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SUPERCONDUCTIVITY

Effect of the Intersite Coulomb Interaction on Chiral Superconductivity at the Noncollinear Spin Ordering

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Abstract—We investigate the effect of the intersite Coulomb interaction in a planar system with the triangular lattice on the structure of chiral order parameter $\Delta(\mathbf{p})$ in the phase of coexisting superconductivity and noncollinear 120° magnetic ordering. It has been established that the Coulomb correlations in this phase initiate the state where the quasi-momentum dependence $\Delta(\mathbf{p})$ can be presented as a superposition of the chiral invariants corresponding to the $d_{x^2-y^2} + id_{xy}$ and $p_x + ip_y$ symmetry types. It is demonstrated that the inclusion of the Coulomb interaction shifts the $\Delta(\mathbf{p})$ nodal point positions and, thereby, changes the conditions for a quantum topological transition.

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1. INTRODUCTION

The interest in superconductivity with the chiral symmetry type of the order parameter is due to the formation of a topologically nontrivial phase and, consequently, edge states [1]. After the discovery of superconductivity in the Na_xCoO_2 compound, it was suggested that the chiral $d_{x^2-y^2} + id_{xy}$ superconducting phase is implemented in quasi-two-dimensional systems with the triangular lattice [2]. In such a superconducting phase, a quantum topological concentration transition can occur [3, 4]. At the point of this transition, the elementary excitation spectrum becomes gapless. These circumstances explain close attention to the systems with the chiral symmetry of the superconducting order parameter.

Recently, it has been established that the long-range magnetic order, along with the spin-orbit interaction, can lead to implementation of the Majorana edge states in a topological superconductor with singlet pairing [5, 6]. This effect was demonstrated on the triangular lattice for the noncollinear magnetic order with the stripe structure in the chiral $d_{x^2-y^2} + id_{xy}$ superconducting phase [5]. Further investigations showed [7] that within the $t-J$ model, the chiral symmetry of the superconducting order parameter is impossible at the stripe magnetic structure. At the same time, the chiral superconductivity still can be implemented for the magnetic structure corresponding to the noncollinear 120° magnetic ordering. This result indicates that the Majorana edge states should be sought in a homogeneous phase of the coexisting chiral superconductivity and 120° spin ordering. Later on, the conditions for implementation of the Majorana modes were established for such a phase using the quadratic Hamiltonian [8].

The absence of exact solutions for the Heisenberg model on the triangular lattice caused the uncertainty in the problem on implementation of a certain magnetic structure. It has been considered for a long time that in the aforementioned model the spin liquid state is implemented [9]. At present, however, most researchers prefer the scenario of implementing the 120° long-range magnetic ordering with three sublattices. This state is the most favorable from the classical point of view at $J_2 < J_1/8$, where J_1 and J_2 are the parameters of exchange between the nearest and next-to-nearest neighbors. In the Hubbard model on the triangular lattice, using the mean field approximation [10] and slave boson representation [11], the phase diagrams were built, which demonstrate the existence of different states, including noncollinear and noncoplanar, with the spin and charge ordering. In these approaches, the ground state with the 120° spin ordering is retained upon near-half-filling doping. The most interesting regime of electron doping at the positive integral of hoppings between the nearest neighbors is considered to qualitatively describe the Na_xCoO_2 electronic structure. Weber et al. [12] showed the implementation of the phase of coexisting chiral superconductivity and 120° magnetic ordering near $n = 1.1$ using the variational Monte Carlo method. In this case, the magnetic phase remained stable up to a concentration of $n = 1.4$. In the framework of this approach, the magnetization at half-filling was slightly decreased (to $M \approx 0.4$). The result from the spin wave theory is $M \approx 0.25$ [13].

As is known, the intersite Coulomb interaction in the t–J model suppresses the d-type superconductivity. Therefore, the superconducting phase is frequently described taking into account additional contributions to the Cooper instability mechanism [14]. In systems with strong intersite repulsion for the first coordination sphere, the superconducting phase can be implemented if there is the pairing interaction in the second coordination sphere. In addition, the superconducting state can be retained at the strong Coulomb repulsion when the pairing and Coulomb interactions have different symmetrical properties because of complexity of a unit cell [15]. This situation is observed, e.g., in cuprate superconductors, and manifests itself in the theoretical description with the use of the spin-fermion model [15] obtained from the Emery model in the strong correlation regime.

