Fibers based on propagating bound states in the continuum

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We show that a circular periodic array of silicon dielectric cylinders supports nearly bound states in the continuum (BICs) propagating along the cylinders. These propagating nearly BICs with extremely large-Q factors are surrounded by resonant modes weakly leaking into the radiation continuum. We present leaky zones in the form of dispersion curves for complex eigenfrequencies dependent on propagation constant k_z, with the wave vector directed along the cylinders in the vicinity of different types of BICs. Symmetry-protected nearly BICs have the resonant width proportional to squared propagation constant $\Gamma \sim k_z^2$; the widths of non-symmetry-protected nearly BICs behave as $\Gamma \sim (k_z - k_c)^2$, where k_c and non-symmetry-protected nearly BICs have the resonant width proportional to k_z^4. The latter propagating nearly BICs can serve for the transmission of a electromagnetic signal paving a way to a different type of optical fiber. We also demonstrate weakly leaking resonant modes which carry orbital angular momentum.

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I. INTRODUCTION

Standard optical fibers guide light using total internal reflection. This restricts their optical properties because only solid or liquid materials can be used for the fiber core. There are no suitable cladding materials which have a sufficiently low refractive index to confine light by total internal reflection in a vacuum or a gas core.

Substantial efforts have been invested over the past years in fabricating photonic-crystal materials that have a periodic modulation of the refractive index on the scale of the optical wavelength. The interest in such materials lies in their ability to strongly reflect light of certain frequencies. Photonic band-gap structures offer the opportunity to design new optical properties into existing materials by wavelength-scale periodic microstructuring of the material morphology [1]. One can imagine that such a structure of the order of ten layers can be rolled up to form cladding that is capable to almost perfectly trap light inside realizing fiber. Another design of two-dimensionally periodic structures in the form of long, fine silica fibers that have a regular array of tiny air holes running down their length constitutes an artificial two-dimensional photonic crystal (PhC) with lattice constants of the order of micrometers [2].

However, the demand of perfectness of such fibers enormously enlarges their cross section. In the present paper, we propose a different design of fibers based on the capability of a periodic array of dielectric cylinders to trap light at certain frequencies. The property is based on a fundamental family of localized solutions of Maxwell's equations, the so-called bound states in the continuum (BICs). Recently, BICs with zero Bloch vector were reported in infinitely long periodic arrays of dielectric cylinders [3–19] (see also the array of metallic wires on a substrate [20]). BICs propagating along the array were also shown to exist [8,9,21–28]. Our primary interest is in the BICs which can propagate along the cylinders [9,28] giving rise to a new family of guided modes with frequencies above the light line.

Physically, the occurrence of BICs in the infinite array of cylinders is the result of the periodicity of the array that quantizes the radiation continua in the form of diffraction continua [8,17]. Obviously, the infinite array of dielectric cylinders is an unrealistic limit. In practice, we deal with a finite number N of cylinders which have material losses given by the imaginary part of the refractive index, structural fluctuations of cylinders, the effect of substrate, etc., transforming the true BIC into a resonant mode with small resonant width [13,16,17,29–31]. Although the full range study of these factors is still far from completion, it was shown that the Q factor of the symmetry-protected quasi-BICs grows quadratically with N [31].

Each cylinder can support guided modes propagating along the cylinder [32] provided that the frequency of the modes lies below the light line. Above the light line, the modes have leakage into the radiation continuum. Assume now that the array of cylinders is rolled into a circle, as shown in Fig. 1, periodically over the azimuth angle with the center lines of the cylinders positioned as $\phi_j = 2\pi j/N,\ j = 0, 1, 2, \ldots, N-1$, where N is the number of cylinders. Then, the Q factor grows exponentially with N for the case of the nearly symmetry-protected BICs [33,34] that is the result of almost perfect destructive interference of the modes' leakage into the radiation continuum. In practice, such Q factors make the nearly BICs in the circular array indistinguishable from true BICs in the infinite array of cylinders [34]. In the present paper, we demonstrate a few examples of the nearly BICs propagating along the cylinders. The property of the nearly BICs to serve as modes with extremely high-Q factors and guide above the light line paves a way to the designs of fibers composed of

