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Green's functions of polaritons in a medium with zero-mean inhomogeneous coupling parameter

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Abstract

Dynamic susceptibilities (Green's functions) of the interacting electromagnetic waves $G''_e(\omega)$ and optical phonons $G''_u(\omega)$ in a medium with zero-mean inhomogeneous coupling parameter have been considered. The calculation was performed using a self-consistent approximation for the two stochastically interacting wave fields.It is shown that on the tops of the resonance maxima of the imaginary parts of the Green functions the fine structure is formed: a minimum (dip) on the top of $G''_e(\omega)$ and narrow maximum (peak) on the top of $G''_u(\omega)$. With increasing the correlation wavenumber of inhomogeneities k_c (i.e., with decreasing the size of inhomogeneities), the width of the peak on $G''_u(\omega)$ decreases, and two resonance maxima in the function $G''_e(\omega)$ are formed.Because of the large difference in the speeds of light and optical phonons, the fine structure of the polaritons is manifested itself more clearly and saved to a much larger values of k_c, than for the studied earlier crossing resonance of spin and elastic waves.

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*Keywords:*electromagnetic waves, optical phonons, crossing resonance, polaritons, Green's functions, self-consistent approximation

1. Introduction

The phenomenon of crossing resonance between two wave fields of different physical nature in a homogeneous medium is well studied. The crossing resonance occurs at the intersection of dispersion curves of these fields and

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leads to lifting of the degeneracyof frequenciesat this point. Coupled oscillations of various physical nature are formed in this area of the strong interaction (polaritons (Kittel' (1978)), magnetoelastic wave (Akhiyezer et al. (1967)), and so on).In this paper, we consider the interaction between electromagnetic waves and transverse optical phonons, which results in the formation of coupled oscillations - polaritons. The crossing resonance between the waves in ionic crystals usually occurs in the long-wavelength range (the wavelength is much larger than the lattice parameters), where the main role plays the Coulomb energy. Thus, we can consider the crystal as a continuous medium and use a macroscopic description. The crossing resonance leads to the formation of the gap in the dispersion relation of waves: electromagnetic waves cannot propagate in the frequency range $\omega_t < \omega < \omega_l$, where ω_t and ω_l are the frequencies of transverse and longitudinal optical phonons.

The dispersion law of waves in the range of the crossing resonance in an inhomogeneous medium was studied in Ignatchenko, Deich(1994) and Deich, Lisyansky, (1996). The research was conducted for the extremely inhomogeneous model of interaction of two wave fields when the coupling parameter between the fields is a random function of the coordinates with zero mean value. In this case, the interaction between the fields is due only to spatial fluctuations in this parameter. The study in Ignatchenko, Deich(1994) and Deich, Lisyansky, (1996) was carried out in the framework of the Bourret approximation (Bourret (1962)) (single scattering of waves from inhomogeneities). In Ignatchenko, Polukhin (2013), the dynamic susceptibilities (Green's functions) of the coupled spin and elastic waves in the crossing resonance range in an inhomogeneous medium with zero mean value of the coupling parameter between the fields were considered. The study was conducted with an approximate into account the processes of multiple scattering of waves from the inhomogeneities. The calculation was performed in the framework of self-consistent approach, taking into account all the diagrams with nonintersecting lines correlations in expansions of the Green functions (Migdal (1958), Kraichnan (1961)).

The objectives of the work are: (i) to investigate the dynamic susceptibilities (Green's functions) of coupled electromagnetic waves and transverse optical phonon (polaritons) in a medium with the zero-mean inhomogeneous coupling parameter between them and (ii) to take into account the processes of multiple scattering of waves from the inhomogeneities in the framework of self-consistent approximation.

2. The system of equations for the Green's functions

We consider a model of an ionic crystal, in which only the dipole moment $d_i(\mathbf{x})$ is non-uniform (Deich, Lisyansky, (1996)). The vector $\mathbf{x}$ determines the position of the dipole moment, and the index i takes the values of x, y, and z. The function $d_i(\mathbf{x})$ can be represented as

$$d_i(\mathbf{x}) = < d_i > + (\Delta d_i)\rho(\mathbf{x}), \tag{1}$$

where $< d_i >$ and Δd_i determined the mean value and root mean square (rms) fluctuation of the coupling parameter between the wave fields, respectively, $\rho(\mathbf{x})$ is the centered ($<\rho>=0$) and normalized ($<\rho^2>=1$) random function of coordinates (angle brackets denotes the average over the ensemble of random functions ρ). In a media with inhomogeneous coupling parameter between the waves, the average value of which is equal to zero, we have