In this work, we investigate the effect of the intersite Coulomb correlations on the formation of the phase of coexisting chiral superconductivity and 120° magnetic order with regard to the exchange interaction within two coordination spheres. Along with the ordinary suppression of the superconducting state, the intersite repulsion at the noncollinear magnetic ordering leads to the nontrivial effect of modifying the quasi-momentum dependence of the superconducting order parameter. As a result, the solution for the order parameter $\Delta(\mathbf{p})$ is determined not only by the chiral $d_{x^2-y^2} + id_{xy}$ invariants, as in the case of the phase without magnetic ordering, but also by the occurring invariants with the $p_x + ip_y$ symmetry type. Admixing of the additional invariants is caused by the induced spin-triplet pairings in the magnetically ordered state [16, 17]. These pairings are characterized by the odd p-type classification with respect to the number of irreducible representation (orbital moment value) if the wave function is even by frequency. In this case, new components have the chiral $p_x + ip_y$ symmetry type due to the triangular lattice symmetry. It should be noted that the triplet pairings are caused by the intersite Coulomb interaction in the magnetically ordered state, since the exchange interaction in the t–J model does not have a triplet superconductivity channel.

The induced contributions of the triplet pairings change the positions of nodal points of the superconducting parameter inside the Brillouin zone. This changes the conditions for forming the gapless excitations in the coexistence phase. As a result, the aforementioned effects will be reflected in the description of topological transitions in the phase of coexisting chiral superconductivity and noncollinear magnetism.

2. HAMILTONIAN OF THE STRONGLY CORRELATED FERMIONS ON THE TRIANGULAR LATTICE

We study the effect of the intersite Coulomb repulsion on the phase of coexisting chiral superconductivity and 120° spin ordering in the framework of the t–J_1–J_2–V model on the triangular lattice from [3, 4] with the description of the Cooper instability with the $d_{x^2-y^2} + id_{xy}$ symmetry type of the order parameter in the intercalated sodium cobaltates. As was shown in [3], there are several factors that require the pairing exchange interaction in the second coordination sphere to be necessarily taken into account. First, the inclusion of such pairings in the consideration of the superconductivity with the $d_{x^2-y^2} + id_{xy}$ symmetry type leads to the occurrence of nodal points in the Brillouin zone, for which the superconducting order parameter turns to zero. This circumstance made it possible to dissolve the contradiction related to the fact that the theoretical calculations for the triangular lattice predicted the $d_{x^2-y^2} + id_{xy}$ Cooper instability channel to be the most favorable, while most experiments pointed out the gapless superconductivity implementation in sodium cobaltates. Second, it is well known that the intersite Coulomb interaction in systems with the strong correlations prevents, to a great extent, the d-type superconductivity. However, the largest contribution is made by the Coulomb interaction between the nearest neighbors, since the correlations between the next-to-nearest neighbors are often suppressed by screening effects. In view of this, it is assumed that when the intersite Coulomb correlations are taken into account, the dominant contribution to the Cooper instability can be made by the pairing interaction between the next-to-nearest neighbors.

We limit the consideration to the upper Hubbard subband with the subspace consisting of the singly $|\sigma\rangle$ and doubly $|2\rangle$ occupied electronic states, as was proposed to qualitatively describe sodium cobaltates. The Hamiltonian of the t–J_1–J_2–V model in the atomic representation is

$$H = \sum_{f\sigma} (\varepsilon - \mu) X_f^{\sigma\sigma} + \sum_f (2\varepsilon + U - 2\mu) X_f^{22} + \sum_{fm\sigma} t_{fm} X_f^{2\bar{\sigma}} X_m^{\bar{\sigma}2} + \sum_{fm} J_{fm} (X_f^{\uparrow\downarrow} X_m^{\downarrow\uparrow} - X_f^{\uparrow\uparrow} X_m^{\downarrow\downarrow}) + \frac{V}{2} \sum_{f\delta} \hat{n}_f \hat{n}_{f+\delta}, \tag{1}$$

where ε is the seed electron energy, μ is the chemical potential, U is the intra-atomic Coulomb repulsion, t_{fm} is the intensity of electron hoppings, and J_{fm} is the exchange interaction parameter. The Hubbard operators are expressed via one-site electronic states as $X^{nm} = |n\rangle\langle m|$, where $n, m = \uparrow, \downarrow, 2$. The operation of the

Hubbard operators on the basis of states is determined by the formula $X^{nm}|p\rangle = \delta_{mp}|n\rangle$. The last term of the Hamiltonian describes the Coulomb interaction between electrons on the nearest sites with the parameter V and $\hat{n}_f = X_f^{\uparrow\uparrow} + X_f^{\downarrow\downarrow} + 2X_f^{22}$ is the operator of the number of electrons on the site.

