Quantum properties of parametric four-wave mixing in a Raman-type atomic system

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We present a study of the quantum properties of two light fields used in parametric four-wave mixing in a Raman-type atomic system. The system realizes an effective Hamiltonian of beam-splitter-type coupling between the light fields, which allows one to control squeezing and amplitude distribution of the light fields, as well as realizing their entanglement. The scheme can be feasibly applied to engineer the quantum properties of two single-mode light fields in properly chosen input states.

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I. INTRODUCTION

Squeezing [1] and entanglement [2] are two important features of quantum light and have no counterparts in the classical framework. They are the cornerstone of quantum computation and quantum communication with continuous-variable light fields, where quantum information is usually encoded into Gaussian states of light [3,4]. Entanglement and squeezing are normally generated through parametric processes such as parametric down conversion [5] and four-wave mixing (FWM) [6–10].

FWM can realize an effective coupling of the beam-splitter type between two light fields [11], which can be applied to generate the important categories of photonic quantum states, such as cat states and symmetric entangled states. In the present work, we consider a Raman-type dispersive FWM to realize a similar beam-splitter-type coupling for other applications including transferring squeezing between different modes, amplifying the amplitude of one squeezed mode, and entangling different quantum modes. There are two degenerate counterpropagating electromagnetic fields as the pump in our considered system, and they realize an effective standing wave creating the spatial modulation of the nonlinearities for the input quantum light fields. This structure behaves like photonic crystals that are widely used for control light propagation [12,13]. To avoid the influence of the quantum noises that are critical to the entanglement generation (see, e.g., [14–16]), we use a dispersive parametric interaction, so that the one-photon detuning and two-photon detuning in the process are large enough to prevent the real excitation of the atomic system from its ground states and only the quantum state of the light fields will be changed during the closed-loop parametric interaction, which involves the two counterpropagating quantum modes and the standing wave of the pump field.

The rest of the paper is organized as follows. We first present a description of the model in Sec. II. The effective Hamiltonian for the system in the dispersive regime is derived in Sec. III. In Sec. IV, we study the properties of quantum light fields, such as the amplitude dynamics, squeezing transferring, and entanglement generation, after finding the evolving light field modes. We summarize the results in Sec. V.

II. THE MODEL

Let us consider a three-level atomic system with the ground state $|a\rangle$ and two upper states $|b\rangle$ and $|c\rangle$; see Fig. 1. Through the system, the input quantum light fields of the single mode (the blue ones in Fig. 1) effectively interact with one another. The classical standing electromagnetic field with the Rabi frequency Ω implements the transition $|a\rangle - |c\rangle$. Such classical standing wave is applied along the z direction and can be decomposed into two running wave with the wave vectors $\pm k$. The two counterpropagating quantum modes that are coupled to the transition $|b\rangle - |c\rangle$ with the coupling parameter g also propagate along the z direction.

In the interaction picture, the Hamiltonian of the system in Fig. 1 takes the following form ($\hbar \equiv 1$):

$$\begin{aligned} H = &-\Delta|c\rangle\langle c| - \delta|b\rangle\langle b| \\ &\times W(|a\rangle\langle c| + |c\rangle\langle a|) + \widehat{s}^{\dagger}|b\rangle\langle c| + \widehat{s}|c\rangle\langle b|, \end{aligned} \tag{1}$$

where

$$\begin{aligned} W &= 2\Omega\cos kz, \\ \widehat{s} &= g(\widehat{a}e^{ik_0z} + \widehat{b}e^{-ik_0z}). \end{aligned}$$

In Eq. (1), we introduce the one-photon detuning $\Delta = \omega - \omega_{ac}$ and the two-photon detuning $\delta = \omega - \omega_0 - \omega_{ab}$, where ω, ω_0 are the optical frequencies of the classical field and quantum

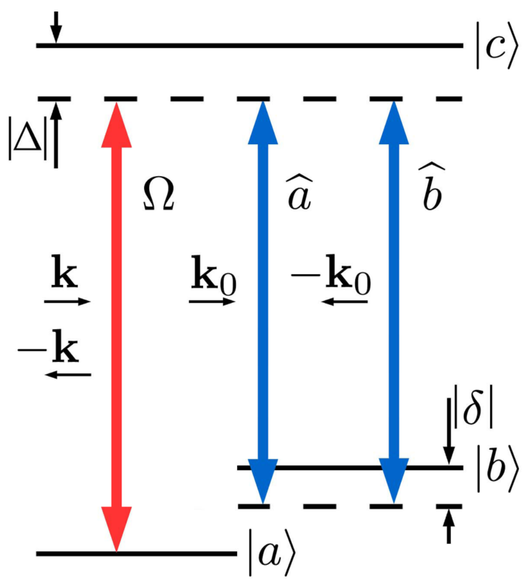

FIG. 1. Scheme of a Raman-type dispersive four-wave mixing of two counterpropagating quantum modes $\widehat{a}$ and $\widehat{b}$ with the wave vectors $\pm k_0$ and the classical standing wave with the Rabi frequency Ω in a media of a three-level quantum systems.

