Effects of Losses and Phase Mismatch on Transient Processes in Optical Parametric Amplification through Three-wave Mixing of Ordinary and Backward Electromagnetic Waves

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Abstract— Three-wave mixing of ordinary electromagnetic waves and the waves with contra-directed phase and group velocities in the pulsed regimes is shown to be accompanied by unusual transient processes. We have investigated how attenuation and phase mismatch of the coupled waves in the metamaterials change the properties of parametric amplification and the threshold intensity of the pump wave which is of critical importance.

1. INTRODUCTION

Nanotechnology has made possible the engineering of the metamaterials (MMs) which enable extraordinary optical electromagnetic waves (EMWs) with the phase and group velocities directed against each other [1]. Such waves are commonly referred to as backward EMWs (BEMWs). The possibility to match phases of the coupled normal and backward waves in the deliberately engineered MMs was shown in ref [2] so that the waves with contra-directed energy fluxes would travel with one and the same or nearly equal phase velocities. The latter is the requirement of a paramount importance for coherent nonlinear optics (NLO) which presents a challenge to achieve. Three-wave mixing (TWM) of ordinary and BEMW was investigated in [3, 4] in the continuous-wave (CW) regime. Extraordinary dependence of the parametrical amplification and of the idler generated in the reflection direction on the pump field intensity was predicted. Such a property has analogs in stimulated scattering on the acoustic waves and in the quasi-phase matched TWM in the periodically modulated nonlinear crystals [5–7]. If intensities of the coupled field vary in time, the extraordinary transient processes emerge as was shown in [8] in the approximation of exact phase matching. As found in [8], the duration of such transient processes may significantly exceed the travel time of the pulse forefront through the metaslab. Attenuation of the coupled waves along the MM, which is inherent to plasmonic MM, and the difference in their phase velocities may have a significant effect on the properties and on the threshold parameters which discriminate different regimes in the time and space domains. In this paper, we present the results of more detailed studies of the effects of both absorption and phase mismatch on transient processes in the parametric interaction of the counterpropagating waves.

2. MATHEMATICAL MODEL

Consider the nonstationary problem of the TWM in a one-dimensional medium of length L with a quadratic nonlinearity. The electric field in each of the waves will be given by the expression $E_j(z,t) = A_j(z,t)\exp\{i(\omega_j t - k_j z)\}$ $j = 1, 2, 3$, where $A_j(z,t)$ is the amplitude; ω_j — are the wave frequencies which satisfy to the relation $\omega_3 = \omega_1 + \omega_2$, k_j — are the wave vectors. The corresponding system of equations, which accounts for the phase mismatch $\Delta k = k_3 - k_2 - k_1$ can be written as [8]

$$\left\{\begin{array}{l} \partial a_1/\partial z + (1/v_1)\,\partial a_1/\partial t = \pm\left(iKa_3a_2^*\exp(i\Delta kz) - (\alpha_1/2)\,a_1\right) \\ \partial a_2/\partial z + (1/v_2)\,\partial a_2/\partial t = \pm\left(iKa_3a_1^*\exp(i\Delta kz) - (\alpha_2/2)\,a_2\right) \\ \partial a_3/\partial z + (1/v_3)\,\partial a_3/\partial t = \pm\left(iKa_1a_2\exp(-i\Delta kz) - (\alpha_2/2)\,a_3\right) \end{array}\right. \tag{1}$$

Here, $a_j = \frac{1}{\sqrt{\omega_j}}\sqrt[4]{\frac{\varepsilon_j}{\mu_j}}A_j$ are the normalized amplitudes of the fields; v_j are group velocities; $K = \chi^{(2)}\frac{8\pi}{c}\sqrt[4]{\frac{\mu_1\mu_2\mu_3}{\varepsilon_1\varepsilon_2\varepsilon_3}}\sqrt{\omega_1\omega_2\omega_3}$ is the coupling factor; $\chi^{(2)}$ is a quadratic nonlinear susceptibility; α_j is the attenuation index of the medium at the frequency ω_j. The system (1) can be employed for investigation of TWM both in the case of counter-propagating and in the case of the co-propagating waves. When all the waves are normal, the right-hand side of all equations in the Eq. (1) has a negative sign. In the case of one of the coupled waves is BEMW, which travels with the phase velocity co-directed with other waves and, therefore, with contra-directed energy flux relative others, the right-hand side of the corresponding equation takes a positive sign. We assume that the pump and the signal wave with frequencies ω_3 and ω_2, accordingly, propagate in the positive direction of the z-axis, and enters the nonlinear medium at the point $z = 0$ (Fig. 1). The amplitude of the idler, which is supposed to be a contra-propagating BW, is taken equal to zero on the right boundary of the medium $a_1(z = L) = 0$ in all calculations.

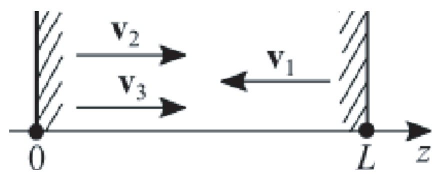


Figure 1: Directional pattern of group velocities.