3. GREEN'S FUNCTIONS FOR THE PHASE OF COEXISTING SUPERCONDUCTIVITY AND NONCOLLINEAR SPIN ORDERING

To solve the formulated problem, we use a method of the diagram technique for Hubbard operators [18] and determine Green's functions in the Matsubara representation

$$D_{\alpha,\beta}(f\tau; f'\tau') = -\langle T_\tau \tilde{X}_f^{\alpha}(\tau)\tilde{X}_{f'}^{-\beta}(\tau')\rangle, \qquad (2)$$

where α and β are the indices designating a pair of one-site states. At the noncollinear magnetic ordering with the average spin value on the site $\langle \mathbf{S}_f\rangle = M(\cos(\mathbf{QR}_f), -\sin(\mathbf{QR}_f), 0)$, the Fourier transform for Green's functions is written in the form

$$D_{\alpha,\beta}(f\tau; f'\tau') = \frac{T}{N}\sum_{\omega_n}\sum_{\mathbf{p}_1\mathbf{p}_2}\exp[-i\omega_n(\tau-\tau') + i\mathbf{p}_1\mathbf{R}_f - i\mathbf{p}_2\mathbf{R}_{f'}]\tilde{D}_{\alpha,\beta}(\mathbf{p}_1,\mathbf{p}_2; i\omega_n). \qquad (3)$$

Solving directly the system of equations for Green's functions, we establish the relation between quasi-momenta $\mathbf{p}_1$ and $\mathbf{p}_2$:

$$\begin{gathered}\tilde{D}_{\sigma2,\sigma2}(\mathbf{p}_1,\mathbf{p}_2;i\omega_n) = \delta(\mathbf{p}_1-\mathbf{p}_2)D_{\sigma2,\sigma2}(\mathbf{p}_1,i\omega_n),\\ \tilde{D}_{\sigma2,\bar{\sigma}2}(\mathbf{p}_1,\mathbf{p}_2;i\omega_n) = \delta(\mathbf{p}_1+\eta_\sigma\mathbf{Q}-\mathbf{p}_2)\\ \times D_{\sigma2,\bar{\sigma}2}(\mathbf{p}_1,\mathbf{p}_2+\eta_\sigma\mathbf{Q};i\omega_n).\end{gathered} \qquad (4)$$

It can be seen that for Green's functions with the invariable projection of the fermion spin moment the connection between the quasi-momenta is determined by the conservation law. For Green's functions describing the spin flip processes, the quasi-momenta are related by vector $\mathbf{Q}$ of the magnetic structure for the fermion with $\sigma = \uparrow$ and $-\mathbf{Q}$ for the fermion with $\sigma = \downarrow$.

For the sake of brevity, we write the matrix Green's function in the block form

$$\hat{D} = \begin{pmatrix} \hat{D}_{1\uparrow}(\mathbf{p},i\omega_n) & \hat{D}_{2\uparrow}(\mathbf{p},\mathbf{p}-\mathbf{Q};i\omega_n)\\ \hat{D}_{2\downarrow}(\mathbf{p}-\mathbf{Q},\mathbf{p};i\omega_n) & \hat{D}_{1\downarrow}(\mathbf{p}-\mathbf{Q},i\omega_n)\end{pmatrix}, \qquad (5)$$

where

$$\begin{gathered}\hat{D}_{1\sigma}(\mathbf{p},i\omega_n) = \begin{pmatrix} D_{\bar{\sigma}2,\bar{\sigma}2}(\mathbf{p},i\omega_n) & D_{\bar{\sigma}2,2\sigma}(\mathbf{p},i\omega_n)\\ D_{2\sigma,\bar{\sigma}2}(\mathbf{p},i\omega_n) & D_{2\sigma,2\sigma}(\mathbf{p},i\omega_n)\end{pmatrix},\\ \hat{D}_{2\sigma}(\mathbf{p}_1,\mathbf{p}_2;i\omega_n)\\ = \begin{pmatrix} D_{\bar{\sigma}2,\sigma2}(\mathbf{p}_1,\mathbf{p}_2;i\omega_n) & D_{\bar{\sigma}2,2\bar{\sigma}}(\mathbf{p}_1,\mathbf{p}_2;i\omega_n)\\ D_{2\sigma,\sigma2}(\mathbf{p}_1,\mathbf{p}_2;i\omega_n) & D_{2\sigma,2\bar{\sigma}}(\mathbf{p}_1,\mathbf{p}_2;i\omega_n)\end{pmatrix}.\end{gathered} \qquad (6)$$