$$\begin{aligned} &< d_i >= 0, \\ &< d_i(\mathbf{x}) d_j(\mathbf{x}') >= (\Delta d)^2 \delta_{ij} < \rho(\mathbf{x})\rho(\mathbf{x}') >, \end{aligned} \tag{2}$$

$$< \rho(\mathbf{x})\rho(\mathbf{x}') >= K(\mathbf{r}), \tag{3}$$

where $K(\mathbf{r})$ is the correlation function, $\mathbf{r} = \mathbf{x} - \mathbf{x}'$, δ_{ij} is the Kronecker delta. Since in our model $< d_i >= 0$, the interaction of two wave fields caused only by stochastic fluctuations in these fields.

The system of the Dyson equation for the averaged Green's functions of electromagnetic waves $G^e_{\mathbf{k}}$ and optical phonons $G^u_{\mathbf{k}}$ in $\mathbf{k}$ - space has the form

$$G_{\mathbf{k}}^{e} = \frac{1}{(2\pi)^3} \frac{1}{\nu_e^2 - k^2 - Q_{\mathbf{k}}^{u}} , \tag{4}$$

$$G_{\mathbf{k}}^{u} = \frac{1}{(2\pi)^3} \frac{1}{\nu_u^2 - k^2 - Q_{\mathbf{k}}^{e}} , \tag{5}$$

where $\nu_u = \sqrt{\omega^2 - \omega_t^2} / \mathrm{v}$, $\nu_e = \omega / \tilde{c}$, $\tilde{c} = c / \sqrt{\varepsilon_\infty}$, $\gamma = \Lambda / \mathrm{v}$, $\Lambda = \sqrt{4\pi (\Delta d)^2 / (\varepsilon_\infty a^3)}$.

Here ω is the frequency, ω_t is the frequency of the transverse optical phonons, c is the velocity of light, ε_∞ is the high-frequency dielectric constant, v is the velocity of optical phonons, Δd is the rms fluctuation of the coupling parameter between electromagnetic waves and optical phonons, a is the average cell size, $Q_{\mathbf{k}}^{e}$ and $Q_{\mathbf{k}}^{u}$ are the mass operators (self-energies) of electromagnetic waves and optical phonons, respectively. A self-consistent system of integral equations for mass operators of the two stochastically interacting fields of any physical nature has essentially the same structure: $Q_{\mathbf{k}}^{e}$ depends on $Q_{\mathbf{k}}^{u}$, and $Q_{\mathbf{k}}^{u}$ depends on $Q_{\mathbf{k}}^{e}$ (see Ignatchenko, Polukhin (2013)):

$$Q_{\mathbf{k}}^{e} = \gamma^2 \nu_e^2 \int \frac{S_{\mathbf{k}-\mathbf{k}_1} d\mathbf{k}_1}{\nu_e^2 - k_1^2 - Q_{\mathbf{k}_1}^{u}} , \tag{6}$$

$$Q_{\mathbf{k}}^{u} = \gamma^2 \nu_e^2 \int \frac{S_{\mathbf{k}-\mathbf{k}_1} d\mathbf{k}_1}{\nu_u^2 - k_1^2 - Q_{\mathbf{k}_1}^{e}} , \tag{7}$$

where $S_{\mathbf{k}-\mathbf{k}_1}$ is the Fourier transform of the correlation function $K(\mathbf{r})$.

Consider the one-dimensional model. Modeling the correlation properties of the random function $\rho(x)$ by an exponential correlation function, we obtain in the one-dimensional case for $K(r)$ and $S(k)$

$$K(r) = \exp(-k_c r), \quad S(k) = \frac{1}{\pi} \frac{k_c}{k_c^2 + k^2} , \tag{8}$$

where k_c is the correlation wave number of inhomogeneities ($r_c = k_c^{-1}$ is the correlation radius).

A cross substitution of equations (6) and (7) into each other resulting in the presentation of each of the mass operator in the form of an infinite continued fraction with integral terms. In the numerical calculation the number of terms of the continued fraction were taken into account, leading to a convergence of the calculated process. The result was substituted in formulas for the Green's functions (4) and (5). Fig. 1 shows the imaginary part of the Green's functions in the crossing resonance point ($k = k_r$, $\omega = \omega_r$).The amplitudes of the functions $G_e''(\omega)$ and $G_u''(\omega)$ are incommensurable because of the large differences between the velocities of light $\tilde{c}$ and optical phonons v in a medium, as they are defined by the ratio (Ignatchenko, Polukhin (2013))