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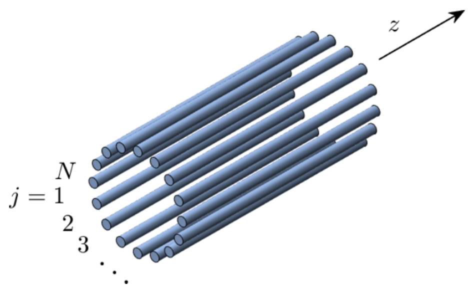

FIG. 1. N infinitely long circular dielectric cylinders with radius a stacked parallel to each other in a circle of radius R.

N dielectric cylinders circularly packed parallel to each other. These nearly BICs fill the core of the fiber and can carry orbital angular momentum (OAM) m. Each type of the above-listed nearly BICs is hosted by a leaky zone with high Q surrounding the nearly BIC.

II. NEARLY BICS PROPAGATING ALONG THE FIBER

Here, for brevity, we omit the details of the calculations. The calculations are based on the theory of scattering by a finite cluster of cylinders [34,35]. We start with the simplest symmetry-protected standing-wave nearly BIC whose coupling with the radiation continuum is exponentially weakened because of symmetry incompatibility [33,34]. This nearly BIC originates from a true standing-wave BIC in the infinite periodic array of dielectric cylinders at the Γ point, first reported by Shipman and Venakides [3] and shown in Fig. 2. The electric field of the BIC solution directed along the cylinders is even relative to the direction perpendicular to the plane of the array (y axis in Fig. 2) and odd relative to $x \to -x$, where x and y are the local coordinate system tied to the center of the cylinder. The z component of the magnetic field of the symmetry-protected BIC equals zero to define the nearly BIC as E polarized.

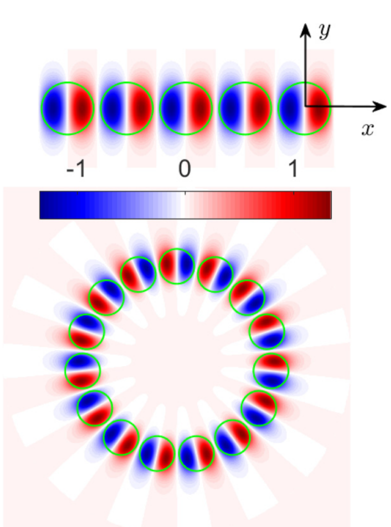

FIG. 2. The z component of the electric field of the symmetry-protected standing-wave BIC in the linear periodic array of silicon dielectric cylinders with $a = 0.44$ and $\epsilon = 15$ with the BIC frequency $k_{0c} = 1.8315$ and its counterpart, i.e., the nearly standing BIC in the circular array of the radius $R = 7.25a$ of 15 cylinders with discrete eigenfrequency $k_{0c} = 1.837$ with $m = 0$ from Ref. [34]. The radius of the cylinders a is given in terms of the azimuth period of circular array $h = 2\pi R/N$, where R is the radius of the circular array. Respectively, the wave numbers are given in terms of inverse h.