As is known, the matrix Green's function can be presented as a product $\hat{D} = \hat{G}\cdot\hat{P}$, where $\hat{P}$ is the force operator matrix, which is written in the loopless approximation

$$\hat{P} = \begin{pmatrix} f_{2\uparrow} & 0 & M & 0\\ 0 & f_{2\downarrow} & 0 & M\\ M & 0 & f_{2\downarrow} & 0\\ 0 & M & 0 & f_{2\uparrow}\end{pmatrix}, \qquad (7)$$

where $f_{2\sigma} = \langle X_f^{\bar{\sigma}\bar{\sigma}}\rangle + \langle X_f^{22}\rangle$. Since in the investigated magnetic phase we have $\langle S_f^z\rangle = 0$, we have $f_{2\sigma} = n/2$, where $n = \langle \hat{n}_f\rangle$. At the transition to the representation for the force operator, we took into account that at the noncollinear spin ordering, the average values of the nondiagonal operators are nonzero: $\langle X_f^{\uparrow\downarrow}\rangle = M\exp(-i\mathbf{QR}_f)$ and $\langle X_f^{\downarrow\uparrow}\rangle = M\exp(i\mathbf{QR}_f)$.

The Dyson–Gorkov equation for function $\hat{G}$ has the standard form

$$\hat{G} = \hat{G}^{(0)} + \hat{G}^{(0)}\hat{\Sigma}\hat{G}, \qquad (8)$$

where $\hat{G}^{(0)}$ is the matrix Green's function for the noncollinear magnetic phase (the diagrams for these functions were presented in [7]). The mass operator $\hat{\Sigma}$ takes into account only the anomalous components responsible for the Cooper instability in the one-loop approximation

$$\Delta_1(\mathbf{p}) = \frac{T}{N}\sum_{\omega_n\mathbf{q}}[J_{\mathbf{p}-\mathbf{q}}G_{2\downarrow,\uparrow2}(\mathbf{q},i\omega_n) - (J_{\mathbf{p}-\mathbf{q}} - V_{\mathbf{p}-\mathbf{q}})G_{2\uparrow,\downarrow2}(\mathbf{q},i\omega_n)], \qquad (9)$$

$$\Delta_2(\mathbf{p}) = \frac{T}{N}\sum_{\omega_n\mathbf{q}}[J_{\mathbf{p}-\mathbf{q}}G_{2\uparrow,\downarrow2}(\mathbf{q},i\omega_n) - (J_{\mathbf{p}-\mathbf{q}} - V_{\mathbf{p}-\mathbf{q}})G_{2\downarrow,\uparrow2}(\mathbf{q},i\omega_n)]. \qquad (10)$$

It follows from the general symmetry relations for the Green's function that $\Delta_2(\mathbf{p}) = -\Delta_1(-\mathbf{p})$. Then, $\Delta_1(\mathbf{p}) \equiv \Delta(\mathbf{p})$. Taking this into account, the solution of Eq. (8) is presented in the form

$$\hat{G}^{-1} = \begin{pmatrix} i\omega_n - \xi_{\mathbf{p}} & -\Delta^*(\mathbf{p}) & -R_{\mathbf{p}-\mathbf{Q}} & 0\\ -\Delta(\mathbf{p}) & i\omega_n + \xi_{\mathbf{p}} & 0 & R_{\mathbf{p}-\mathbf{Q}}\\ -R_{\mathbf{p}} & 0 & i\omega_n - \xi_{\mathbf{p}-\mathbf{Q}} & \Delta^*(-\mathbf{p}+\mathbf{Q})\\ 0 & R_{\mathbf{p}} & \Delta(-\mathbf{p}+\mathbf{Q}) & i\omega_n + \xi_{\mathbf{p}-\mathbf{Q}}\end{pmatrix}, \qquad (11)$$

where $\xi_{\mathbf{p}} = \varepsilon + U - \mu + J_0(1 - n/2) + V_0 n + nt_{\mathbf{p}}/2$, $J_0 = 6J_1 + 6J_2$, $V_0 = 6V$, and $t_{\mathbf{p}}$ is the Fourier image of the hopping integral. We introduced the parameters determining the self-consistent field $R_{\mathbf{p}} = M(t_{\mathbf{p}} - J_{\mathbf{Q}})$ and $R_{\mathbf{p}-\mathbf{Q}} = M(t_{\mathbf{p}-\mathbf{Q}} - J_{\mathbf{Q}})$, where $J_{\mathbf{Q}} = -3J_1 + 6J_2$ is the

Fourier image of the exchange integral for the wave vector $\mathbf{Q} = 2\pi/a(1/\sqrt{3},\ 1/3)$ of the 120° magnetic structure. The occurrence of the terms proportional to the hopping integrals in the effective exchange field is caused by the account for the kinematic interaction in the noncollinear magnetic phase [7].

