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Electronic spin polarization in the Majorana bound state in one-dimensional wires

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ABSTRACT

We have studied the effect of magnetic field and disorder on the electronic z-spin polarization at the ends of the one-dimensional wire with strong Rashba spin–orbit coupling deposited on an s-wave superconductor. It was shown that in the topologically nontrivial phase the polarization as well as the energy of the Majorana bound state oscillate as a function of the magnetic field. Despite being substantially nonzero in the low transversal and longitudinal fields the polarization at one of the wire's ends is significantly suppressed at a certain range of the magnitudes and angles of the canted magnetic field. Thus, in this case the polarization cannot be regarded as a local order parameter. However, the sum of the absolute values of the polarization at both ends remains significantly nonzero. It was demonstrated that Anderson disorder does not seriously affect observed properties but leads to the appearance of the additional areas with weak spin polarization at the high magnetic fields.

1. Introduction

Since the works of Alexei Kitaev about the appearance of Majorana fermions (MFs) in solid state systems [1,2] the number of studies in this area has been increasing remarkably. The reasons why this topic became so popular are two. The first one is fundamental due to very unusual properties of MFs: Majorana particle and its antiparticle are the same, they are real-valued solutions of the Dirac equation and carry no charge [3]. In particular, in particle physics neutrinos are treated as MFs, but that still needs an experimental proof. The second issue which explains big activity around Majoranas is that emerging in solid state systems they are non-local. Thus quantum information encoded in the Majorana state is robust against decoherence by local perturbation. Moreover, because an MF obeys so-called non-Abelian statistics [4], such a qubit can be manipulated by braiding operations [5,6]. So it opens the road to use MFs in topological quantum computations.

Since Majorana creation and annihilation operators are the same in solid state systems so-called Majorana bound state (MBS) can be defined as an equal-weight superposition of electron and hole states with zero energy. It means that MBSs can be found in systems with superconducting (SC) order. But this is not the only condition. Following Kitaev's idea it is necessary to engineer an effective “spinless” pairing. Different candidates were proposed for the experimental observation of MBSs such as superfluid ^{3}He-B [7], quantum spin Hall systems [8], magnetic chains [9,10].

In 2010 two groups independently found out that semiconducting nanowires with strong spin–orbit interaction deposited on the surface of an s-wave SC can be driven into a topologically nontrivial phase by applying an external magnetic field as it is depicted at Fig. 1a [11,12]. Without SC and magnetic field the spin–orbit interaction causes splitting of the spin subbands of the wire (see blue and red lines in Fig. 1b). In the magnetic field the energy gap emerges in the spectrum (solid curves in Fig. 1b). If the chemical potential lies in the gap we obtain the desired situation with only one pair of the Fermi points in the half of the Brillouin zone: spin σ corresponds to right mover and the opposite spin $-\sigma$ corresponds to left mover. Induced s-wave SC pairing such that $V_z > \sqrt{\Delta^2 + \mu^2}$ leads to an effective “spinless” pairing and the two MBSs appear at the ends of the wire as it is schematically shown by the red circles at Fig. 1a.

One of the possible ways to probe Majorana states is to provide transport experiments using, for example, tunnel spectroscopy tools. And initially in theoretical studies it was shown that MBS could reveal itself as Zero Bias Peak (ZBP) in differential conductance of height $2e^2/h$. The coupling between the two MBSs results in the ZBP splitting [13–15]. First experiment presented the appearance of the ZBP as a proof of the existence of MBSs in 1D semiconducting nanowires was done in 2012 by Kouwenhoven's group [16]. The noise measurements can be also used to detect MBSs [13,17]. In one-lead geometry quantum transport is defined by Majorana fermion induced resonant Andreev reflection when the Fano factor equals 2. In two-lead configuration there is a competition of two mechanisms depending on the coupling strength between MBSs: resonant Andreev reflection and