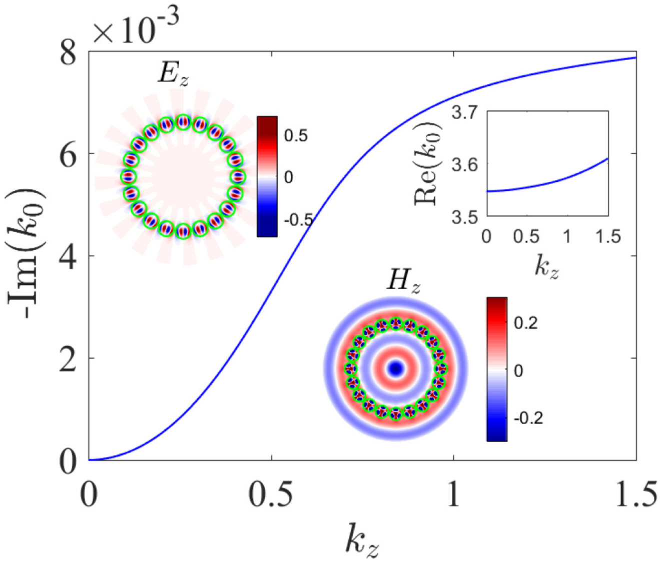

FIG. 3. Leaky zone (dispersion of the complex eigenfrequency) of resonant modes propagating along the fiber consisting of 20 silicon cylinders with $\epsilon = 15$ and radius $a = 0.44 = 0.1382R$. The insets show the z components of the electric and magnetic fields at $k_0 = 3.6086 - 0.007866i$, $k_z = 1.5$, $m = 0$.

For the case of an infinite periodic linear array of dielectric cylinders, there are the BICs propagating along the array with finite value of Bloch wave number $k_n = 2\pi n/L$, $n = 0, \pm 1, \pm 2, \ldots$, where L is the period of the array [8,9,21,23–28]. Such propagating BICs are the result of the avoided crossing of modes with different n [36,37]. However, the BIC propagating along the cylinders with the propagation constant k_z occurs when the symmetry-protected BIC coalesces with the non-symmetry-protected BIC protected by topological charge [9,28]. When N cylinders are rolled up into a circle periodically by the angular period $2\pi/N$, the linear Bloch number k_n is substituted by azimuth Bloch number $k_m = 2\pi m/N$, $m = 0, 1, 2, \ldots, N-1$. Respectively, the BICs with finite Bloch vector k_n are substituted by the nearly BICs with azimuth Bloch number k_m.

The radius of the cylinders a is given in terms of the azimuth period of circular array $h = 2\pi R/N$, where R is the radius of circle. Respectively, the wave numbers are given in terms of inverse h. The dispersion curves are computed by solving the dispersion equation $f(k_0, k_z) = 0$ through analytical continuation of k_0 into the complex plane, where $k_0 = \omega h/c$ is the vacuum wave number and k_z is the propagating constant, the wave number along the cylinders. Figure 3 shows the real and imaginary parts of complex eigenfrequencies for the case of 20 silicon cylinders. The resonant width and frequency depend on k_z quadratically for small k_z, as seen from Fig. 3. Such a behavior is typical for the guided modes in the vicinity of the Γ point in infinite arrays [26,27,29,38]. The Q factor of the

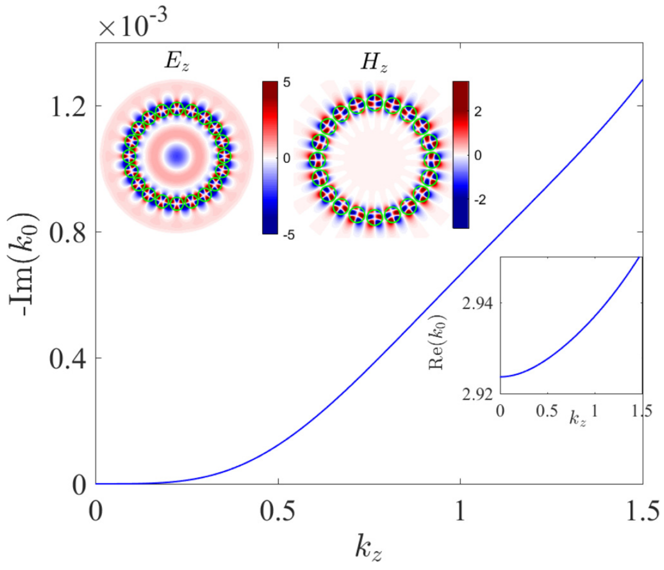

FIG. 4. Leaky zone of resonant mode of the circular array of cylinders with $a = 0.43084 = 0.1354R$, which converts into the non-symmetry-protected E-polarized nearly BIC at $k_z = 0$ and $a = 0.43084 = 0.1354R$. The insets show the z components of electric and magnetic fields at $k_0 = 2.9508 - 0.00128i$, $m = 0$, $k_z = 1.5$.

