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Local density of states in one-dimensional photonic crystals and sinusoidal superlattices

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Abstract

We have calculated the local density of states (LDOS) for four Brillouin zones of a superlattice for a plane source depending on its location relative to the change in the profile of dielectric permeability $\varepsilon(z)$ of the superlattice. It is shown that the LDOS for the cases of sinusoidal and rectangular profiles of $\varepsilon(z)$ are close to each other in the first and second Brillouin zones, and sharp differences between them appear beginning with the third zone. Radical changes in the LDOS occur in a rectangular superlattice with different thicknesses of adjacent layers. In this case, the function LDOS has a sharp jump at the edges of the allowed bands in the transition from one layer to another. The effects studied theoretically in this paper, can be detected and studied experimentally by the intensively developing currently methods of nanooptics.

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1. Introduction

Sources of light in photonic crystal structures were intensively studied in recent decades. LEDs that are based on electroluminescence, are the most promising light sources for practical applications (Novotny and Hecht (2012), Mottier (2009), David et al. (2012)). Doped nanoparticles in a substance emitting by laser excitation, are used for studies of the effect of photonic crystal structures on the radiation pattern, and the relaxation (Sapienza et al. (2011),

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Kuroda et al. (2010)). Photonic crystals, that are periodically structured optical media, are used to control the radiative properties of these light sources. The use of photonic crystals based on the presence of forbidden zones in which there is suppression of the spontaneous emission (Yablonovitch (1987)). In the one-dimensional photonic crystal, a sharp increase in output power at frequencies close to the edges of the band gap is observed (Tocci et al. (1996)).

The local density of states (LDOS) (Moroz (1999), Wubs and Lagendijk (2002), Yeganegi et al. (2014)) is an essential characteristic of inhomogeneous media. In the work (Moroz (1999)) the LDOS was calculated for the superlattice with a rectangular profile of modulation of the dielectric permeability (a one-dimensional photonic crystal). It is known that the spontaneous emission rate of atoms and molecules embedded in a photonic crystal is directly proportional to the LDOS (Novotny and Hecht (2012)). Therefore, the spontaneous emission can either be reduced down to zero (at zeroes of the LDOS) or substantially enhanced at points which correspond to the maxima of the LDOS) with respect to the vacuum case. Suppression of the spontaneous emission may have applications for semiconductor lasers, solar cells, heterojunction bipolar transistors, and thresholdless lasers.

The aim of this work is to study changes of the LDOS depending on the position of the radiation source relative to the profile of the superlattice dielectric permeability $\varepsilon(z)$, as well as a comparison of the LDOS for superlattices with the rectangular and the sinusoidal profiles of modulation of $\varepsilon(z)$.

2. Green's function and LDOS

In the sinusoidal superlattice the scalar Green's function satisfies the equation

$$\nabla^2 G(\mathbf{r}, \mathbf{r}_0) + \left(\frac{\omega}{c}\right)^2 [\varepsilon + \Delta\varepsilon \cos(qz + \psi)] G(\mathbf{r}, \mathbf{r}_0) = -\delta(\mathbf{r} - \mathbf{r}_0), \quad (1)$$

where $q = 2\pi/l$, l is the superlattice period, ψ is the phase, ε is the dielectric permeability, ω and c are the frequency and speed of light, respectively. In the work (Ignatchenko and Tsikalov (2015), Ignatchenko and Tsikalov (2016)) we obtain an analytic expression for the Green's functions in the \mathbf{k} -space in the form of continued fractions, which have fast convergence, so that expression is useful in the study of the Green's function in the \mathbf{k} -space and in the \mathbf{r} -space.

$$G(\mathbf{k}_\perp, k_z, z_0) = -\frac{\exp(-ik_z z_0)}{(2\pi)^3} \frac{1 + P_1^+ + P_1^-}{L_0}. \quad (2)$$