4. EXCITATION SPECTRUM AND CHIRAL ORDER PARAMETER OF THE COEXISTENCE PHASE

The elementary excitation spectrum in the phase of coexisting superconductivity and noncollinear magnetic order is determined from the expression

$$E_{\lambda\mathbf{p}} = \sqrt{\frac{1}{2}(\xi_{\mathbf{p}}^2 + \xi_{\mathbf{p-Q}}^2 + |\Delta(\mathbf{p})|^2 + |\Delta(-\mathbf{p}+\mathbf{Q})|^2) + R_{\mathbf{p}}R_{\mathbf{p-Q}} + (-1)^{\lambda}\nu_{\mathbf{p}}^2}, \quad (12)$$

where

$$\nu_{\mathbf{p}}^2 = \Big\{\frac{1}{4}(\xi_{\mathbf{p}}^2 - \xi_{\mathbf{p-Q}}^2 + |\Delta(\mathbf{p})|^2 - |\Delta(-\mathbf{p}+\mathbf{Q})|^2)^2 + R_{\mathbf{p}}R_{\mathbf{p-Q}}[(\xi_{\mathbf{p}} + \xi_{\mathbf{p-Q}})^2 + |\Delta(\mathbf{p}) + \Delta(-\mathbf{p}+\mathbf{Q})|^2]\Big\}^{1/2}. \quad (13)$$

It should be noted that this spectrum has the property $E_{\lambda\mathbf{p}} = E_{\lambda,\,-\mathbf{p}+\mathbf{Q}}$.

The analytical expressions for Green's functions are determined from reciprocal matrix (11). Note that to find the anomalous function $G_{2\downarrow,\uparrow 2}(\mathbf{p},\ i\omega_n)$ from expression (9) for the superconducting order parameter, we can use the above formulas by making the replacement $\mathbf{p} \to \mathbf{p} + \mathbf{Q}$ in (11). Summating over the Matsubara frequencies in (9), we obtain the integral self-consistency equation, the solutions of which determine the quasi-momentum dependence of the superconducting order parameter

$$\Delta(\mathbf{p}) = \frac{1}{N}\sum_{\mathbf{q}}\Phi(\mathbf{p},\mathbf{q})\Delta(\mathbf{q}), \quad (14)$$

where the equation core is determined as

$$\Phi(\mathbf{p},\mathbf{q}) = \sum_{\lambda=1}^{2}\Big\{\Big[(J_{\mathbf{p+q}} + J_{\mathbf{p-q}} - V_{\mathbf{p-q}})(E_{\lambda\mathbf{q}}^2 - \xi_{\mathbf{q-Q}}^2 - |\Delta(-\mathbf{q}+\mathbf{Q})|^2) + (J_{\mathbf{p+q-Q}} + J_{\mathbf{p-q+Q}} - V_{\mathbf{p+q-Q}}) \times R_{\mathbf{q}}R_{\mathbf{q-Q}}\Big]\frac{(-1)^{\lambda}\tanh(E_{\lambda\mathbf{q}}/2T)}{2E_{\lambda\mathbf{q}}(E_{2\mathbf{q}}^2 - E_{1\mathbf{q}}^2)}\Big\}.$$

As we showed previously [7], in the approximation linearized by the parameter $\Delta(\mathbf{p})$ valid near the critical temperature, the solution of Eq. (14) at the 120° magnetic ordering corresponds to the linear superposition of the $d_{x^2-y^2} + id_{xy}$ and $p_x + ip_y$ invariants

$$\Delta(\mathbf{p}) = 2\Delta_{21}\phi_{21}(\mathbf{p}) + 2\Delta_{22}\phi_{22}(\mathbf{p}) + 2\Delta_{11}\phi_{11}(\mathbf{p}), \quad (15)$$

where the chiral invariants have the form

$$\phi_{21}(\mathbf{p}) = \cos(p_y) - \cos(\sqrt{3}p_x/2)\cos(p_y/2) + i\sqrt{3}\sin(\sqrt{3}p_x/2)\sin(p_y/2), \quad (16)$$

$$\phi_{22}(\mathbf{p}) = \cos(\sqrt{3}p_x) - \cos(\sqrt{3}p_x/2)\cos(3p_y/2) - i\sqrt{3}\sin(\sqrt{3}p_x/2)\sin(3p_y/2), \quad (17)$$

and

$$\phi_{11}(\mathbf{p}) = \sin(p_y) + \cos(\sqrt{3}p_x/2)\sin(p_y/2) + i\sqrt{3}\sin(\sqrt{3}p_x/2)\cos(p_y/2). \quad (18)$$

The functions $\phi_{21}(\mathbf{p})$ and $\phi_{22}(\mathbf{p})$ determine the chiral $d_{x^2-y^2} + id_{xy}$ invariants corresponding to the pairing interactions between the nearest and next-to-nearest neighbors, respectively, and $\phi_{11}(\mathbf{p})$ is the chiral $p_x + ip_y$ invariant for the first coordination sphere.