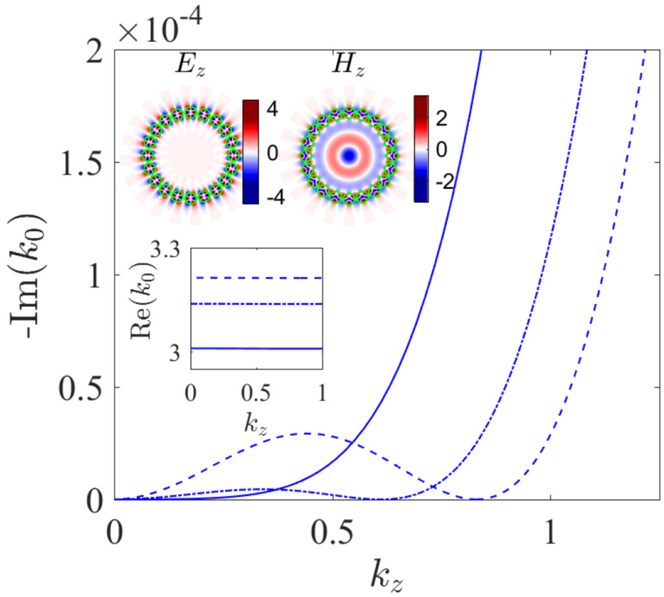

FIG. 5. Two examples of leaky zones of the resonant mode at $a = 0.418 = 0.1313R$ with two nearly BICs with $m = 0$ at $k_z = 0$ and $k_z = 0.83$ (dashed line) and at $a = 0.43 = 0.1351R$ with two nearly BICs at $k_z = 0$ and $k_z = 0.605$ (dash-dotted line) which finally collapses into the mode with single nearly BIC at $k_z = 0$, whose resonant width has an asymptote k_z^4 at $a = 0.453 = 0.1382R$ (solid line). The insets show the z components of the electric and magnetic fields at $k_0 = 3.2124 - 3.95 \times 10^{-8}i$, $k_z = 0.83$ for $a = 0.418$.

eigenmode is given by the equation $Q = -\mathrm{Re}(k_0)/2\mathrm{Im}(k_0)$. The insets in Fig. 3 show profiles of electromagnetic fields (z components of electric and magnetic field) at $k_z = 1.5$. This mode converts into a standing-wave E-polarized nearly BIC with $H_z = 0$ at $k_z = 0$ (see Fig. 5 in Ref. [34]). Therefore, when the wave number k_z moves away from zero, not only does the Q factor reduce, but also both polarizations are mixed, as seen from the insets in Fig. 3. One can see that the magnetic field fills the whole inner space of the fiber as different from the electric field, which is mostly localized inside the cylinders. That is related to the electric field being odd relative to $x \to -x$ and mostly localized around the cylinders, while the magnetic field is even to fill the whole inner space of the fiber [34].

Recently, the asymptotic scaling law for the coupling of the resonant mode with the lowest resonant width with the radiation continuum in the infinite array of dielectric spheres was derived in the vicinity of the Γ point [39]. It was shown that the coupling behaves linearly with the propagation constant for the symmetry-protected BIC at the Γ point. Respectively, the resonant width behaves quadratically with the propagation constant k_z, as demonstrated in Fig. 3.

However, for the non-symmetry-protected nearly BIC, the coupling constant can be presented in the general case as $w \sim a_0 + a_2k_z^2$ [39] with a_0 extremely small to have the quaternion behavior of the resonant width $-\mathrm{Im}(k_0) = |w|^2 \sim |a_0 + a_2k_z^2|^2$, as shown in Fig. 4. The resonant mode originates from the non-symmetry-protected standing-wave E-polarized nearly BIC at $k_z = 0$. This nearly BIC is symmetry protected in respect to the magnetic field and, due to tuning, the cylinder radius acquires exponentially small coupling with the radiation continuum in respect to the electric field to achieve $Q = 2.6 \times 10^8$. When the propagation constant k_z moves away from zero, the resonant mode mixes both polarizations. For the infinite array of cylinders, the electromagnetic field of this non-symmetry-protected BIC was localized around the cylinders. In the circular array, the leaky mode has the even electric field filling the whole core of the fiber, while the odd magnetic field remains localized around the cylinders, as shown in the insets in Fig. 4. Such a quaternion behavior of the resonant width was shown in Ref. [28] relative to k_z and in Refs. [29,40] relative to the Bloch wave number along the infinite periodic array.

