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Topological Phase of Coexisting Superconductivity and 120-Degree Magnetic Order on a Triangular Lattice

V. V. Val'kov[a], * and A. O. Zlotnikov[a]

[a]*Kirensky Institute of Physics, Krasnoyarsk Scientific Center, Russian Academy of Sciences, Siberian Branch, Krasnoyarsk, 660036 Russia*

**e-mail: vvv@iph.krasn.ru*

Abstract—Using the t–J–V model, self-consistent calculations of the superconducting order parameter described by a linear combination of $d_{x^2-y^2} + id_{xy}$ and $p_x + ip_y$ chiral invariants are performed in the phase of coexistence with 120-degree magnetic ordering. Sublattice magnetization is determined in the spin-wave approximation for the case of half-filling. A nontrivial topology of the coexistence phase is demonstrated, testifying to the possibility of obtaining edge states and Majorana modes.

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INTRODUCTION

It is well known that a topologically nontrivial phase with the formation of edge states can be obtained in a superconductor with chiral symmetry of the order parameter [1]. In a triangular lattice with $d_{x^2-y^2} + id_{xy}$ chiral symmetry of the superconducting phase, the topological transition from the trivial to nontrivial phase coexists with the quantum topological concentration transition between two phases with nontrivial topology [2]. As was shown in [3], the conditions for obtaining edge Majorana states can be fulfilled when there is noncollinear magnetic order with a stripe structure in the chiral superconducting phase. This is of special importance, since the mechanism of inducing Majorana modes in topological superconductors, which is associated with noncollinear magnetism, evokes great interest [4, 5].

It is assumed that the phase of the coexistence of superconductivity and noncollinear magnetism forms in ternary rare-earth borides and chalcogenides, rare-earth intermetallic compounds, and sodium cobaltites, the layers of which form a triangular lattice. We therefore studied the effect magnetic structure has on Cooper instability for an ensemble of Hubbard fermions on a triangular lattice [6]. We showed that the chiral symmetry of the superconducting order parameter is broken during the formation of a magnetic stripe structure. If the magnetic structure corresponds to noncollinear 120° ordering, a uniform phase of coexisting chiral superconductivity and magnetic order (the SC + 120° phase) can form. The conditions for obtaining Majorana modes for the SC + 120° phase were later established using a quadratic Hamiltonian, the topological invariants were calculated, and the topological phase diagram was determined [7, 8].

The question of SC + 120° phase topology with regard to the strong electron correlations typical of the investigated materials remains unanswered. Topological invariant $\tilde{N}_3$, expressed via Green's functions for 2D systems with coupling, was proposed in [9]. In this work, we use the t–J–V model to find superconducting pairing amplitudes using self-consistency equations for 120° magnetic order. We calculate the topological $\tilde{N}_3$ invariant in the SC + 120° phase and demonstrate the nontrivial topology of the ground state.

MODEL

The Hamiltonian of the t–J_1–J_2–V model when considering the upper Hubbard subband in the atomic representation has the form

$$H = (\varepsilon - \mu)\sum_{f\sigma} X_f^{\sigma\sigma} + (2\varepsilon + U - 2\mu)\sum_{f} X_f^{22} + \sum_{fm\sigma} t_{fm} X_f^{2\bar{\sigma}} X_m^{\bar{\sigma}2} + \sum_{fm} J_{fm}\left(X_f^{\uparrow\downarrow} X_m^{\downarrow\uparrow} - X_f^{\uparrow\uparrow} X_m^{\downarrow\downarrow}\right) + \frac{V}{2}\sum_{f\delta} n_f n_{f+\delta}, \quad (1)$$

where ε is the seed energy of electrons, μ is the chemical potential, U is the one-site Coulomb repulsion parameter, t_{fm} is the amplitude of electron hoppings, and J_{fm} is the exchange coupling value. The last term in the Hamiltonian describes the Coulomb interaction between electrons on the nearest sites with parameter V, and n_f is the operator of the number of electrons on the site.

In [10], we found Green's matrix Matsubara function for the SC + 120° phase:

$$G^{-1} = \begin{bmatrix} i\omega_n - \xi_p & -\Delta_p^* & -R_{p-Q} & 0 \\ -\Delta_p & i\omega_n + \xi_p & 0 & R_{p-Q} \\ -R_p & 0 & i\omega_n - \xi_{p-Q} & \Delta_{-p+Q}^* \\ 0 & R_p & \Delta_{-p+Q} & i\omega_n + \xi_{p-Q} \end{bmatrix}, \quad (2)$$

where $\xi_p = \varepsilon(n) - \mu + nt_p/2$; $n = \langle n_f \rangle$; t_p is the Fourier image of the hopping integral; Δ_p is the superconducting order parameter; $R_p = M(t_p - J_Q)$; $R_{p-Q} = M(t_{p-Q} - J_Q)$; M is the magnetic order parameter; and J_Q is the Fourier image of the exchange integral for magnetic structure vector $\vec{Q}$.

