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Majorana zero modes in the coexistence phase of chiral superconductivity and 120°-type magnetic order on the triangular lattice

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ABSTRACT

We discuss the conditions under which Majorana zero modes can be implemented in the coexistence phase of chiral superconductivity and 120°-type noncollinear spin structure on the triangular lattice. It is shown that the gapless elementary excitations exist on the finite region in the parameter space consisting of the effective exchange field, the chemical potential, and the superconducting order parameter. The range of parameters supporting Majorana edge states with exactly zero excitation energy has also been found.

1. Introduction

In the high-energy physics a Majorana fermion has its own antiparticle [1]. In recent years the interest in the Majorana problem has grown due to implementing the elementary excitations in the condensed matter which are similar to the Majorana fermions.

Among the promising systems supporting Majorana zero modes, there are quantum superfluid liquids [2] and topological superconductors [3–5]. In the majority of topological superconductors a spin–orbit interaction plays a significant role in the formation of Majorana bound states [6].

Recently, an alternative mechanism for the creation of Majorana zero modes has been proposed which is not connected with a spin–orbit interaction but is caused by the coexistence of superconductivity and magnetism [7,9,10]. In Ref. [7] the coexistence phase of chiral $d_{x^2-y^2}+id_{xy}$-wave superconductivity and noncollinear stripe-type spin ordering on the triangular lattice has been considered. It has been shown that the edge states with almost zero excitation energy appear when the Fermi contour crosses the nodal points of the chiral superconducting order parameter. However, in the $t-J-V$ model the solution of the self-consistent integral equation for the superconducting order parameter in the presence of stripe-type magnetic ordering does not have the chiral structure [11]. This means that the state considered in [7] with coexisting chiral superconducting and noncollinear magnetic orders does not satisfy the self-consistent equations for the order parameters due to symmetry reasons. Furthermore, consideration of the real quasimomentum dependence of the superconducting order parameter in the above-mentioned coexistence phase significantly complicates searching for the Majorana zero modes.

The chiral structure of the superconducting order parameter remains valid in the case of 120°-type magnetic order on the triangular lattice [11]. Such structure of noncollinear magnetic ordering has also been considered in [8] to create the Majorana modes, but only in the particular case of the model parameters. Therefore, in this paper the formation of the Majorana zero modes in the coexistence phase of chiral $d_{x^2-y^2}+id_{xy}$-wave superconductivity and 120°-type noncollinear spin order is analyzed. As a result, a wide range of the parameters supporting the Majorana zero modes is obtained. At the borders of this parameter region the bulk spectrum is gapless.

2. Model and method

We consider the Hamiltonian describing the coexistence phase of chiral superconductivity and noncollinear magnetic order on the triangular lattice in the mean-field approximation:

$$H=-\mu\sum_{f\sigma}c_{f\sigma}^{\dagger}c_{f\sigma}+\sum_{fm\sigma}t_{fm}c_{f\sigma}^{\dagger}c_{m\sigma}+h(\mathbf{Q})\sum_{f}(\exp(i\mathbf{Q}\mathbf{R}_f)c_{f\uparrow}^{\dagger}c_{f\downarrow}+\exp(-i\mathbf{Q}\mathbf{R}_f)c_{f\downarrow}^{\dagger}c_{f\uparrow})+\sum_{fm}(\Delta_{fm}c_{f\uparrow}c_{m\downarrow}+\Delta_{fm}^{*}c_{m\downarrow}^{\dagger}c_{f\uparrow}^{\dagger}),\qquad(1)$$

where μ is the chemical potential, t_{fm} is the electron hopping amplitude. By analogy with Ref. [7], the formation of noncollinear magnetic order is considered in the mean-field approximation where the on-site spin average is $\langle\mathbf{S}_f\rangle=M(\cos(\mathbf{Q}\mathbf{R}_f),-\sin(\mathbf{Q}\mathbf{R}_f),0)$. The effective exchange field parameter is defined as follows:

$$h(\mathbf{Q})=M/2\sum_{m}I_{fm}\exp(-i\mathbf{Q}(\mathbf{R}_f-\mathbf{R}_m)),\qquad(2)$$

where M is the magnetization, I_{fm} is the exchange integral. Hereinafter, the coordinates in real and quasimomentum space are

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defined in the basis: $\mathbf{R}_f = n\mathbf{a}_1 + m\mathbf{a}_2$, $\mathbf{k} = k_1\mathbf{b}_1 + k_2\mathbf{b}_2$, where $\mathbf{a}_i$ are the basic vectors of the triangular lattice, $\mathbf{b}_i$ are the basic reciprocal vectors.