We considered the leaky zones (dispersion curves for complex eigenfrequencies) of the resonant modes in Figs. 3 and 4 originated from standing-wave nearly BICs, which are suitable for signal transmission along the fiber because of slow velocity of the signal. However, it is more interesting that the constant a_2 in the coupling strength $w \sim a_0 + a_2k_z^2$ can be negative to give rise to the nearly BIC at finite propagation constant. Figure 5 shows the leaky zones which hold the BIC point with $k_{zc} \neq 0$. One can see the evolution of the resonant width vs the propagation constant with increase of the cylinder's radius a. For the first two choices $a = 0.418$ and $a = 0.43$, there are two points where the resonant width nearly turns to zero (dashed and dash-dotted lines in Fig. 5). The first point $k_{zc} = 0$ corresponds to the symmetry-protected E-polarized standing-wave nearly BIC with $Q = 1.6 \times 10^7$ for $a = 0.418$ and $Q = 5 \times 10^7$ for $a = 0.43$. The second point corresponds to the propagating nearly BIC with mixed polarizations. The propagation constant k_{zc} turns to zero with the increase of the cylinder's radius with the two BICs coalescing at $a = 0.453$ at $k_{zc} = 0$. The leaky resonant modes hosting this standing-wave nearly BIC at the point of coalescence acquire quaternary dependence of the resonant width $-\mathrm{Im}(k_0) \sim k_z^4$, as shown

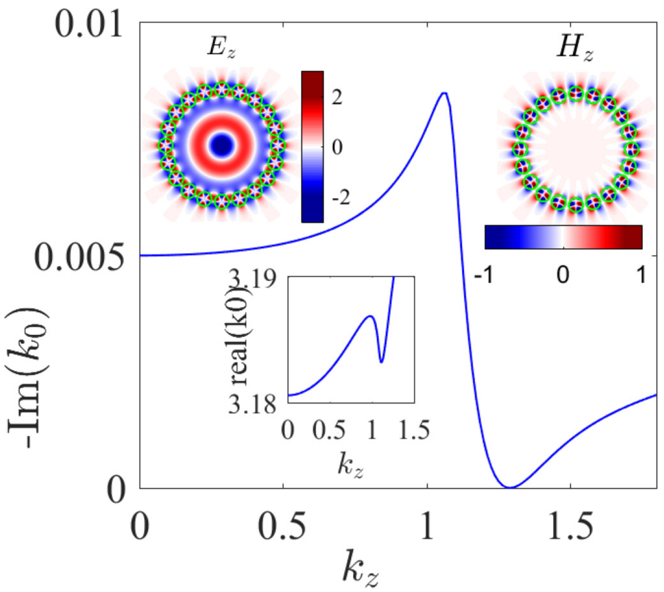

FIG. 6. Leaky zone of the resonant mode which converts into the symmetry-protected nearly BIC for $a = 0.4 = 0.1257R$. The insets show the z components of the electric and magnetic fields at the complex eigenvalue $k_0 = 3.1916 - 5 \times 10^{-9}i$ and the wave number $k_z = 1.29, m = 0$.