modes, correspondingly, while ω_{ac} and ω_{ab} are the frequencies of the corresponding transitions.

III. EFFECTIVE HAMILTONIAN

In what follows, we will consider a regime of the dispersive interaction, where the large detunings of the electromagnetic fields prevent the real transitions of the three-level system from its ground state. So, the atomic system remains the same initial state $|a\rangle$ during the interaction, and only the quantum states of the optical fields can be changed. The classical field is assumed to be much stronger than the quantum ones. In order to derive the effective Hamiltonian in the dispersive regime, we use an adiabatic elimination technique [11,17,18]. We start from the Schrödinger equation $i\hbar\frac{d\Psi(t)}{dt} = H\Psi(t)$ and project it onto atomic states $|a\rangle$, $|b\rangle$, and $|c\rangle$:

$$i\frac{d}{dt}\langle a|\Psi(t)\rangle = W\langle c|\Psi(t)\rangle, \tag{2}$$

$$i\frac{d}{dt}\langle b|\Psi(t)\rangle = -\delta\langle b|\Psi(t)\rangle + \widehat{s}^{\dagger}\langle c|\Psi(t)\rangle, \tag{3}$$

$$i\frac{d}{dt}\langle c|\Psi(t)\rangle = -\Delta\langle c|\Psi(t)\rangle + W\langle a|\Psi(t)\rangle + \widehat{s}\langle b|\Psi(t)\rangle, \tag{4}$$

following the derivations detailed in [11,18] under the conditions (n is the maximal photon number in the quantum modes)

$$\left|\frac{W}{\Delta}\right| \ll 1, \tag{5}$$

$$\left|\frac{g\sqrt{n}W}{\Delta\delta}\right| \ll 1, \tag{6}$$

which allows one to avoid the one-photon and two-photon transitions so that the system remains in its ground state $|a\rangle$ and only the state of the light fields will be changed. Then we can eliminate the states $|c\rangle$ and $|b\rangle$ from the dynamical equations, to obtain the effective dynamical evolution,

$$i\frac{d}{dt}\langle a|\Psi(t)\rangle = \widetilde{H}_{\text{eff}}\langle a|\Psi(t)\rangle, \tag{7}$$

where

$$\widetilde{H}_{\text{eff}} = \frac{\widehat{s}^{\dagger}\widehat{s}W^2}{\Delta^2\delta}$$

is the effective Hamiltonian.

Furthermore, we can simplify the present Hamiltonian and eliminate the z dependence. By omitting the fast oscillation terms and making transformation $T_\delta = \exp i\Delta k(a^{\dagger}a + b^{\dagger}b)z$, we obtain a beam-splitter-type Hamiltonian,

$$H_{\text{eff}} = \chi_0(\widehat{a}^{\dagger}\widehat{a} + \widehat{b}^{\dagger}\widehat{b}) + \sigma_0(\widehat{a}\widehat{b}^{\dagger} + \widehat{a}^{\dagger}\widehat{b}), \tag{8}$$

where

$$\chi_0 = \frac{2\Omega^2 g^2}{\Delta^2\delta} - \Delta kc, \tag{9}$$

with $\Delta k = k - k_0$, is due to a self-phase modulation and

$$\sigma_0 = \frac{\Omega^2 g^2}{\Delta^2\delta} \tag{10}$$

is the cross coupling between the quantum modes.