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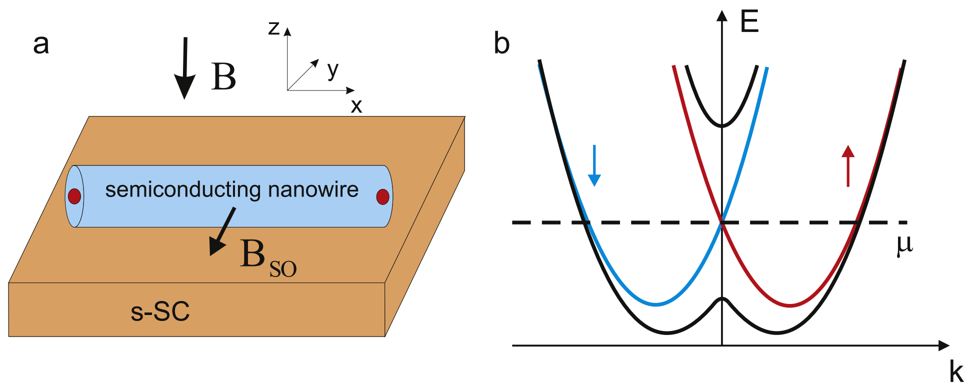

Fig. 1. (a) A semiconducting nanowire with strong spin–orbit interaction deposited on an s-wave SC in an external magnetic field; (b) band structure of the semiconducting nanowire for **B**=0 (blue and red lines) and **B** ≠ 0 (black lines). (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

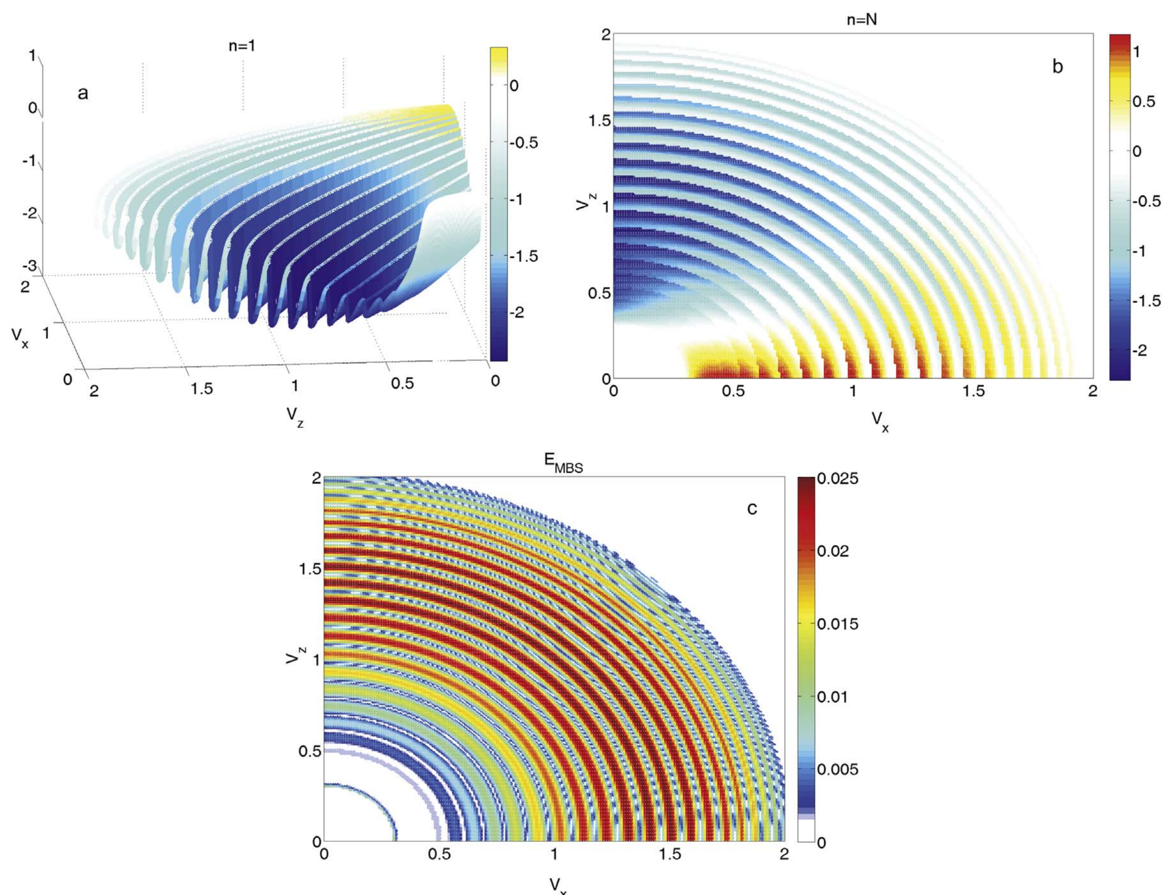

Fig. 2. The electronic z-spin polarization in the MBS on the first (a) and last (b) sites of the 1D wire and the MBS energy (c) as functions of the magnetic-field magnitude and orientation. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

crossed Andreev reflection. Therefore the Fano factor changes from 0 to 1.