In the case of the 120°-type magnetic order (3×3-type) the magnetic structure vector is $\mathbf{Q} = (Q, Q)$, $Q = 2\pi/3$ and $h(\mathbf{Q}) = M/2(-3I_1 + 6I_2)$. Here I_1 and I_2 are the exchange parameters for the first and second coordination spheres, respectively.

It is assumed that a pairing interaction leading to superconductivity with the anomalous amplitude Δ_{fm} is developed between the next nearest neighbors. This can be achieved when the pairing interaction between nearest neighbors is suppressed by the inter-site Coulomb interaction [7,12]. Then the superconducting order parameter is well defined by the $d_{x^2-y^2} + id_{xy}$-wave chiral invariant for the second coordination sphere on the triangular lattice:

$$\Delta_{2,\mathbf{k}} = 2\Delta_{22}\left[\cos(2k_1 + k_2) + e^{i2\pi/3}\cos(2k_2 + k_1) + e^{i4\pi/3}\cos(k_1 - k_2)\right]. \quad (3)$$

In order to simplify calculations, we will consider the periodic boundary conditions along $\mathbf{a}_2$. This corresponds to description of the triangular lattice folded on a cylinder. Then the task of calculation of elementary excitations (for the fixed value k_2) is solved using the Bogoliubov transformation:

$$\alpha_{k_2 j} = \sum_{n=1}^{N_1}\left(A_{jk_2}(n)c_{nk_2\uparrow} + B_{jk_2}(n)c_{n,k_2-Q_2\downarrow} + C_{jk_2}(n)c^{\dagger}_{n,-k_2\downarrow} + D_{jk_2}(n)c^{\dagger}_{n,-k_2+Q_2\uparrow}\right), \quad (4)$$

where $j = 1, 2, \ldots, 4N_1$, N_1 is a total number of sites along $\mathbf{a}_1$.

3. The parameter region for the bulk gapless excitations

The parameter regions supporting Majorana zero modes are well described based on the analysis of the bulk spectrum. At the borders of the different topological phases a gap in the bulk excitation spectrum should disappear [13]. To find such region we analyze the expression for the bulk spectrum in the presence of coexisting superconductivity and noncollinear magnetic order:

$$E^{\pm}_{\mathbf{k}} = \sqrt{\frac{1}{2}\left(\xi^2_{\mathbf{k}} + \xi^2_{\mathbf{k-Q}} + 2h^2 + |\Delta_{\mathbf{k}}|^2 + |\Delta_{\mathbf{k-Q}}|^2\right) \pm \nu^2_{\mathbf{k}}}, \quad (5)$$

where

$$\nu^2_{\mathbf{k}} = \left\{\frac{1}{4}\left(\xi^2_{\mathbf{k}} - \xi^2_{\mathbf{k-Q}} + |\Delta_{\mathbf{k}}|^2 - |\Delta_{\mathbf{k-Q}}|^2\right)^2 + h^2\left[(\xi_{\mathbf{k}} + \xi_{\mathbf{k-Q}})^2 + |\Delta_{\mathbf{k}} + \Delta_{\mathbf{k-Q}}|^2\right]\right\}^{1/2}. \quad (6)$$

The following notations have been introduced: $\xi_{\mathbf{k}} = t_{\mathbf{k}} - \mu$, $t_{\mathbf{k}}$ is a Fourier-transform of the hopping integral.