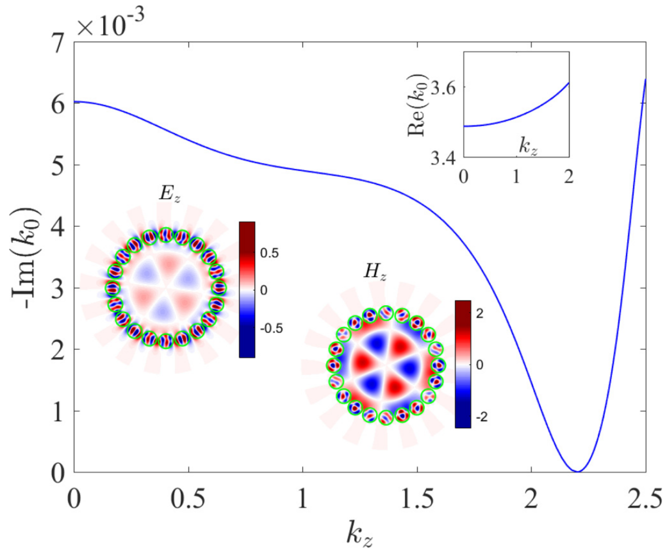

FIG. 8. Leaky zone of the resonant mode which converts into the nearly BIC with OAM $m = 3$ for $a = 0.4327 = 0.136R$. The insets show the z component of electric and magnetic fields of the propagating nearly BIC at $k_0 = 3.6544 - 2 \times 10^{-7}i, k_z = 2.2$.

in Fig. 5 by the solid line. That phenomenon was studied in detail for the case of the infinite array of cylinders and spheres in Refs. [24,27] as a result of preservation of topological charge in two-dimensional space of two polarizations of the BIC. It is remarkable that for k_z in the wide range, the resonant width is smaller than 2.5×10^{-4}, as shown in Fig. 5 by the dashed and dash-dotted lines. A weak dependence of the Q factor on the wave number allows one to use this nearly BIC for signal transmission with high efficiency.

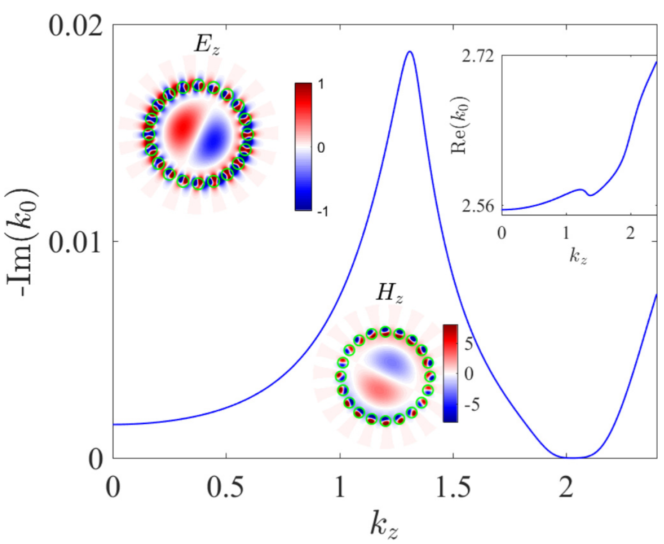

FIG. 7. Resonant mode which converts into the nearly BIC with OAM $m = 1$ for $a = 0.369 = 0.116R$. The insets show the z component of the electric field of the propagating nearly BIC at $k_0 = 2.644 - 5 \times 10^{-9}i, k_z = 2.027$.

Figure 6 shows the resonant mode which holds only the propagating nearly BIC at finite values of the wave number, but not a standing-wave nearly BIC.

This nearly BIC has mixed polarizations with the even electric field E_z filling the whole core of the fiber and the odd magnetic field localized in the vicinity of cylinders. The mode has a Q factor of the order of 3×10^2 at $k_z = 0$. The Q factor decreases when the wave number goes away from zero, but then again goes to an extremely large value of 3.2×10^8 when k_z reaches $k_z = 1.29$, as shown in Fig. 6. The dispersion curve shows a nonmonotonic behavior that is related to an avoided crossing of two neighboring resonances.

The former cases with $m = 0$ do not need tuning of the cylinder radius. Once $m \neq 0$, the propagating-wave nearly BICs with the OAM $m \neq 0$ need tuning of the radius, as shown in Ref. [34]. These propagating-wave nearly BICs with OAM are shown in Figs. 7 ($m = 1$) and 8 ($m = 3$).

III. DISCUSSION AND CONCLUSIONS

First, it is interesting to compare propagating nearly BICs in the circular array of cylinders with guided modes propagating along an isolated dielectric cylinder [32]. That comparison is given in Fig. 9. One can see that the frequencies that are dependent on the propagation constant k_z behave very similarly to one another, while the resonant widths are strikingly different. If the frequency of the guided mode in the isolated cylinder is below the line of light, the mode can propagate along the cylinder without leakage. As soon as the frequency is above the line, the mode becomes leaky, as shown in Fig. 9(b) by a solid line, while the widths of the nearly BICs including the resonant modes surrounded the BIC have extremely small leakage above the light line in a rather wide domain of the propagation constant.