The alternative way to detect MBS is to analyze the components of the electronic spin polarization of the 1D nanowire. In articles [18,19] the Majorana polarization was introduced and it was shown that this characteristic is not equal to zero at the ends of the wire exactly in the topologically nontrivial phase. As a result, the Majorana polarization can be regarded as a local order parameter and measured by means of spin-polarized scanning tunneling microscopy. The features of the Majorana polarization will be discussed in another article of the authors [20]. In this study we analyze the behavior of the z-component of the electronic spin polarization in the MBS, P_z, which was also considered in [18]. In particular, we study its dependence on the magnitude and direction of the magnetic field in the xz plane and disorder. It is shown that in the general case, when the magnetic field is oriented at an arbitrary angle to the wire and perpendicular to the Rashba effective field, P_z at opposite ends of the wire can have both different sign and magnitude. Along with the MBS energy the z-axis spin polarization demonstrates oscillating behavior which is determined by the structure of the MBS wave function. It is demonstrated that there is a range of the magnetic-field magnitudes and angles at which the P_z is significantly suppressed or, moreover, is equal to zero in the topological phase. It allows to conclude that the z-spin projection cannot be treated as a local order parameter in the canted magnetic field.

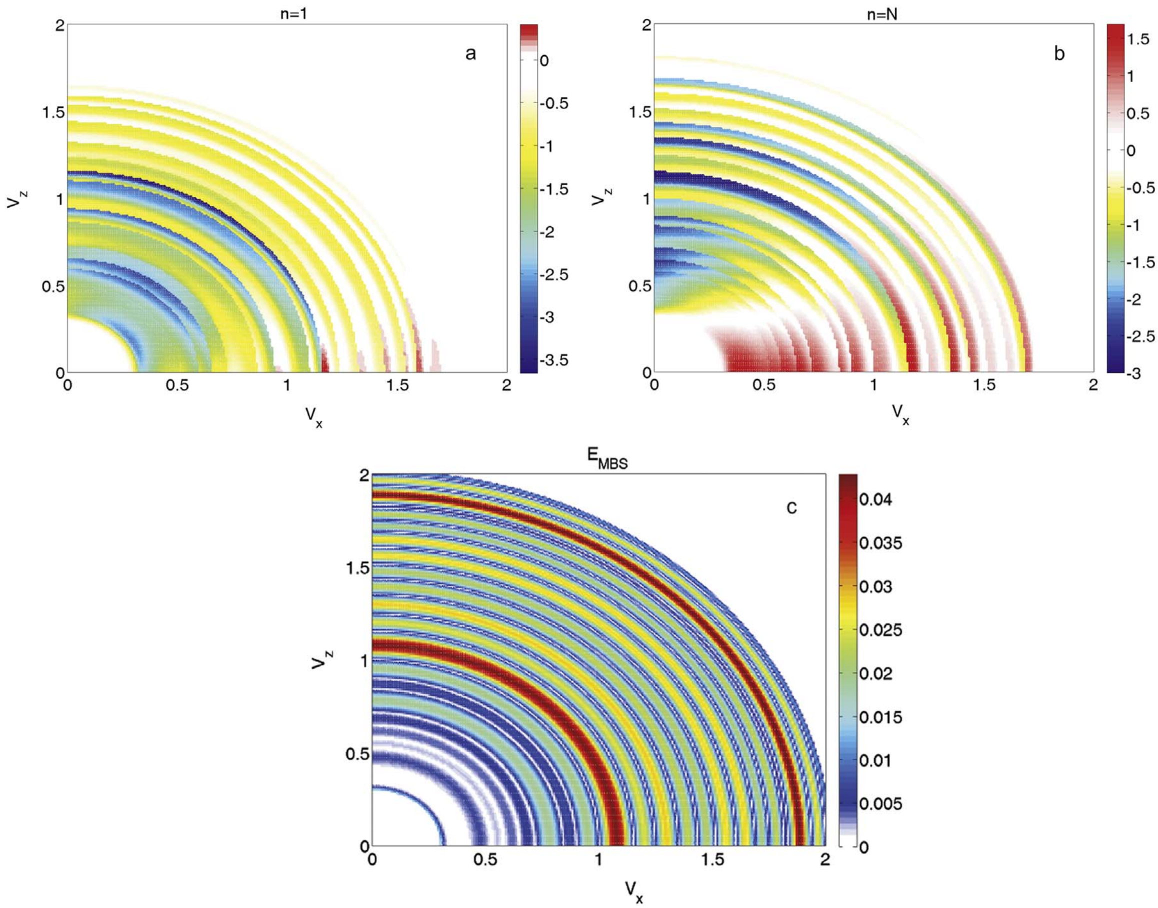

Fig. 3. The influence of disorder on the electronic z-spin polarization (a, b) and the MBS energy (c), $W = 1/2$.

2. Model Hamiltonian

Let us consider the 1D semiconductor wire deposited on the surface of the s-wave SC along the x-axis and characterized by strong Rashba spin–orbit interaction as depicted in Fig. 1a. But now we suppose that the magnetic field is oriented at an arbitrary angle in the xz plane and perpendicular to the Rashba effective field, B_{SO}. Induced Cooper pairing of electrons described by the potential Δ occurs in the wire due to the proximity effect. The model Hamiltonian is given by