Remarkably, that the quasimomentum dependence of the superconducting order parameter (3) with regard to the 120°-type spin ordering satisfies the relation $\Delta_{2,\mathbf{k-Q}} = \Delta_{2,\mathbf{k}}$. Then, the parameters at which the bulk excitation spectrum $E^{\pm}_{\mathbf{k}}$ is equal to zero are defined by the equation:

$$h^2 = \xi_{\mathbf{k}}\xi_{\mathbf{k-Q}} + |\Delta_{2,\mathbf{k}}|^2. \quad (7)$$

In the case $k_2 \neq -2\pi/3$ in Eq. (7), the quasimomentum components k_1 and k_2 of the gapless points in the Brillouin zone are connected by the relation:

$$\cos(k_1) = \frac{\sin^2(k_2) - \cos^2(k_2) + \sqrt{3}\sin(k_2) - \cos(k_2) + 1/2}{\sqrt{3}\sin(k_2) + \cos(k_2) + 2}. \quad (8)$$

In the case $k_2 = -2\pi/3$, the bulk spectrum is equal to zero for the family of curves (7) each of which is labeled by the quasimomentum component $k_1 \in [-\pi, \pi)$. For further consideration the equation for the envelope curve is quite essential:

$$\widetilde{h}^2 = (t_{\mathbf{k}} - \widetilde{\mu}_{\mathbf{k}})^2 + |\Delta_{2,\mathbf{k}}|^2,$$
$$\widetilde{\mu}_{\mathbf{k}} = \frac{t_{\mathbf{k}}\cdot\partial t_{\mathbf{k}}/\partial k_1 + (\partial|\Delta_{2,\mathbf{k}}|^2/\partial k_1)/2}{\partial t_{\mathbf{k}}/\partial k_1}. \quad (9)$$

4. Majorana zero modes on the triangular lattice

The system of equations for the coefficients of the Bogoliubov transformation (4) has a form:

$$\begin{aligned}
(\varepsilon + \mu - t_k)A_k(n) = {} & T_kA_k(n-1) + T^*_kA_k(n+1) + \Gamma_kA_k(n-2) \\
& + \Gamma^*_kA_k(n+2) + he^{-iQ_1n}B_k(n) \\
& - \Delta_{22}\left[e^{ik}C_k(n-2) + \Psi_kC_k(n-1) \right.\\
& \left. + \Psi_{-k}C_k(n+1) + e^{-ik}C_k(n+2)\right], \\
(\varepsilon + \mu - t_{k-Q_2})B_k(n) = {} & he^{iQ_1n}A_k(n) + T_{k-Q_2}B_k(n-1) + T^*_{k-Q_2}B_k(n+1) \\
& + \Gamma_{k-Q_2}B_k(n-2) + \Gamma^*_{k-Q_2}B_k(n+2) \\
& + \Delta_{22}\left[e^{i(k-Q_2)}D_k(n-2) + \Psi_{k-Q_2}D_k(n-1) \right.\\
& \left. + \Psi_{-k+Q_2}D_k(n+1) + e^{-i(k-Q_2)}D_k(n+2)\right].
\end{aligned} \quad (10)$$

Here
$t_k = 2t_1\cos(k) + 2t_3\cos(2k)$,
$T_k = t_1(1 + \exp(ik)) + t_2(\exp(i2k) + \exp(-ik))$,
$\Gamma_k = t_2\exp(ik) + t_3(1 + \exp(i2k))$,
$\Psi_k = \exp(i2\pi/3)(\exp(i2k) + \exp(i2\pi/3 - ik))$, t_1, t_2, t_3 are the electron hopping amplitudes for the first three coordination spheres. We now use for brevity the notation $k = k_2$. The equations for the coefficients $C_k(n)$, $D_k(n)$ can be obtained from the presented ones by the replacement: $A_k(n) \leftrightarrow C_k(n)$, $B_k(n) \leftrightarrow D_k(n)$, $\varepsilon \to -\varepsilon$, $\Delta_{22} \to -\Delta^*_{22}$, $\Psi_k \to \Psi^*_{-k}$.

In order to study the conditions supporting the Majorana zero modes we solve the system of Eq. (10) at $\varepsilon = 0$. The most wide parameter region, in which the zero modes are realized, corresponds to $k = -2\pi/3$. In what follows, only hoppings between nearest neighbors are considered for simplicity. Taking into account the next nearest neighbors hoppings does not change the obtained results on the qualitative level.