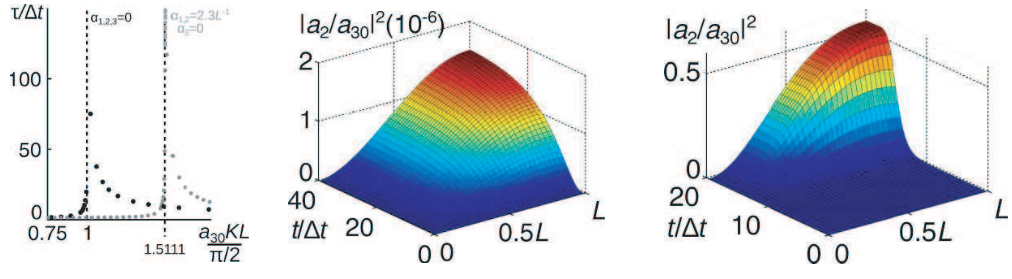
3. EFFECT OF LOSSES ON THE PROPERTIES OF THE TRANSIENT PROCESS

As noted, all MMs, which support BEMW, introduce significant losses. Hence, it becomes necessary to consider the effect of the wave attenuation as an important factor that may significantly influence the TWM transient processes. Obviously, phase mismatch also causes changes in the spatiotemporal properties of TWM. In order to account for these effects, parametric amplification was investigated for the setting as follows. The pump is assumed a CW with $a_3(z = 0) = a_{30}$. A semi-infinite signal pulse with the sharp forefront enters the metaslab at $z = 0$. Its shape is given by the equation $a_2(z = 0) = (a_{20}/2)[1 - \tanh(-t/t_f)]$, where $a_{20} = 10^{-4}a_{30}$ is the amplitude of the pulse at the metaslab entrance, $t_f = 0,05L/v_3$ is the width of the leading edge of the pulse. Generated counter-propagating idler is initially absent: $a_1(t = 0, z = L) = 0$. The problem was solved numerically by using Eq. (1).

First, consider the effect of attenuation for the case of precise phase matching $\Delta k = 0$ and compare the results with the case of the loss-free medium $\alpha_1 = \alpha_2 = \alpha_3 = 0$. In the latter case, the appearance of the process varies significantly depending on the magnitude of the parameter $a_{30}KL$. As shown in [8], if $a_{30}KL < \pi/2$, the signal at the exit $z = L$ is well described by the equation $a_{2L} = A(1 - e^{-(t-t_0)/\tau})$ where A is the signal magnitude at the exit in the stationary mode, whereas its temporal properties dramatically change at $a_{30}KL > \pi/2$. A duration of the transient process τ is defined as the time interval for which normalized amplitude of the signal at the exit from the medium changes from zero to the fraction of the stationary mode level given by $1 - e^{-1} \approx 0.63$. Corresponding dependence on the parameter $a_{30}KL$ is depicted in Fig. 2(a) with black dots. Here, $\Delta t = L/v_3$ is the travel period through the metaslab for the pump. It is seen from the graph that when $a_{30}KL < \pi/2$ the transient period increases with the increase of $a_{30}KL$ and decreases for $a_{30}KL > \pi/2$. Thus, $a_{30}KL = \pi/2$ — is an important characteristic value, the threshold which discriminates two BW TWM regimes. It was found that at $a_{30}KL < \pi/2$ a duration of the transient process is well described as $\tau \sim 1/\cos(a_{30}KL)$. The effect of losses can be predicted with the aid of the stationary solution to Eq. (1) in the approximation of neglected pump field depletion [3, 4]:

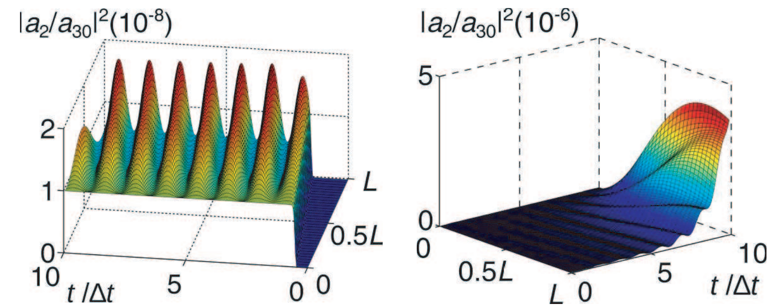
$$\frac{a_{2L}}{a_{20}} = \frac{\exp\left((\alpha_1 - \alpha_2)\,L/2\right)}{\cos(RL) + \sin(RL)\,(\alpha_1 + \alpha_2)\,/4R}, \quad R = \sqrt{K^2 - (\alpha_1 + \alpha_2)^2/16} \tag{2}$$