$$\widehat{H}_W = \sum_{n=1}^{N}\left[\sum_{\sigma}\xi_\sigma a_{n\sigma}^{+}a_{n\sigma} + (\Delta a_{n\uparrow}a_{n\downarrow} - V_x a_{n\uparrow}^{+}a_{n\downarrow} + \text{h. c.})\right] - \sum_{\sigma;n=1}^{N-1}\left[\frac{t}{2}a_{n\sigma}^{+}a_{n+1,\sigma} + \frac{\alpha}{2}\sigma a_{n\sigma}^{+}a_{n+1,\bar{\sigma}} + \text{h. c.}\right], \quad (1)$$

where $a_{n\sigma}^{+}$ ($a_{n\sigma}$)—the creation (annihilation) electron operator on the nth site with spin σ; $\xi_\sigma = t + \sigma V_z - \mu$—the on-site energy of the electron with spin σ in the transverse magnetic field V_z; μ—the chemical potential of the system; $V_{x(z)} = \frac{1}{2}\mu_B g B_{x(z)}$—the x- and z-components of the Zeeman energy; t —the hopping matrix element between nearest sites; α—the intensity of the Rashba spin–orbit interaction. In the tight-binding approximation we have $t = \hbar^2/2m^*a^2$, where m^*—the effective mass of electrons, a—the lattice spacing. The Rashba parameter, α_R, is defined as $\alpha = \alpha_R/a$. Consequently, for the InSb-nanowire we get $m^* = 0.015 m_e$, $\alpha_R = 0.2\,\text{eV}\cdot\text{Å}$, $\Delta \sim 10^{-4}\,\text{eV}$, $g \approx 50$, $B_{x,z} = 0.01 - 1\,\text{T}$ (i.e. $V_{x,z} \sim 10^{-5} - 10^{-3}\,\text{eV}$) [16]. If $a \sim 1\,\text{nm}$ that $t \sim 1\,\text{eV}$, $\alpha \sim 10^{-2}\,\text{eV}$. Thus, $t \gg \alpha, V_{x,z}, \Delta$.

3. Electronic z-spin polarization

We utilize the Bogolubov transformation to study the MBS's properties,

$$\beta_l = \sum_{n=1}^{N}[u_{ln}a_{n\uparrow} + v_{ln}a_{n\downarrow}^{+} + w_{ln}a_{n\downarrow} + z_{ln}a_{n\uparrow}^{+}]. \quad (2)$$

Following [18], we derive z-component of the electronic spin polarization on the nth site using the coefficients of the transformation (2):

$$P_z(n, \omega) = \sum_{l=1}^{4N}\left\langle \Psi_{ln}|\sigma_z\frac{\tau_0+\tau_z}{2}|\Psi_{ln}\right\rangle\delta(\omega - E_l). \quad (3)$$

where E_l —lth eigenvalue of the wire's Hamiltonian $\widehat{H}_W$; $|\Psi_{ln}\rangle = (u_{ln}, v_{ln}, w_{ln}, z_{ln})$—$l$th eigenvector; σ_z, τ_0, τ_z—the Pauli matrices acting in the spin and particle-hole spaces, respectively. In further numerical calculations main parameters of the system will be taken on the basis of the above-mentioned estimations: t=1, $\Delta = 0.3$, $\mu = 0$, $\alpha = 0.2$, N=30.