In Fig. 1 the set of curves $h(\mu)$ is shown, for which the gapless elementary excitations occur. We consider the triangular lattice folded on a cylinder with the length $L_1 = 30a$ and $k = -2\pi/3$. The superconducting order parameter is $\Delta_{22} = 0.3|t_1|$. The borders of the region within which there exist the bulk gapless elementary excitations are shown by the bold red curve. This curve is described by Eqs. (7) and (9). The edge states with exactly zero energy corresponding to the Majorana zero modes are found for the parameter lines which lie beyond this region. In the figure one of the points supporting the

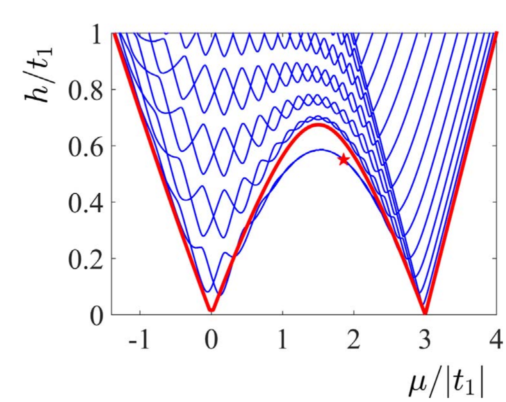

Fig. 1. Conditions for the implementation of the zero excitation energy on the triangular lattice with open boundaries along the unit vector $\mathbf{a}_1$ (blue lines). Here h is the effective exchange field induced by noncollinear magnetic order, μ is the chemical potential, t_1 is the nearest-neighbors hopping amplitude. $L_1 = 30a$ is the cylinder length, $\Delta_{22} = 0.3|t_1|$. Quasimomentum $k = -2\pi/3$ is connected with the periodic boundary conditions along $\mathbf{a}_2$. The bold red line shows the border of the region for the bulk gapless excitations. For the parameter line beyond this area depicted by a red star the Majorana zero modes occur. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Majorana zero mode is depicted by a star. For those lines, which are bounded by the above-mentioned region, the states with exactly zero energy have a band character and their densityprofile is distributed on the whole cylinder length.

The Majorana zero modes exist also for $k = \pi/3$. However, in this case the Majorana zero modes are found only in the vicinity of $\Delta_{22} = |t_1|$.

Due to the fact that the Fermi operators $c_{n,-k+Q_2\sigma}$, $c_{n,k\sigma}$ for the values $k = -2\pi/3$, $\pi/3$ describe fermions with equivalent quasimomentum it is possible to introduce the Majorana operators in the well-known form [14]:

$$\gamma_{An\sigma} = c_{nk\sigma} + c^{\dagger}_{nk\sigma},\ \gamma_{Bn\sigma} = -i(c_{nk\sigma} - c^{\dagger}_{nk\sigma}). \quad (11)$$

As a result, the spatial distribution of the Majorana operators can be analyzed, using the new Bogoliubov coefficients: $w_{n\sigma}$ is for $\gamma_{An\sigma}$ and $z_{n\sigma}$ is for $\gamma_{Bn\sigma}$.

The density profile of the Majorana modes on the lattice sites is shown in Fig. 2 for the parameters h and μ depicted in Fig. 1 by a star. It is seen that the distribution corresponds to the edge states with exactly zero energy. It can also be seen that the zero excitation energy can be implemented even when the distributions of the Majorana modes belonging to the different edges of the cylinder overlap. With increasing the cylinder length the tendency of the Majorana modes to be localized near the edges becomes more apparent. This is confirmed by Fig. 3 where the density of the Majorana states for the length $L_1 = 120a$ is shown.

It should be noted that recently [15] the parameter lines supporting the Majorana zero modes have been obtained in the framework of the Kitaev model [14]. It is shown that a passage through these lines is accompanied by a change of the fermionic parity of the ground state.

In the considered model as well as in the Kitaev model there exist the elementary excitations which energy exponentially decreases with increasing the length L_1. The parameter region for such excitations is much wider compared with the region for which the Majorana zero modes are found. Moreover these conditions are not restricted by the quasimomentum values $k = -2\pi/3$, $\pi/3$. One of such examples is given in [8].

The parameter lines supporting the Majorana zero modes for different values of the superconducting order parameter are shown in Fig. 4. The solid red (upper) line is for $\Delta_{22} = 0.5|t_1|$, the solid purple line is for $\Delta_{22} = 0.3|t_1|$, and the results for $\Delta_{22} = 0.1|t_1|$ are shown by the solid blue (lower) line. The dashed red (upper) and blue (lower) lines in the figure enclose the parameter regions, for which the bulk spectrum does not exhibit an energy gap in the cases of $\Delta_{22} = 0.5|t_1|$ and $\Delta_{22} = 0.1|t_1|$, respectively. These lines are obtained from Eq. (9). It is seen that in the case $\Delta_{22} = 0.5|t_1|$ the solid line, supporting the realization of the Majorana zero modes, lies well below the border of such region.

