:

$$h_i = (1 - \delta l_{i-1} - \delta l_i)\lambda_n/2, \tag{19}$$

where $\lambda_n = c/(f_0 n)$ is the wavelength in the dielectric at f_0. In this way, we have obtained all the formulas that allow for the design of the multilayer passband filter.

The frequency response of the designed filter may be computed by multiplication of the transfer matrices **M** for every grating and every dielectric layer in the multilayer structure [2]. Matrix **M** is also called the characteristic matrix or the ABCD matrix [7]. It relates the tangential components of the electric- and magnetic-field strengths on the first and second surface of a planar structure according to the following equation:

$$\begin{pmatrix} E_1 \\ Z_0 H_1 \end{pmatrix} = \mathbf{M} \begin{pmatrix} E_2 \\ Z_0 H_2 \end{pmatrix}, \tag{20}$$

where the free-space characteristic impedance is given by $Z_0 \equiv (\mu_0/\varepsilon_0)^{1/2}$.

The transfer matrix **M** of the ith dielectric layer has the following form [2]:

$$\mathbf{M} = \begin{pmatrix} \cos\theta_i & \frac{-i}{n}\sin\theta_i \\ -in\sin\theta_i & \cos\theta_i \end{pmatrix}, \qquad \theta_i \equiv \frac{2\pi f}{c} n h_i, \tag{21}$$

whereas the transfer matrix **M** of the grating may be obtained from Eq. (8) by the following equation [7]:

$$\mathbf{M} = \begin{pmatrix} \dfrac{1 + S_{11} - S_{22} - \det[S_{ik}]}{2S_{21}} & \dfrac{1 + S_{11} + S_{22} + \det[S_{ik}]}{2S_{21}} \\ \dfrac{1 - S_{11} - S_{22} + \det[S_{ik}]}{2S_{21}} & \dfrac{1 - S_{11} + S_{22} - \det[S_{ik}]}{2S_{21}} \end{pmatrix}. \tag{22}$$

The required scattering matrix **S** of the entire layered structure is related to the product (**M**) of the transfer matrices for every structure component as follows [7]:

$$\mathbf{S} = \begin{pmatrix} \dfrac{M_{11}+M_{12}-M_{21}-M_{22}}{M_{11}+M_{12}+M_{21}+M_{22}} & \dfrac{2\det[M_{ik}]}{M_{11}+M_{12}+M_{21}+M_{22}} \\ \dfrac{2}{M_{11}+M_{12}+M_{21}+M_{22}} & \dfrac{-M_{11}+M_{12}-M_{21}+M_{22}}{M_{11}+M_{12}+M_{21}+M_{22}} \end{pmatrix}. \tag{23}$$

Equations (22) and (23) are concerned with the particular case in which both media surrounding the filter are air.

We should note that the filter being considered, as well as its components (dielectric layers and gratings of strip conductors), are reciprocal two-port networks. Thus their matrices **M** and **S** satisfy the equations $\det[M_{ik}] = 1$ and $S_{12} = S_{21}$ [7]. Moreover, the neglect of dielectric and ohmic loss in the structure results in the following equation: $|S_{11}| = |S_{22}|$.

We shall illustrate the accuracy of our derived design formulas with the following example. Let the desired filter have the order $n_f = 5$; let the passband, edge frequencies be $f_l = 0.975$ THz and $f_h = 1.025$ THz; let the minimum return loss level be $L_r = 14$ dB; and let the refractive index be $n = 1.87$. From these parameters, we obtain a center frequency of $f_0 = 1$ THz, a fractional bandwidth of $w = 0.050$, an external quality factor of $Q_e = 26.00$, coupling coefficients of $k_{1,2} = 0.03781$ and $k_{2,3} = 0.02956$, and a wavelength of $\lambda_n = 160.2$ µm in the dielectric. We shall suppose that all the gratings have the same period: namely, $T_i = \lambda_n/4$ (i.e., $T_i = 40$ µm). In this way, we obtain the spacing of the gratings and the thicknesses of the layers. Their values, measured in micrometers, are presented in the row labeled as "Formulas" in Table 1.