4. Numerical calculations

We plot the z-component of electronic spin polarization in the MBS ($E_l = E_{MBS}$ in (3)) at both wire's edges as a function of the components of the magnetic field (Fig. 2a, b). If the magnetic field does not satisfy the inequity [20]

$$\mu^2 + \Delta^2 < V_x^2 + V_z^2 < (2t - \mu)^2 + \Delta^2, \quad (4)$$

the wire is in the topologically trivial phase. In this case P_z is close to zero with the corresponding low- and high-field white segments at Fig. 2a, b. As it is clearly seen at Fig. 2a in the topological SC phase the P_z oscillates with changing the magnetic-field magnitude. Such a behavior agrees with one of the MBS energy, E_{MBS} (Fig. 2c). In general the oscillations can be explained by the structure of the MBS wave function [21,22]. The MBS wave function has an exponentially decaying envelope with certain localization length and a fast-oscillating part depending on the magnetic field. When the field exceeds a critical value, the two MBSs at the wire's ends overlap and their energy split away from zero (see the colored areas in Fig. 2c). At the same time the MBS energy can return to zero if the fast-oscillating part vanishes (see the white-color arches at Fig. 2c, do not confuse with the white-color segments corresponding to the trivial SC). As a result we obtain the MBS energy and MBS probability density oscillations. The last leads to the oscillating behavior of the P_z. It is worth to note that at high fields the oscillations approach to small and zero values of P_z. Another important feature is that at one of the ends the P_z has a different sign if the magnetic-field orientation changes from the transversal to the longitudinal one (see Fig. 2b). Consequently, there is an area at the low canted fields where the z-spin polarization is virtually absent. At the high fields the oscillations give rise to the alternation of the weak- and strong-polarization regimes. Thus, for the arbitrary magnetic-field angles at the xz plane the electronic z-spin polarization can be close to zero even in the topologically nontrivial phase and it becomes inconvenient to use this parameter to describe the topological phase transition [18]. At the same time, as it follows from Figs. 2a, b the sum of the P_z absolute values at both ends remains significantly nonzero.

5. The influence of disorder

Let us consider the influence of disorder on P_z and E_{MBS}. For modeling Anderson disorder we introduce an on-site random potential W_n which takes values with an uniform distribution in the interval [−W/2, W/2]. The effect of disorder on the electronic z-spin polarization in the MBS at both ends and the MBS energy is shown in Fig. 3. It follows from the graphs that the qualitative changes in the behavior of P_z and E_{MBS} as a result of random term addition to the on-site energy do not occur. Regions with different sign and, consequently, the region of weak spin polarization at one of the ends are maintained (see Fig. 3a,b). The disorder suppresses the polarization at the high fields as it results from a comparison of Figs. 2b and 3b. The MBS energy has narrow arch-shaped areas where $E_{MBS} \simeq 0$ and wide arch-shaped areas where $E_{MBS} \neq 0$, due to the coupling between the MBSs at opposite ends. But the disorder causes to the nonmonotonic change of the maxima height of the E_{MBS} when $V_{x,z}$ is swept.

6. Conclusions

We numerically studied the dependence of the z-component of the electronic spin polarization at the ends of the topological superconducting wire on the magnetic field and diagonal disorder. It was shown that the P_z oscillates as a function of the external magnetic field. This behavior correlates with the one of the MBS energy and is explained by the features of the MBS wave function. It was demonstrated that the P_z oscillations reach close-to-zero values with increasing the magnetic field. Additionally, there is a range of the magnetic-field angles where the P_z becomes weak in the topologically nontrivial phase since this characteristic has different sign in the transversal and longitudinal fields. Consequently, it becomes unpractical to use this parameter to describe the topological phase transition in the canted field. However, the superposition of the electronic spin polarization at both ends can still be considerably nonzero in this case. Anderson disorder does not seriously affect the above-mentioned features but leads to the appearance of the additional areas with weak spin polarization.

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