Due to the splitting of the core $\Phi(\mathbf{p},\ \mathbf{q})$, integral equation (14) is reduced to the system of nonlinear algebraic equations that allow the temperature dependences of anomalous amplitudes of the superconducting order parameter $\Delta(\mathbf{p})$ to be established:

$$\begin{aligned}(1 - a_{21})\Delta_{21} - a_{22}\Delta_{22} - a_{11}\Delta_{11} &= 0,\\ -b_{21}\Delta_{21} + (1 - b_{22})\Delta_{22} - b_{11}\Delta_{11} &= 0,\\ c_{21}\Delta_{21} + c_{22}\Delta_{22} + (1 + c_{11})\Delta_{11} &= 0.\end{aligned} \quad (19)$$

The coefficients of this system of equations depend on the anomalous amplitudes and are determined as

$$a_{ij} = \Big(J_1 - \frac{V}{2}\Big)\frac{1}{N}\sum_{\mathbf{q}\lambda}\frac{(-1)^{\lambda}\phi_{ij}(\mathbf{q})\tanh(E_{\lambda\mathbf{q}}/2T)}{E_{\lambda\mathbf{q}}\nu_{\mathbf{q}}^2} \times\{\cos(q_y)[E_{\lambda\mathbf{q}}^2 - \xi_{\mathbf{q-Q}}^2 - |\Delta(-\mathbf{q}+\mathbf{Q})|^2] + \cos(q_y - Q_y)R_{\mathbf{q}}R_{\mathbf{q-Q}}\}, \quad (20)$$

$$b_{ij} = J_2\frac{1}{N}\sum_{\mathbf{q}\lambda}\frac{(-1)^{\lambda}\phi_{ij}(\mathbf{q})\tanh(E_{\lambda\mathbf{q}}/2T)}{E_{\lambda\mathbf{q}}\nu_{\mathbf{q}}^2} \times\{\cos(\sqrt{3}q_x)[E_{\lambda\mathbf{q}}^2 - \xi_{\mathbf{q-Q}}^2 - |\Delta(-\mathbf{q}+\mathbf{Q})|^2] + \cos(\sqrt{3}(q_x - Q_x))R_{\mathbf{q}}R_{\mathbf{q-Q}}\}, \quad (21)$$

and

$$c_{ij} = \frac{V}{2}\frac{1}{N}\sum_{\mathbf{q}\lambda}\frac{(-1)^{\lambda}\phi_{ij}(\mathbf{q})\tanh(E_{\lambda\mathbf{q}}/2T)}{E_{\lambda\mathbf{q}}\nu_{\mathbf{q}}^2} \times\{\sin(q_y)[E_{\lambda\mathbf{q}}^2 - \xi_{\mathbf{q-Q}}^2 - |\Delta(-\mathbf{q}+\mathbf{Q})|^2] - \sin(q_y - Q_y)R_{\mathbf{q}}R_{\mathbf{q-Q}}\}. \quad (22)$$

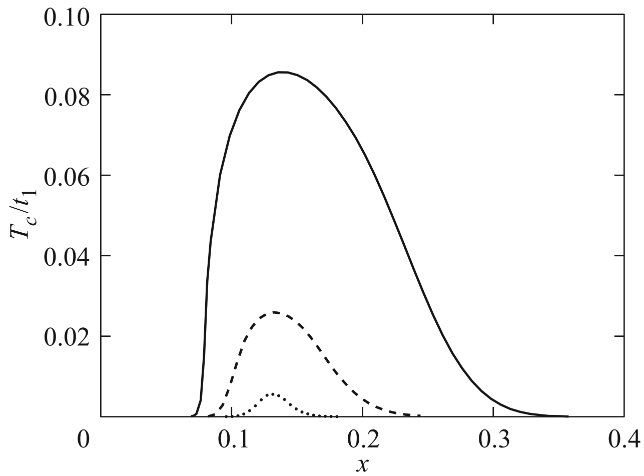

Fig. 1. Dependences of the temperature of the transition to the phase of coexisting chiral superconductivity and 120° spin ordering on the concentration $n = 1 + x$ at different parameters V of the intersite Coulomb repulsion: $V = 0.5t_1$ (solid line), $V = 0.8t_1$ (dashed line), and $V = 2J_1 = t_1$ (dotted line).