$$G_e(\omega_r) / G_u(\omega_r) = (\tilde{c} / \mathrm{v})^2 . \tag{9}$$

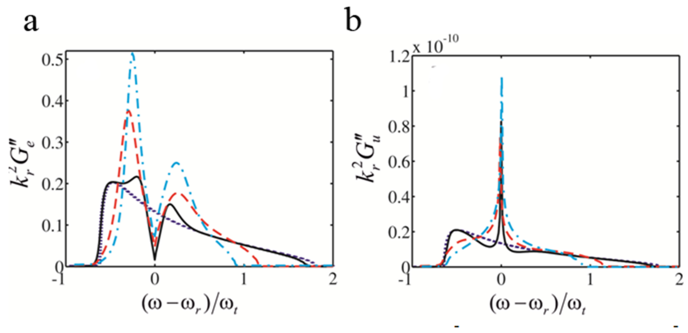

Fig. 1.The imaginary parts of the electromagnetic waves G_e'' (*a*) and optical phonons G_u'' (*b*) Green's functions at $u_c = k_c/k_r = 0.05$ (solid black curves), 0.5 (dashed red curves), 1 (blue dot-dash curves) and $u_c = 0$ (dashed black curves)

Therefore, the scales of Figures 1*a* and *b* differ by ten orders of magnitude. In Fig. 1 the black dotted curves show the limiting case of $k_c \to 0$, $S_{k-k_1} \to \delta(k-k_1)$, corresponding to the model of independent grains in a polycrystal with random values of the coupling parameter Δd between the two wave fields in each grain. In this limit the width of the peaks of $G_e''(\omega_r)$ and $G_u''(\omega_r)$ is the same and is determined by the stochastic dispersion of the parameter Δd. Peaks of the functions $G_e''(\omega_r)$ and $G_u''(\omega_r)$ are not symmetrical with respect to the crossing resonance point $\omega = \omega_r$.When k_c becomes not equal to zero, the interaction between the grains appears and a thin structure is formed in the center of each peak: minimum (dip) on top of the $G_e''(\omega_r)$ and narrow maximum on the $G_u''(\omega_r)$. Because of the large difference in the velocities of $\tilde{c}$ and v, the shape of the fine structure is very different from that which was obtained earlier for crossing resonance of spin and elastic waves (Ignatchenko, Polukhin (2013)). With the increase k_c (i.e., a decrease in the size of the inhomogeneities) the peak widths decrease, the right edges of the peaks are compressed more than the left ones. The peaks are narrowed due to the fact that with increasing k_c, wave less and less feels inhomogeneities; at sufficiently small size of inhomogeneities, a single-mode peak in $G_u''(\omega_r)$ becomes narrow as in a homogeneous medium without damping (Fig.1*b*). Another picture is observed for the dynamic susceptibility of electromagnetic waves (Fig.1*a*). Here, with an increase in k_c the minimum (dip) on top of the $G_e''(\omega_r)$ is transformed into a resonance structure with two peaks. It should be noted that the fine structure in the case of polaritons is saved to a much larger k_c, than in the case of magnetoelastic waves (Ignatchenko, Polukhin (2013)), which increases the chances of its experimental detection.

3. Conclusion

In the framework of a self-consistent approximation, dynamic susceptibilities (Green's functions) of the interacting electromagnetic waves $G_e''(\omega)$ and optical phonons $G_u''(\omega)$ in a medium with zero-mean inhomogeneous coupling parameter have been considered. It is shown that on the tops of the resonance maxima of the imaginary parts of the Green functions the fine structure is formed: a minimum (dip) on the top of $G_e''(\omega)$ and narrow maximum (peak) on the top of $G_u''(\omega)$. In the limiting case of when the correlation wave number of inhomogeneitiesis equal to zero ($k_c = 0$), corresponding to the model of independent grains in a polycrystals, the width of the peaks of $G_e''(\omega)$ and $G_u''(\omega)$ is the same, and the peaks are not symmetrical with respect to the crossing resonance point $\omega = \omega_r$. When k_c becomes not equal to zero, the interaction between the grains appears and a thin structure is formed in the center of each peak: minimum (dip) on top of the $G_e''(\omega_r)$ and narrow maximum on the $G_u''(\omega_r)$. With increasing the correlation wavenumber of inhomogeneities k_c (i.e., with decreasing the size of inhomogeneities) the width of the peak on $G_u''(\omega)$ decreases, and the function $G_e''(\omega)$ is formed two resonance maxima. Because of the large difference in the speeds of light and optical phonons, the fine structure of the polaritons is manifested itself more clearly and saved to a much larger values of k_c, than for the studied earlier crossing resonance of spin and elastic waves.

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