The tendency for the Majorana modes to be localized near the edges is more pronounced when their parameter line greatly differs from the border of the region of bulk gapless excitations. It is seen that the lines in the case of $\Delta_{22} = 0.1|t_1|$ describing the implementation of the zero excitation energy for the periodic and open boundary conditions are almost the same (compare lower solid and dashed lines). Then the vast majority of the Majorana zero modes on the triangular lattice are realized in the parameter region enclosed by the dashed lines in Fig. 4.

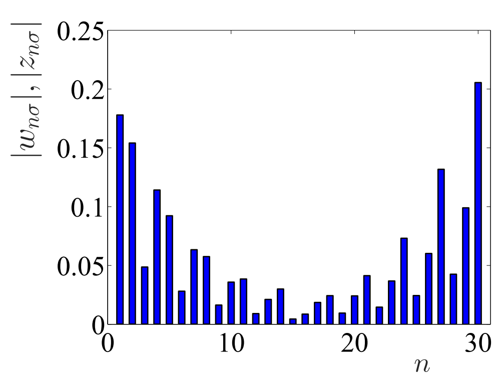

Fig. 2. The distribution of the Majorana modes on the lattice sites along $\mathbf{a}_1$ for the parameters depicted in Fig. 1 by a star.

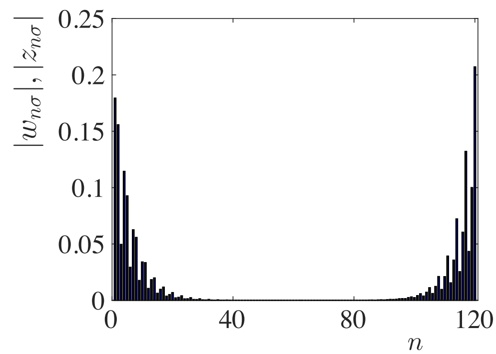

Fig. 3. The distribution of the Majorana modes for the parameters: $\Delta_{22} = 0.3|t_1|$, $L_1 = 120a$, $h = 0.55t_1$, $\mu = 1.86|t_1|$.

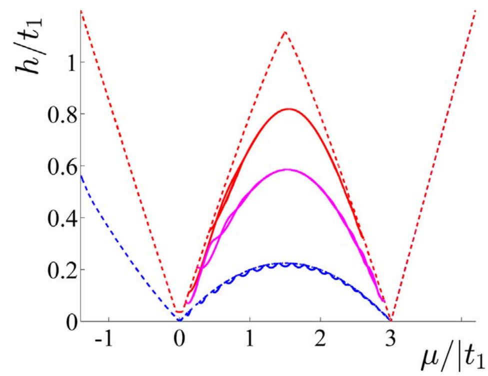

Fig. 4. The evolution of the parameter lines supporting Majorana zero modes (solid lines) upon changing the superconducting order parameter amplitude: $\Delta_{22} = 0.5|t_1|$, $\Delta_{22} = 0.3|t_1|$, $\Delta_{22} = 0.1|t_1|$ (from the upper line to the lower line), $L_1 = 30a$, $k = -2\pi/3$. The dashed lines bound the region of the gapless excitations of the bulk spectrum. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

5. Conclusions

As has been shown in [11], the coexistence phase of chiral superconductivity and noncollinear spin ordering is realized in the case of 120°-type magnetic structure. In the presence of stripe-type magnetic order the superconducting order parameter is modified and it does not correspond to the chiral $d_{x^2-y^2} + id_{xy}$-wave symmetry of the triangular lattice. Thus, in this paper we consider the 120°-type spin state in the coexistence phase for searching for the Majorana zero modes.

We describe the conditions under which the edge states with exactly zero excitation energy appear in the coexistence phase of chiral superconductivity and 120°-type noncollinear magnetic structure on the triangular lattice. Such edge states correspond to the Majorana zero modes. The edge states have zero excitation energy even in the case when distributions of the Majorana modes belonging to different edges overlap. The expressions describing the conditions for the implementation of the Majorana zero modes have been obtained analytically.

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