IV. ENGINEERING OF THE QUANTUM LIGHT FIELDS

The field operators' evolution is described by the Heisenberg equation as the two coupled propagation ones,

$$\frac{d}{dz}\widehat{a} = i\chi\widehat{a} + i\sigma\widehat{b},$$
$$\frac{d}{dz}\widehat{b} = -i\sigma\widehat{a} - i\chi\widehat{b},$$

where the parameters $z = \alpha_0 z_0$ (the replacement $t \rightarrow z_0/c$ has been used), $\chi = -\chi_0 c/\alpha_0$, and $\sigma = -\sigma_0 c/\alpha_0$ are renormalized with the unperturbed absorption coefficient α_0, and the counterpropagation geometry of the quantum modes is taken into account. Since the two modes have a counterpropagating geometry, the boundary conditions become $\widehat{a}(z=0) = \widehat{a}_0$ and $\widehat{b}(z=L) = \widehat{b}_L$, where L is the length of the medium. The solution for the output modes $\widehat{a}(z=L) = \widehat{a}_L$ and $\widehat{b}(z=0) = \widehat{b}_0$ can be written in the following form:

$$\widehat{a}_L = S_1(L)\widehat{a}_0 + S_2(L)\widehat{b}_L, \tag{11}$$

$$\widehat{b}_0 = S_2(L)\widehat{a}_0 + S_1(L)\widehat{b}_L, \tag{12}$$

where

$$S_1 = \left[\cos sL - i\frac{\chi}{s}\sin sL\right]^{-1}, \tag{13}$$

$$S_2 = iS_1\frac{\sigma}{s}\sin sL, \tag{14}$$

$$s = \sqrt{\chi^2 - \sigma^2}. \tag{15}$$

This result is in agreement with that obtained by treating the currently considered system as a classical one [19].

During the process, the commutation relations $\langle \widehat{a}_L \widehat{a}_L^\dagger \rangle - \langle \widehat{a}_L^\dagger \widehat{a}_L \rangle = \langle \widehat{b}_0 \widehat{b}_0^\dagger \rangle - \langle \widehat{b}_0^\dagger \widehat{b}_0 \rangle = |S_1|^2 + |S_2|^2 = 1$ for the photon modes are always preserved, as it should be.

An interesting property of the system is that the parameter s can be imaginary when $\chi^2 < \sigma^2$. This situation can happen in the range of the parameters

$$\tfrac{1}{3} < P < 1, \tag{16}$$

where we have introduced a notation for the dimensionless parameter,

$$P \equiv \frac{\Omega^2 g^2}{\Delta^2 \delta(\Delta k) c}. \tag{17}$$

This situation is analogous to the presence of the band gap in photonic crystal [12,13], as predicted in [20].

In the following, we will apply the above-mentioned dynamics to study the properties of propagation, such as the conversion, and squeezing transferring between the light fields, as well as entanglement generation under the different input parameters.

A. Field-mode swapping

We start with the discussion of how the amplitude of the quantum modes can be changed in our process. We can find the photon number in the modes $A_a = \langle \widehat{a}^\dagger \widehat{a} \rangle$ and $A_b = \langle \widehat{b}^\dagger \widehat{b} \rangle$ from Eqs. (11) and (12),

$$A_a^L = |S_1|^2 A_a^0 + |S_2|^2 A_b^L,$$
$$A_b^0 = |S_1|^2 A_b^L + |S_2|^2 A_a^0,$$

where it is assumed that initially the two modes $\widehat{a}$ and $\widehat{b}$ are not correlated, $\langle \widehat{a}_0^\dagger \widehat{b}_L \rangle = \langle \widehat{a}_0 \widehat{b}_L^\dagger \rangle = 0$. In the numerical calculation, we consider the case when the input state for the mode $\widehat{a}$ is in the coherent state $|\alpha\rangle$ and the second mode $\widehat{b}$ is in the vacuum state $|0\rangle$. In this case, the amplitudes of photon modes take a simple form,

$$A_a^L = |S_1|^2 |\alpha|^2,$$
$$A_b^0 = |S_2|^2 |\alpha|^2.$$

In Fig. 2(a) we present normalized quantum modes amplitudes as the function of a normalized length of the medium. When the function s is real (inequality (16) is not satisfied) there are oscillations between the modes Figs. 2(a) and 2(b). As the parameter P gets closer to the boundary of the condition Eq. (16) the oscillations become stronger Fig. 2(b). The maximum energy transfer from one mode to another is at the point $|s|L = \pi/2$. Theirs amplitudes at this point are $A_a^L = \frac{s^2}{\chi^2}|\alpha|^2$ and $A_b^0 = \frac{\sigma^2}{\chi^2}|\alpha|^2$. In Fig. 2(c) the case is presented when condition Eq. (16) is satisfied. The oscillatory behavior is changed to the exponential one and the total reflection of the forward wave into the backward one takes place. This situation is analogous to the presence of the bandgap in photonic crystals [20].