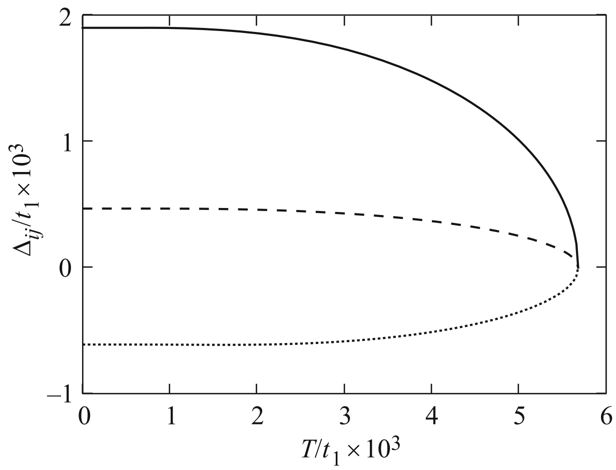

Fig. 2. Temperature dependences of the amplitudes Δ_{22} (solid line), Δ_{21} (dashed line), and Δ_{11} (dotted line) of the chiral superconducting order parameter at $V = 0.96t_1$ and $n = 1.12$.

5. RESULTS OF THE NUMERICAL CALCULATIONS

Figure 1 shows concentration dependences of the temperature corresponding to the onset of superconductivity with the chiral symmetry of the order parameter at the 120° magnetic ordering for different intersite Coloumb interaction parameters V. It is assumed that the temperature of the onset of magnetic order is much higher than the critical temperature of superconductivity. The magnetization value used is $M = 1 - n/2$. The parameters $J_1 = 0.5t_1$ and $J_2 = 0.06t_1$ are the constants of exchange between the nearest and next-to-nearest neighbors and t_1 is the parameter of hopping between the nearest neighbors ($t_1 \sim 0.1$ eV). To simplify the analysis, the effects of long-range hoppings are ignored. It can be seen that the increase in the parameter of intersite repulsion leads to the expected superconductivity suppression. At $V = 2J_1$ and larger, the Cooper instability is induced by the pairing interaction in the second coordination sphere. Since $J_2 \ll J_1$, with the intersite Coulomb correlations taken into account the transition to the coexistence phase occurs at fairly low temperatures (several kelvins).

As we showed above, the effect of intersite Coulomb correlations in the phase of coexisting superconductivity and 120° spin ordering is not only reduced to renormalization of the anomalous amplitude Δ_{21}, but also manifests itself in the fact that the solution for the superconducting order parameter is determined by superposition of the chiral $d_{x^2-y^2} + id_{xy}$ and $p_x + ip_y$ invariants. The intensity of admixing the chiral $p_x + ip_y$ invariant is determined by the amplitude Δ_{11}. The occurrence of triplet pairings in the seed singlet superconducting phase at the antiferromagnetic ordering was discussed previously [16, 17]. These pairings are determined as dynamically induced by the magnetic order. It means that they are developed separately from the pairings in the triplet superconductivity channel, which can exist without magnetic ordering. In the investigated model, the last processes are ignored.

Figure 2 presents temperature dependences of the anomalous amplitudes of the chiral superconducting order parameter at $V = 0.96t_1$ and $n = 1.12$. The chosen doping concentration is close to optimal. It can be seen that for the presented Coulomb repulsion parameter, the contribution of the pairing interaction in the first coordination sphere specified by the amplitude Δ_{21} is significantly suppressed as compared with the contribution Δ_{22} determining the pairing interaction between the next-to-nearest neighbors. The amplitude Δ_{11} characterizing the induced triplet states takes negative values. It means that these processes do not additionally contribute to the formation of Cooper instability against the background of the long-range magnetic order and open an additional channel for superconductivity suppression. This effect, however, only leads to the minor decrease in the temperature of superconductivity onset, since the Δ_{11} value is essentially lower than the amplitude Δ_{22}.

Figure 3 shows a decrease in the amplitude Δ_{22} with increasing parameter V. It can be seen that the Coulomb repulsion between the nearest neighbors does not sharply suppress the superconductivity, since the main contribution to the Cooper instability at large V values is made by the pairing interaction between the next-to-nearest neighbors. The growth of Coulomb

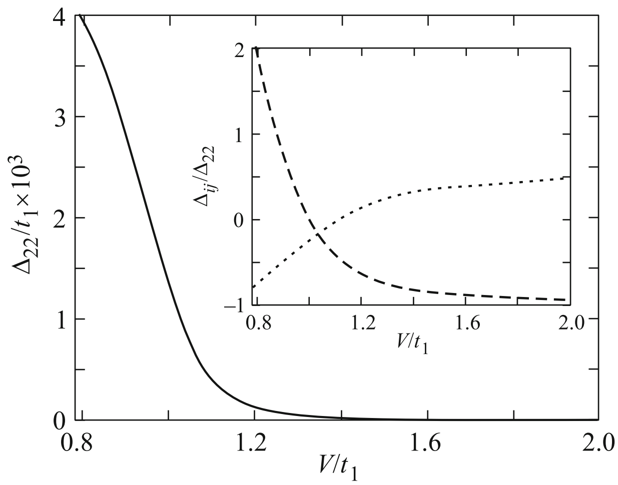

Fig. 3. Dependence of the anomalous amplitude Δ_{22} on parameter V of the intersite Coulomb repulsion in the limit of zero temperature at a concentration of n = 1.12. Inset: dependences of amplitudes Δ_{21} and Δ_{11} on V with respect to the Δ_{22} value.