Here P_1^\pm are ascending continued fractions, determined by the recursive formula

$$P_n^\pm = -\frac{\Delta\varepsilon}{2} \left(\frac{\omega}{c}\right)^2 \exp(\pm i\psi) \frac{\exp(\pm inqz_0) + P_{n+1}^\pm}{L_n^\mp}, \quad (3)$$

and L_0, L_n^\pm are ordinary continued fractions defined by the formulas

$$L_0 = \varepsilon \left(\frac{\omega}{c}\right)^2 - k^2 - \frac{\Delta\varepsilon^2}{4} \left(\frac{\omega}{c}\right)^4 \left[\frac{1}{L_1^+} + \frac{1}{L_1^-} \right], L_n^\pm = \varepsilon \left(\frac{\omega}{c}\right)^2 - k_\perp^2 - (k_z \pm nq)^2 - \frac{\Delta\varepsilon^2}{4} \left(\frac{\omega}{c}\right)^4 \frac{1}{L_{n+1}^\pm}. \quad (4)$$

The function of the LDOS is proportional to the imaginary part of the Green's function at the point of the radiation source, and is defined by expression (see., e.g. Tocci et al. (1996), Moroz (1999), Wubs and Lagendijk (2002), Yeganegi et al. (2014))

$$\rho_l(\omega, \mathbf{r}_0) = \frac{2\omega}{\pi c^2} \text{Im}[G(\omega, \mathbf{r}_0, \mathbf{r}_0)]. \quad (5)$$

We investigate the LDOS for a sinusoidal superlattice using the expression (2) for the calculation of the Green's function. The function of the LDOS for a plane source in a sinusoidal superlattice has calculated by us in the coordinates ω and ψ (Fig. 1). It is assumed, that the plane source is located at the point $z = z_0$, and its position relative to the superlattice is characterized by the superlattice phase ψ .

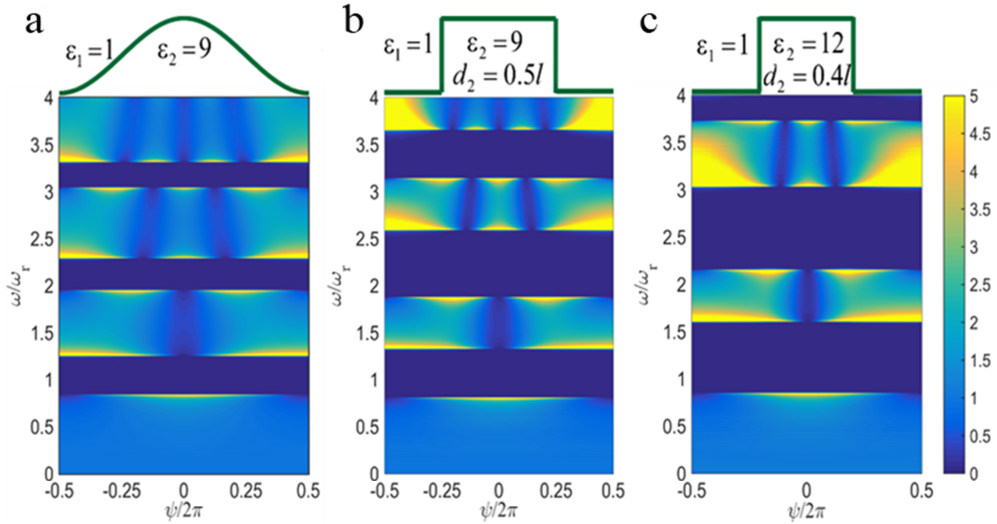


Fig. 1. Relief maps of the normalized LDOS $\rho_l(\omega, \psi) / \rho_0(\omega)$ for the sine superlattice (a) and the rectangular superlattices with equal (b) and different (c) adjacent layer thicknesses. The attenuation $\omega'' / \omega_r' = 0.001$ was introduced in the calculations.