B. Squeezing transferring

We define the quadratures of the quantum modes,

$$X_{a,b} = [\widehat{a}_L(\widehat{b}_0) + \widehat{a}_L^\dagger(\widehat{b}_0^\dagger)]/2, \tag{18}$$

$$Y_{a,b} = [\widehat{a}_L(\widehat{b}_0) - \widehat{a}_L^\dagger(\widehat{b}_0^\dagger)]/2i, \tag{19}$$

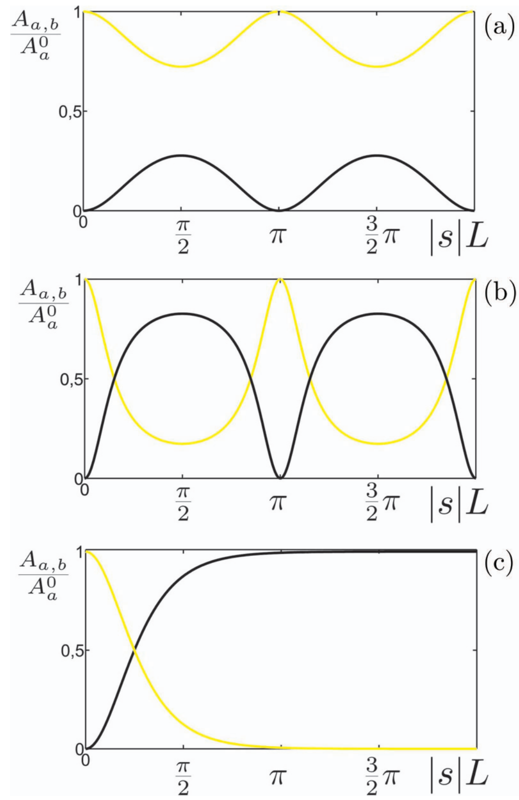

FIG. 2. Dynamics of the normalized over $A_0 = |\alpha|^2$ quantum field's amplitude as a function of the dimensionless parameter $|s|L$, where s is given in Eq. (15) and L is the interaction length. The red (light gray) line is the function A_a/A_a^0 and the black one is the function A_b/A_a^0. (a) $P = 10$, (b) $P = 1.1$, (c) $P = 0.4$.

and their fluctuations $\langle \Delta X_{a,b}\rangle^2 = \langle X_{a,b}^2\rangle - \langle X_{a,b}\rangle^2$ and $\langle \Delta Y_{a,b}\rangle^2 = \langle Y_{a,b}^2\rangle - \langle Y_{a,b}\rangle^2$.

If the fluctuation in one of the quadratures is less than 1/4, the field will be in a squeezed state. For the input coherent states in the considered system, the output is always in coherent states and there is no way to get a squeezed state,

$$\langle \Delta X_{a,b}\rangle^2 = \langle \Delta Y_{a,b}\rangle^2 = \tfrac{1}{4}\big(|S_1|^2 + |S_2|^2\big) = \tfrac{1}{4}.$$

The situation will be different in the case where one of the modes is initially in a squeezed state. As an example, we assume that mode $\widehat{a}$ is in the coherent state $|\alpha\rangle$ and mode $\widehat{b}$ is in the squeezed state $|\xi\rangle$ with squeezing parameter r. The expressions for the four quadratures fluctuations are

$$\langle \Delta X_{a,b}\rangle^2 = \tfrac{1}{4} + \tfrac{1}{4}\big\{2|S_{2,1}|^2 \sinh^2 r - \big[S_{2,1}^2 + (S_{2,1}^*)^2\big]\sinh r \cosh r\big\},$$
$$\langle \Delta Y_{a,b}\rangle^2 = \tfrac{1}{4} + \tfrac{1}{4}\big\{2|S_{2,1}|^2 \sinh^2 r + \big[S_{2,1}^2 + (S_{2,1}^*)^2\big]\sinh r \cosh r\big\}.$$

In this case, the amplitude of the squeezed mode can be sufficiently amplified due to the parametric energy transferring from the coherent mode. In Fig. 3(a) about $P = 10$ [see Eq. (17)], there is the effective squeezing transferring between two quadratures of the same mode. At the same time, in