Allowing for the exchange interaction between the nearest and next-to-nearest neighbors and the interstitial Coulomb interaction, we can write the superconducting order parameter in the SC + 120° phase as a linear superposition of the chiral invariants with the $d_{x^2-y^2} + id_{xy}$ and $p_x + ip_y$ types of symmetry:

$$\Delta_p = 2\Delta_{21}\varphi_{21}(p) + 2\Delta_{22}\varphi_{22}(p) + 2\Delta_{11}\varphi_{11}(p). \quad (3)$$

The energy spectrum in the SC + 120° phase has the form

$$E_{1,2p} = \sqrt{\frac{1}{2}\left(\xi_p^2 + \xi_{p-Q}^2 + |\Delta_p|^2 + |\Delta_{-p+Q}|^2\right) + R_p R_{p-Q} \mp \nu_p}, \quad (4)$$

$$\nu_p = \sqrt{\frac{1}{4}\left(\xi_p^2 - \xi_{p-Q}^2 + |\Delta_p|^2 - |\Delta_{-p+Q}|^2\right)^2 + R_p R_{p-Q}\left[(\xi_p + \xi_{p-Q})^2 + |\Delta_p + \Delta_{-p+Q}|^2\right]}. \quad (5)$$

MAGNETIZATION AT HALF-FILLING

It should be noted that obtained spectrum (4) can assume complex values, depending on the parameters. This is because the effective exchange fields in the noncollinear magnetic phase R_p and R_{p-Q} are alternating-sign quasi-momentum functions with regard to the non-Fermi character of Hubbard operator rearrangements. The presence of imaginary features in the spectrum is in this case due to the magnetic order parameter in particular.

Let us calculate the magnetization at half-filling. The average spin operator for sublattice magnetizations lying in the plane perpendicular to the quantization axis is determined as $\langle \vec{S}_f \rangle = M\left(\cos(\vec{Q}\vec{R}_f), -\sin(\vec{Q}\vec{R}_f), 0\right)$. As was shown in [11], this choice of the quantization axis greatly simplifies calculations.

We first determine the Fourier image of Green's function $-\left\langle T_\tau \tilde{S}_f^z(\tau)\tilde{S}_{f'}^z(\tau')\right\rangle$ in the spin-wave (loopless) approximation for the exchange coupling corrections:

$$D^{zz}(p, i\omega_n) = \frac{2M^2\left[(J_{p-Q} + J_{p+Q})/2 - J_Q\right]}{(i\omega_n)^2 - \omega_p^2}, \quad (6)$$

where $\omega_p = 2M\sqrt{(J_p - J_Q)\left[(J_{p-Q} + J_{p+Q})/2 - J_Q\right]}$ is the spin-wave excitation spectrum. With half-filling, the spin-fluctuation corrections from the term describing Hubbard fermion hoppings are completely suppressed. To satisfy condition $\left(S_f^z\right)^2 = 1/4$, which is valid at half-filling, magnetization M must satsify the equation

$$M(T) = \frac{1}{2}\frac{1}{\sum_p \sqrt{\frac{(J_{p-Q} + J_{p+Q})/2 - J_Q}{(J_p - J_Q)}}\coth\left(\frac{\omega_p}{2T}\right)}. \quad (7)$$

This expression allows us to calculate the temperature dependence of the magnetic order parameter for different types of lattice, which is specified by function J_p, and the magnetic structure determined by vector $\vec{Q}$ on the condition that the average magnetic moment projection onto one of the axes is always zero. In particular, we can easily obtain from (7) the well-known results of the so-called Tyablikov approximation for a ferromagnet with $\vec{Q} = (0,0,0)$ and an antiferromagnet with $\vec{Q} = (\pi,\pi,\pi)$ on a simple cubic lattice.

Below, we consider a 2D triangular lattice with 120° ordering, for which we have $\vec{Q} = (2\pi/3, 2\pi/3)$ in the zero temperature limit. We assume the stability of magnetic ordering at finite temperatures is determined by the weak exchange between the quasi-two-dimensional structure planes. We then have $M(0) = 0.298$.

SELF-CONSISTENT CALCULATIONS OF THE SUPERCONDUCTING AMPLITUDES

Let us consider a case where the SC + 120° phase is obtained near half-filling. We assume the fluctuation corrections weakly affect the magnetic order in this range of concentrations. In addition, we consider hoppings to the second and third coordination spheres of the triangular lattice.

Self-consistency equations for the superconducting order parameter amplitudes were derived in [10]. Figure 1 shows concentration dependences of anoma-