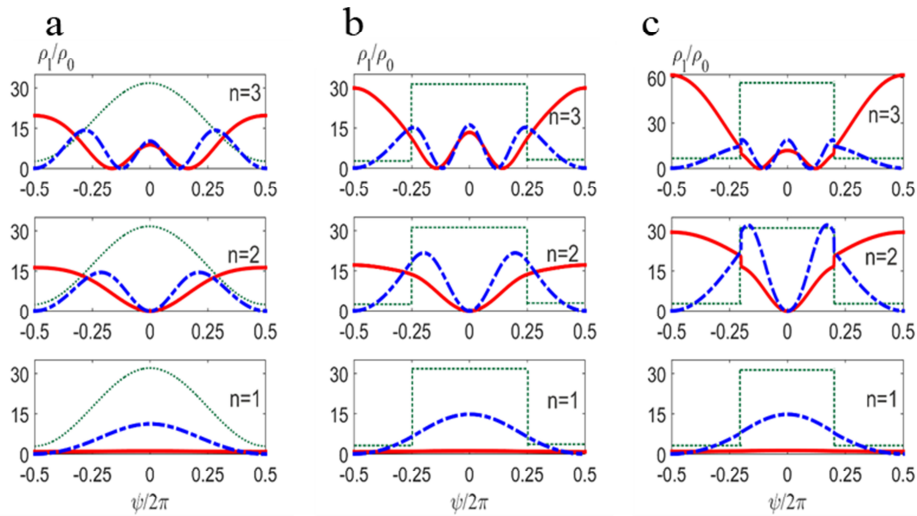


Fig. 2. Dependences of the normalized LDOS $\rho_l(\omega, \psi) / \rho_0(\omega)$ on ψ at fixed frequencies for three Brillouin zones ($n = 1, 2, \text{ and } 3$). Solid red curves at $n = 2$ and 3 correspond to the frequencies near the lower edge of the corresponding band, and at $n = 1$ to the middle of the first zone. Dashed blue curves in all three figures correspond to frequencies near the upper edge of each zone. Profiles of modulation $\varepsilon(z)$ are also shown (dotted green curves). The graphs correspond to the sine superlattice (a) and the rectangular superlattices with equal (b) and different (c) adjacent layer thicknesses.

Thus, $\psi = 0$ corresponds to the position of the source in the maximum value of the function $\varepsilon(z)$ and $\psi = \pi$ and $\psi = -\pi$ correspond to the minima of the function $\varepsilon(z)$.

Relief map of the normalized function ρ_l/ρ_0 , where ρ_0 is the LDOS for a plane source in vacuum, are shown in Fig. 1a for four Brillouin zones. Dependences of the function ρ_l/ρ_0 on ψ at fixed frequencies are shown in Fig. 2a for three Brillouin zones ($n=1, 2$, and 3). Solid red lines at $n=2$ and 3 correspond to the frequencies near the lower edge of the corresponding band, and at $n=1$ to the middle of the first zone. Dashed blue curves in all three figures correspond to frequencies near the upper edge of each zone. The LDOS for a superlattice with the rectangular profile of the function $\varepsilon(z)$ was calculated in (Tocci et al. (1996)) for the third Brillouin zone. Calculated by us LDOS for a rectangular superlattice also is shown in the Figs. 1 and 2. Figs. 1b and 2b corresponds to the superlattice with equal and Figs. 1c and 2c with unequal thicknesses of adjacent layers. A comparison of the LDOS for the cases of sinusoidal and rectangular profiles shows that LDOS close to each other in the first and second Brillouin zones, and significant differences between them appear beginning with the third zone. Radical changes in the LDOS occur in a rectangular superlattice with different thicknesses of adjacent layers. In this case, the function of the LDOS has a sharp jump at the edges of the allowed bands in the transition from one layer to another.

Optical and electron-optical methods for experimental studies of the LDOS in heterogeneous structures are intensively developed at present (Tocci et al. (1996), Garcia and Kociak (2008), Birowosuto et al. (2010)).

3. Conclusion

Changes in the LDOS of the superlattice for the plane source, depending on its location relative to the profile of the dielectric permeability $\varepsilon(z)$ of the superlattice have been investigated. Comparison of functions of the LDOS for superlattices with the sinusoidal and the rectangular modulation profile has been carried out. It is shown that LDOS for the cases of sinusoidal and rectangular profiles $\varepsilon(z)$ are close to each other in the first and second Brillouin zones, and significant differences between them appear beginning from the third zone. Radical changes in the LDOS occur in a rectangular superlattice with different thicknesses of adjacent layers. In this case, the function of LDOS has a sharp jump at the edges of the allowed bands in the transition from one layer to another. The effects studied theoretically in this paper, can be detected and studied experimentally by intensively currently developing methods of nanooptics (Novotny and Hecht (2012)).

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