Table 1. Structure Parameters of the Filter (all units: µm)

Parameter	s_0	s_1	s_2	h_1	h_2	h_3
Formulas	33.91	16.92	15.08	70.04	77.47	77.79
Optimal	33.95	16.86	15.08	70.14	77.53	77.85

The dotted lines in Fig. 2 show the computed frequency dependencies of the transmission coefficient $|S_{21}|$ and the reflection coefficient $|S_{11}|$ for the bandpass filter that was designed by the proposed formulas. One can see that the designed filter requires slight adjustments. Using physical rules of filter optimization [3], we obtained the refined structure parameters, which are presented in the row labeled as "Optimal" in Table 1.

We should note that the accuracy of the proposed design formulas decays with an increase in the fractional bandwidth w. This accuracy decay appears in any filter that is designed with use of its lumped-element prototype [7]. Furthermore, the inaccuracy is due to the difference of frequency dispersion of the resonator-coupling coefficients in the filter and the dispersion of the corresponding coefficients in its lumped-element prototype [13,14].

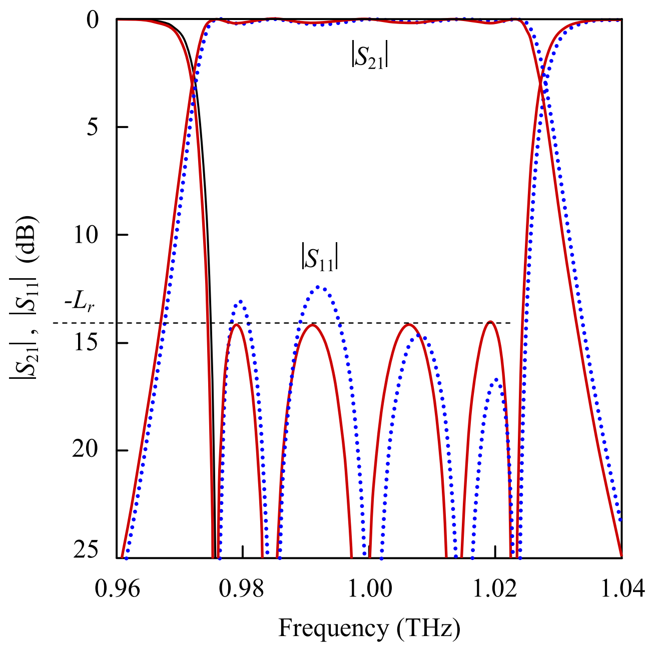

Fig. 2. Computed frequency responses of the bandpass filter for the structure parameters obtained by the formulas derived in this Letter (dotted lines) and by optimization (solid lines).

Funding. Ministry of Education and Science of the Russian Federation (14.607.21.0039).

REFERENCES

1. B. A. Belyaev and V. V. Tyurnev, Opt. Lett. **40**, 4333 (2015).
2. H. A. Macleod, *Thin-Film Optical Filters* (CRC Press, 2010).
3. B. A. Belyaev, V. V. Tyurnev, and V. F. Shabanov, Opt. Lett. **39**, 3512 (2014).
4. B. A. Belyaev, V. V. Tyurnev, and V. F. Shabanov, Dokl. Phys. **59**, 73 (2014).
5. B. A. Belyaev, V. V. Tyurnev, and V. F. Shabanov, Dokl. Phys. **59**, 245 (2014).
6. B. A. Belyaev and V. V. Tyurnev, Russ. Phys. J. **58**, 646 (2015).
7. G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Artech House, 1980).
8. V. V. Tyurnev, J. Commun. Technol. Electron. **53**, 554 (2008).
9. K. C. Gupta, R. Garg, and R. Chadha, *Computer-Aided Design of Microwave Circuits* (Artech House, 1981).
10. B. A. Belyaev and V. V. Tyurnev, Microw. Opt. Technol. Lett. **55**, 1613 (2013).
11. V. V. Tyurnev, J. Commun. Technol. Electron. **47**, 1 (2002).
12. J.-S. Hong, *Microstrip Filters for RF/Microwave Applications* (Wiley, 2011).
13. V. V. Tyurnev, J. Commun. Technol. Electron. **54**, 298 (2009).
14. V. V. Tyurnev, Prog. Electromagn. Res. B **21**, 47 (2010).