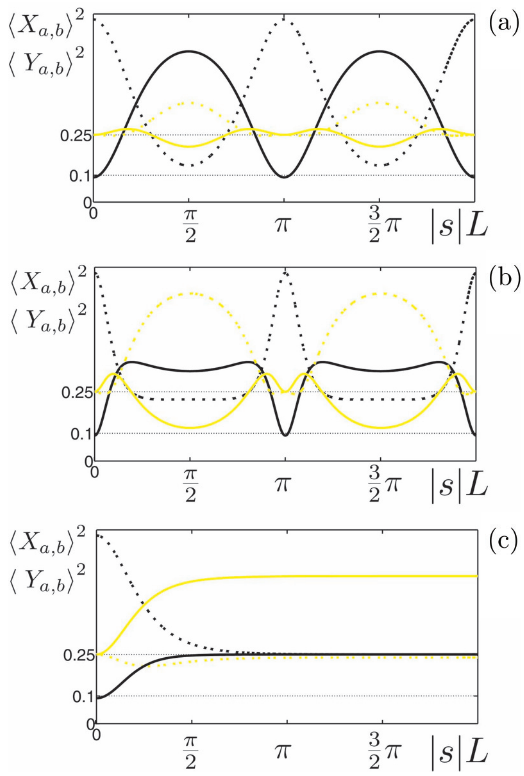

FIG. 3. Dynamics of the fields quadratures fluctuations as a function of the dimensionless parameter $|s|L$, where s is given in Eq. (15) and L is the interaction length. $\langle\Delta X_a\rangle^2$: red (light gray) solid line; $\langle\Delta X_b\rangle^2$: black solid line; $\langle\Delta Y_a\rangle^2$: red (light gray) dotted line; $\langle\Delta Y_b\rangle^2$: black dotted line. (a) $P = 10$, (b) $P = 1.1$, (c) $P = 0.4$.

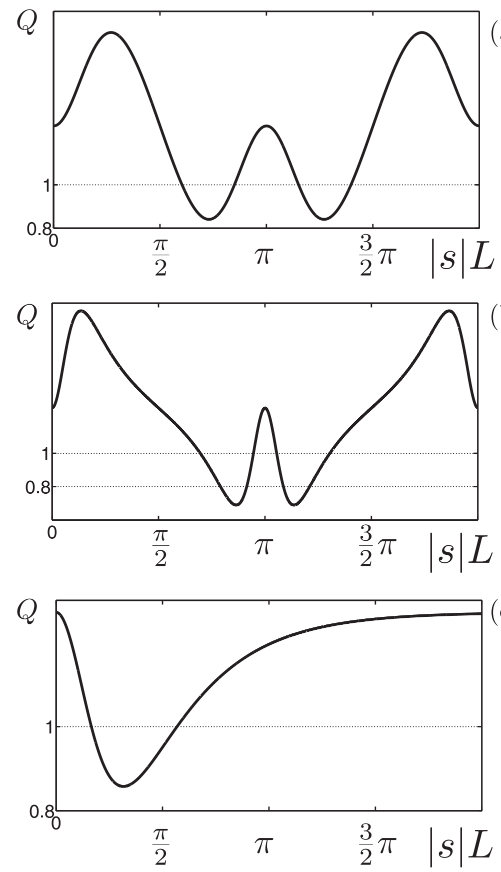

FIG. 4. The function Q that is an entanglement criteria as a function of dimensionless parameter $|s|L$, where s is given in Eq. (15) and L is the interaction length. (a) $P = 10$, (b) $P = 1.1$, (c) $P = 0.4$.

Fig. 2(a), we can see that at the point $L = \frac{\pi}{2}$, the mode, which is initially weak and squeezed, gets sufficiently amplified due to the parametric interaction with another quantum mode and preserves almost the initial level of the squeezing. Next we consider the case when the squeezing transfers from one mode to another [$P = 1.1$ as in Fig. 3(b)]. The quantum mode, which is initially a coherent state and has a large amplitude quantum mode around the point $L = \frac{\pi}{2}$, will become squeezed [see Fig. 3(b)] and its amplitude will still be sufficiently large [see Fig. 2(b)]. For the case when the parameters lie in the region where the band gap exists, $P = 0.4$ [see Eq. (17)], there will not be sufficient squeezing in any quadrature as in Fig. 3(c). The possibility of the squeezing transferring by a coherent process involving electromagnetically induced transparency in an atomic system was studied in [21], but principally it is impossible to obtain more than 25% of the initial squeezing in that way. Here we demonstrate more than 90% of the squeezing transferring between the modes. The amplitude of the squeezed mode and the amount of squeezing are easy to control by the intensity of the pump field.