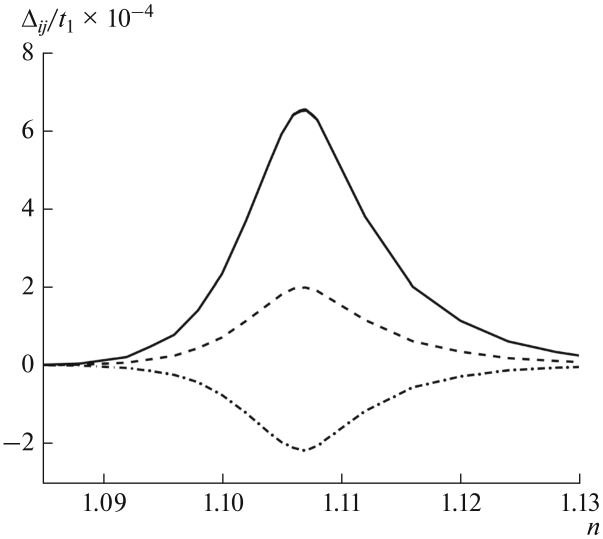

Fig. 1. Dependence of amplitudes Δ_{22} (solid line), Δ_{21} (dashed line), and Δ_{11} (dashed-and-dotted line) of the superconducting order parameter on the electron density in the zero temperature limit.

lous amplitudes in the zero temperature limit at $M(0) = 0.298$. It was shown that the elementary excitation spectrum in the SC+120° phase is in this case always real. The rest parameters were chosen to be $J_1 = 0.5\ t_1$; $J_2 = 0.06\ t_1$; $V = 0.96\ t_1$, $t_2 = -0.2\ t_1$, and $t_3 = -0.18\ t_1$, where t_1 is the parameter of hopping between nearest neighbors. The maximum temperature of critical superconductivity is reached at a concentration of $n = 1.107$ and is $T_c = 0.00188t_1$.

TOPOLOGICAL INVARIANT FOR THE SC + 120° PHASE

In systems with coupling, nontrivial phases are determined by the Green function's topology [9]. The expression for the topological invariant of the ground state is then

$$N_3 = \frac{\varepsilon_{\mu\nu\lambda}}{24\pi^2} \times \int_{-\infty}^{+\infty} d\omega \iint dp_1 dp_2 \mathrm{Tr}\left(G\partial_\mu G^{-1} G\partial_\nu G^{-1} G\partial_\lambda G^{-1}\right), \quad (8)$$

where $\varepsilon_{\mu\nu\lambda}$ is the Levi–Civita symbol and G is Green's matrix function (2).

Upon the quantum transition to the SC + 120° phase as the concentration rises, the topological invariant changes from $N_3 = 0$ to $N_3 = 3$, which testifies to the transition from the topologically trivial phase (with 120° ordering) to the topologically nontrivial (SC + 120°) phase. Yet another topological transition to the SC + 120° phase with an invariant of $N_3 = 2$ occurs later, near a concentration of $n = 1.118$.

It is worth noting that topological invariant $N_3 = 2$ is typical of the chiral superconducting phase on a triangular lattice [2]. Allowing for the magnetic order in this range of concentrations thus does not qualitatively change the features of edge state formation in a system. On the other hand, the concentration range with $N_3 = 3$ is obtained exclusively due to the 120° spin ordering. It was assumed that in regions with odd N_3 values, special edge states with zero excitation energy (i.e., Majorana bound states) can form [8, 12].

CONCLUSIONS

We determined the electron densities at which the phase of coexisting chiral superconductivity and 120-degree magnetic ordering can form in the t–J–V model on a triangular lattice. We determined the sublattice magnetization in the spin-wave approximation. Using this approximation, we calculated the topological invariant expressed via Green's functions. We demonstrated the existence of a nontrivial topology of the ground state in the coexistence phase when strong electron correlations are allowed for.

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REFERENCES

1. Volovik, G.E., *JETP Lett.*, 1997, vol. 66, p. 522.
2. Val'kov, V.V., Val'kova, T.A., and Mitskan, V.A., *JETP Lett.*, 2015, vol. 102, p. 361.
3. Lu, Y.-M. and Wang, Z., *Phys. Rev. Lett.*, 2013, vol. 110, p. 096403.
4. Martin, I. and Morpurgo, A.F., *Phys. Rev. B*, 2012, vol. 85, p. 144505.
5. Nakajima, Y., Hu, R., Kirshenbaum, K., et al., *Sci. Adv.*, 2015, vol. 1, no. 5, p. e1500242.
6. Val'kov, V.V. and Zlotnikov, A.O., *JETP Lett.*, 2016, vol. 104, p. 483.
7. Val'kov, V.V., Zlotnikov, A.O., Fedoseev, A.D., and Shustin, M.S., *J. Magn. Magn. Mater.*, 2017, vol. 440, p. 37.
8. Val'kov, V.V., Zlotnikov, A.O., and Shustin, M.S., arXiv:1707.04422 [cond-mat.supr-con].
9. Volovik, G.E., *The Universe in a Helium Droplet*, Oxford Univ. Press, 2003.
10. Val'kov, V.V. and Zlotnikov, A.O., *Phys. Solid State*, 2017, vol. 59, p. 2120.
11. Katanin, A.A., Irkhin, V.Yu., and Igoshev, P.A., *Model'nye podkhody k magnetizmu dvumernykh zonnykh sistem* (Model Approaches to the Magnetism of Two-Dimensional Band Systems), Moscow: Fizmatlit, 2013.
12. Ghosh, P., Sau, J.D., Tewari, S., and Das Sarma, S., *Phys. Rev. B*, 2010, vol. 82, p. 184525.

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