At $\alpha_1 = \alpha_2 = 0$, the denominator in expression (2) tends to zero at $RL = a_{30}KL = \pi/2$ which corresponds to the threshold described above. It is reasonable to anticipate that in the case of non-zero attenuation, the position of the peak can be estimated in a similar way. For example, in the case of $\alpha_1 = \alpha_2 = 2.3L^{-1}$, $\alpha_3 = 0$; the threshold is estimated at $(a_{30}KL)_{th} \approx 1.5111\pi/2$ which is in good agreement with the data obtained from the numerical solution to Eq. (1) (Fig. 2(a) gray points). It is seen from Fig. 2 that duration of the transient process reaches a maximum as the intensity of the pump tends to its characteristic threshold value. Both for a loss-free medium and a lossy metaslab, the signal temporal properties drastically change near the threshold value. For the given losses, it exhibits an asymptotic behavior $a_{2L} = A(1 - e^{-(t-t_0)/\tau})$ below the threshold for values $a_{30}KL < 1.5111\pi/2$ (Fig. 2(b)) and a jump-like dependence above the threshold for $a_{30}KL > 1.5111\pi/2$ (Fig. 2(c)). Thus, the threshold value $(a_{30}KL)_{th}$ increases with the increase of losses at the frequencies of the coupled waves. The offset of the threshold with an increase of extinction is explained by the fact that a greater amplification is required to compensate losses. Note, that the photon conversion rate exceeds 50% at the time instants exceeding 20 travel time intervals.



(a) (b) (c)

Figure 2: (a) Dependence of the duration of the transient process on the parameter $a_{30}KL$ for the cases of $\alpha_1 = \alpha_2 = \alpha_3 = 0$ (the left pick, black dots) and for the case of $\alpha_1 = \alpha_2 = 2.3L^{-1}$ (the right pick, gray dots). (b) and (c) Spatiotemporal dependence of the signal in the metaslab for $\alpha_1 = \alpha_2 = 0.1L^{-1}$, $\alpha_3 = 0$. (b) $a_{30}KL = 0.955\pi/2$, (c) $a_{30}KL = 1.3\pi/2$.



(a) (b)

Figure 3: Spatiotemporal variations of intensity of the signal wave in the metaslab at $\Delta k = 10L^{-1}$ and the pump intensity below and above the threshold value. (a) $a_{30}KL = 0.955\pi/2$. (b) $a_{30}KL = 1.3\pi/2$.

4. EFFECT OF PHASE MISMATCH ON THE PROPERTIES OF THE TRANSIENT PROCESSES

The effect of phase mismatch on the transient processes was investigated through numerical solutions to the Eq. (1) for the loss-free case with the aid of the same model as the one used above. It is found that a change in the magnitude of the phase mismatch Δk does not cause a change of the magnitude of threshold $(a_{30}KL)_{th}$ but it does cause changes in the transient processes. It is found that such an effect is distinctly different for the pump intensities below the threshold (Fig. 3(a)) and above it (Fig. 3(b)). As seen from the spatiotemporal dependencies presented in Fig. 3, the output signal splits into series of pulses lasting even after 10 travel times in the case of lower pump intensity (Fig. 3(a)).

Calculations show that for the given parameters and the time interval exceeding $400\Delta t$, the maximum of the signal output stabilizes at the level of about 7% above its input value. With the decrease of phase mismatch, it increases as $|a_{2L}/a_{20}|^2 = 1.6$ for $\Delta k = 3L^{-1}$, $|a_{2L}/a_{20}|^2 = 2.9$ for $\Delta k = 2L^{-1}$, and $|a_{2L}/a_{20}|^2 = 10.3$ for $\Delta k = 1L^{-1}$. Series of pulses become less pronounced in the case of Fig. 3(b) due to the huge enhancement in the amplification rate. As noted the dependence of amplification on the phase mismatch ΔkL diminishes significantly as the pump intensity exceeds its threshold value.

5. CONCLUSIONS

The effects of losses and phase mismatch on the optical parametric amplification and on the transient processes in the pulse regime are investigated by the numerical simulation methods in the case of the idler is a backward counter-propagating wave. The pump is supposed a continuous wave, whereas the input weak signal is of the step-wise shape with a sharp forefront. Both waves are supposed normal. Unlike a common case of co-directed energy fluxes, the key feature inherent to the phase matched TWM of the waves with contra-directed group velocities is that amplification of the signal grows infinitely in the approximation of a given pump field as its amplitude approaches a certain threshold value. With account for depletion of the pump wave and losses, we show that the maximum conversion efficiency and amplification of the signal gradually increase as the pump intensity approaches the threshold, which increases with the increase in losses and then turns to the parametric generation for the pump intensities above the threshold. The shape of the output amplified signal is shown to significantly differ from that at the metaslab entrance. Duration of the transient period on the forefront of the output signal pulse also varies significantly as the pump intensity approach and exceeds the threshold value. The effect of the phase mismatch appears more complex. Along with the decrease of the maximum achievable conversion rate, phase mismatch causes spatiotemporal oscillations of the signal inside the medium. The oscillations in the output signal vanish after a certain period. Corresponding dependencies on the phase mismatch magnitude are investigated. With an arbitrary choice of the length of the medium, a steady-state gain mode establishes which indicates the presence of the accumulation effect in spite of the existing phase mismatch.

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