C. Entanglement generation

In this section, we demonstrate the possibility of generating the entanglement between quantum modes. As the criteria of entanglement, we use the inequality $Q \equiv (\Delta u)^2 + (\Delta \upsilon)^2 < 1$ [22], where $u = X_a + X_b$ and $\upsilon = Y_a - Y_b$. The quadratures $X_{a,b}$ and $Y_{a,b}$ are given in Eqs. (18) and (19). When both modes are in the coherent state, there will be no possibility to generate entanglement, and function Q is simply larger than 1. When mode $\widehat{a}$ is in a coherent state $|\alpha\rangle$ and mode $\widehat{b}$ is in a squeezed state $|\xi\rangle$, the system will be able to generate an entanglement between the modes in a certain range of system parameters. For the considered system, we have

$$\begin{aligned} Q &= (\Delta u)^2 + (\Delta \upsilon)^2 \\ &= (1 + \sinh^2 r)\big(|S_1|^2 + |S_2|^2\big) \\ &\quad - \tfrac{1}{2}\sinh r \cosh r\big[S_1^2 + S_2^2 + (S_1^*)^2 + (S_2^*)^2\big]. \end{aligned}$$

Through numerical analysis, we find a few regimes where two quantum modes can be entangled.

In Fig. 4, we plot the function Q compared with the entanglement criteria [22]. We see that for $P = 10$ and $P = 1.1$, there are two dips around $|s|L = \pi + \pi l$ for $l = 0,1,2\ldots$, to have the two modes entangled; see Figs. 4(a) and 4(b). And when $P = 0.4$, we have only one dip where these modes are entangled; see Fig. 4(c). This parametrically induced beam splitter is capable of entangling two light fields like the usual beam splitter [23].

D. Example of possible implementation

In order to let the parameter P be close to or larger than unity, the proper requirements on the pump and the quantum fields frequencies, as well as on the two-photon detuning, should be satisfied. Equation (17) can be rewritten as $P = (\frac{\Omega g}{\Delta\delta})^2 \frac{\delta}{\Delta kc}$, where the first term is the second-order small parameter in Eq. (6), so the second factor $\delta/(\Delta kc)$ should be sufficiently large. It is possible to achieve such conditions by using two neighboring Zeeman sublevels of alkali vapors as the two ground states $|a\rangle$ and $|b\rangle$ in Fig. 1. For example, one can obtain an energy spacing about $\omega_{ab} = 50$ MHz between two neighboring Zeeman sublevels of the state $5^2S_{1/2}$ of rubidium vapor in a magnetic field, while the excited state can be $5^2P_{1/2}$, to which the pump and quantum electromagnetic with the different polarizations π and σ^+ undergo the transition (D1 line). The level scheme in Fig. 1 implies that

$$kc - \Delta_1 = k_0c + \omega_{ab} - \Delta_2,$$

from which one has the relation

$$\Delta kc = \omega_{ab} + \delta.$$

The frequency kc of the pump fields and the frequency k_0c of the quantum fields can become very close, since their respective detuning, Δ_1 and Δ_2 ($\Delta_1 = \Delta$ in Fig. 1), can be flexibly chosen to have a two-photon detuning $\delta = \Delta_1 - \Delta_2$ comparable with the energy gap ω_{ab}. Therefore, the parameter $P = (\frac{\Omega g}{\Delta\delta})^2 \frac{\delta}{\Delta kc}$ can be adjusted to the proper values considered in the current scheme. In the considered case, we can set two-photon detuning $\delta = -50.5$ MHz and obtain $\frac{\delta}{\Delta kc} \approx 100$.

V. CONCLUSION

In summary, we have studied the engineering of two quantum modes counterpropagating in a medium of a three-level atomic system also in the presence of a strong classical standing electromagnetic field. The interaction between the atomic system and the electromagnetic modes is via dispersive FWM, to have all fields well detuned from one-photon and two-photon resonances, so that the FWM process is purely dispersive and the atomic system itself will always be in the ground state $|a\rangle$. Under such condition, there will be no loss in the system in the presence of decays and Langevin noises. The derived effective Hamiltonian has a form that is similar to a beam-splitter Hamiltonian [5]. We demonstrate that there are possibilities for the parametric amplitude amplification of a squeezed state, as well as the squeezing transferring and the entanglement generation, with such type of interaction. Also we have found that in a certain range of the parameters such as in Eq. (16), the features similar to those in a photonic-crystal band gap can appear.

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