Apparently, the choice of the fiber of N dielectric cylinders of circular cross section is not the best from a technology view-

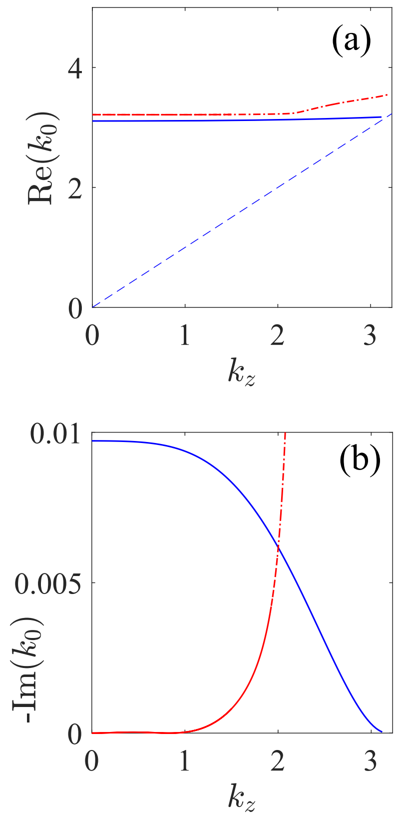

FIG. 9. (a) Dispersion and (b) resonant width of the guided mode in an isolated dielectric cylinder with $a = 0.418$ and $m = 0$ (solid lines) compared to the case shown in Fig. 5 (red dash-dotted lines). The thin dashed line shows the light line.

point. In general, there can be any circular dielectric structure which possesses a symmetry relative to the azimuth discrete rotations $\phi \to \phi + 2\pi n/N$, where $n = 1, 2, 3, \ldots, N$ and N is an integer. In practice, the fiber can be chosen in the form of a single dielectric cylinder with periodical grating on its surface. The present type of fiber composed of N dielectric cylinders has a unique property to exponentially enlarge the Q factor with N [33,34] for specific solutions, i.e., that is nearly BICs. These solutions are localized within the fiber even though the frequency of the solution is embedded into the radiation continuum. The fiber can support various nearly BICs, mostly standing waves. These BICs are surrounded by weakly leaking resonant modes with the Q factor proportional to the inverse of k_z^2. The symmetry-protected nearly BICs do not need tuning of the cylinder radius, which makes them interesting from a technological point of view. There are also non-symmetry-protected nearly BICs which occur via tuning the cylinder radius. The resonant modes surrounding these nearly BICs have extremely weak quaternion dependence of the resonant width on the propagation constant, which is especially interesting for signal transmission. The non-symmetry-protected propagating nearly BICs surrounded by resonant modes with Q factor inversely proportional to $(k_z - k_{zc})^2$ are the most interesting for signal processing in the fiber. They do not need tuning of the cylinder radius, in contrast to nearly BICs which carry the orbital angular momentum (OAM).

It is clear that the transmission of electromagnetic signals over the fiber requires some finite range of frequencies. Because of discreteness of the BIC frequency, propagation of signals will be accompanied by leakage. However, the majority of resonant widths do not exceed one percent of the frequency. The propagation length is given by the decay rate of nearly BICs into the radiation continuum. Its value can be accessed as [23]

$$\frac{L}{\lambda} = \left|\frac{d\mathrm{Re}(k_0)}{dk_z}\right| \frac{k_0}{2\pi \mathrm{Im}(k_0)}. \quad (1)$$

For example, we obtain $L/\lambda \approx 10^4$ for $a = 0.418$ (see Fig. 5) and $L/\lambda \approx 5 \times 10^6$ for $a = 0.4$ (see Fig. 6), where λ is the wavelength. Therefore, the propagating nearly BICs can serve for the propagation of electromagnetic signals with high quality. That prompts one to use the circular array of cylinders as a different type of optical fiber.

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