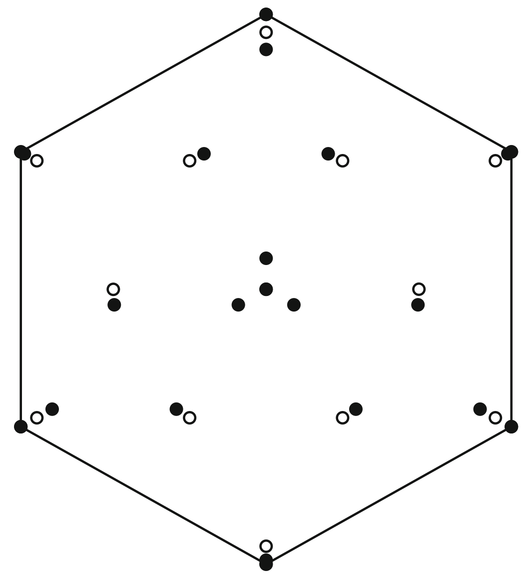

Fig. 4. Nodal points of the superconducting order parameter $\Delta(\mathbf{p})$ with regard to the chiral $p_x + ip_y$ invariant (closed circles) and with disregard of it (open circles).

repulsion results in the fast drop of the anomalous amplitude Δ_{21}. At $V = 2J_1$, this amplitude turns to zero. With a further increase in V, the amplitude Δ_{21} becomes negative. This behavior is illustrated in the inset in Fig. 3, where the dependences of the amplitudes Δ_{21} and Δ_{11} on the intersite repulsion parameter relative to the Δ_{22} value are shown. Interestingly, the amplitude Δ_{11} also changes its sign and becomes positive with increasing V at $\Delta_{21} < 0$. However, this occurs at the very low critical temperatures of superconductivity (tenths of kelvins).

The account for the Coulomb repulsion modifies the quasi-momentum dependence of the superconducting order parameter by admixing of the triplet invariant. This leads to the change in the positions of nodal points in the Brillouin zone, where the superconducting order parameter turns to zero. Open circles in Fig. 4 show the $\Delta(\mathbf{p})$ nodal points at $\Delta_{11} = 0$ and closed circles are obtained taking into account the contributions of triplet pairings. The Δ_{21} and Δ_{22} values were chosen at a concentration of n = 1.12 near zero temperature (Fig. 2). At $\Delta_{11} = 0$, the superconducting order parameter contains nodal points at the center and at the crossing of the hexagonal Brillouin zone boundaries, as well as 12 points inside the zone: six of them are located near the boundaries, which is caused by the smallness of amplitude Δ_{21}, and the rest six points are slightly shifted relative to nodal points of the invariant $\phi_{22}(p)$. It can be seen that with regard to the triplet invariant with $\Delta_{11} = -0.32\Delta_{22}$, the nodal points located in the Brillouin zone shift and three new points near (0, 0) arise.

6. CONCLUSIONS

As is known, the intersite Coulomb interaction usually suppresses the d-type superconductivity due to renormalization of the effective pairing interaction parameter. For the investigated phase of coexisting chiral superconductivity and noncollinear 120° spin ordering, the additional effect occurs on the triangular lattice. This effect is related to the fact that the Coulomb interaction initiates the formation of a nontrivial structure of the superconducting order parameter; specifically, $\Delta(\mathbf{p})$ is presented in the form of linear superposition of the chiral $d_{x^2-y^2} + id_{xy}$ and $p_x + ip_y$ invariants. Admixing of the triplet invariant is only caused by the Coloumb interaction and only for the coexistence phase. In this case, the anomalous amplitude determining the intensity of the contribution of triplet pairing has negative values in a wide range of parameters. This evidences for the occurrence of an additional contribution leading to suppression of the phase of coexisting superconductivity and noncollinear magnetic ordering. In the investigated case, this contribution does not significantly affect the critical temperature of the transition to the coexistence phase; nevertheless, the result obtained changes the qualitative picture of symmetry classification of the superconducting order parameter and is important for studying the topological properties of the superconducting state in the presence of the 120° magnetic ordering.

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