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18 ABSTRACT

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We analyze a new type of plasmon system arising from small metal nanoparticles linked by narrow conductive molecular bridges. In contrast **19** 20 to the well-known charge-transfer plasmons, the bridge in these systems consists only of a narrow conductive molecule or polymer in which 21 the electrons move in a ballistic mode, showing quantum effects. The plasmonic system is studied by an original hybrid quantum-classical model accounting for the quantum effects, with the main parameters obtained from first-principles density functional theory simulations. We 22 have derived a general analytical expression for the modified frequency of the plasmons and have shown that its frequency lies in the near-23 24 infrared (IR) region and strongly depends on the conductivity of the molecule, on the nanoparticle-molecule interface, and on the size of the 25 system. As illustrated, we explored the plasmons in a system consisting of two small gold nanoparticles linked by a conjugated polyacetylene molecule terminated by sulfur atoms. It is argued that applications of this novel type of plasmon may have wide ramifications in the areas of 26 27 chemical sensing and IR deep tissue imaging.

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³⁰ I. INTRODUCTION

31 Surface plasmons (SPs) are delocalized collective oscillations of 32 free electrons relative to the positive ions at the interface between 33 negative and positive permittivity materials, such as metal-dielectric 34 interfaces. SPs are associated with oscillations of charge density 35 coupled with electromagnetic fields (EMF) created by the coher-36 ent motion of the free charges. These oscillations are called surface 37 plasmon polaritons in the case of planar interfaces or localized sur-38 face plasmons (LSPs) in the case of metal nanoparticles (NPs) with 39 a closed surface.^{1,2} The plasmon polaritons are characterized by a 40 specific frequency (surface resonant frequency, SRF) when the inci-41 dent electromagnetic radiation is resonant with the surface plasmon

resonance (SPR) frequency. The SRF is highly sensitive to the permittivity of a surrounding chemical environment, the change of which due to chemical composition and morphology often leads to substantial shifts of the SRF.^{3–6}

Since the SRF can be readily measured by different optical techniques,³ LSPs are nowadays intensively used for the creation of nanoscale sensors for chemical and biological substances.^{4–7} Moreover, the strong local electromagnetic field enhancements offered by plasmon excitation has rendered a wide use of plasmonic materials in different fields, such as high resolution imaging,⁸ plasmon lasers,⁹ chemical synthesis,¹⁰ water splitting,¹¹ optical waveguid-ing,^{12–15} biomedical and telecom applications,^{16,17} and photovoltaic cells,^{1,18–21} just to name a few.

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Gold is the most known substance for plasmonic applica-55 56 tions due to its light absorption in the visible region. It is often 57 used in the form of separate nanoparticles (NPs), nanorods or 3-D materials built from NPs.^{22,23} Besides Au NPs, silver and cop-58 per nanoparticles are also employed, albeit to a lesser extent due 59 to their instability.^{1,4,7,8,10,19,21} A variety of different forms of plas-60 61 monic materials are conditioned by the fact that the LSP is con-62 centrated at the edges of the nanoparticles, that the strength of 63 the electromagnetic field strongly depends on the interaction and 64 the interparticle distance, and that small changes are very influ-65 ential on the measured signal. From this point of view, the SRF also depends on the size and shape of the nanoparticles¹⁻⁶ as well 66 as the type of material that links them.⁵ In the case of conduc-67 68 tive materials between two nanoparticles a new mode, called charge 69 transfer plasmon (CTP), emerges. Such plasmons, observed in the 70 case of conductively coupled metallic nanoparticles, are expected 71 to expand their applications from molecular sensing to nanoscale 72 wave-guiding.

An example of charge transfer plasmons was experimentally 73 74 observed in Ref. 5 in a system consisting of two gold NPs with 75 diameter $D \ge 40$ nm joined by a cylindrical gold bridge with radius 76 from 10 to 20 nm. The CTP was described by classical Maxwell's 77 electrodynamics as quantum effects were not expected to be signif-78 icant due to the large system size. Quantum effects of CTPs have 79 subsequently been investigated in systems consisting of two NPs separated by subnanometer interparticle gaps,^{5,25-27} where the coupling 80 81 of the nanoparticles is conditioned by tunneling between them and 82 by screening effects. However, CTPs have not been investigated for systems consisting of NPs connected by narrow conductive bridges 83 84 (conducting organic molecules, COMs), where the importance of 85 quantum effects is expected—this is precisely the topic of the present 86 study. We thus consider here a hybrid model that exhibits CTPs con-87 sisting of two metallic NPs connected by a thin COM and analyze 88 the implications of such systems, which thus are novel for plasmonic 89 generation.

90 II. APPLIED APPROACHES FOR CALCULATING 91 OF ELECTRON TRANSPORT AND PLASMON 92 OSCILLATIONS

93 Generally, theoretical descriptions of plasmon resonances in 94 bulk materials can be based on Maxwell's equations. At least, 95 measured plasmon peak positions and spectral areas in bulk 96 metals are found to be in good agreement with classical the-97 ory predictions. However, surface plasmon resonance widths in 98 conductive nanostructures are significantly influenced by quantum confinement effects. Thus, in the case of small nano-objects, 99 100 they cannot be reproduced using the dielectric functions of bulk 101 materials.⁴

¹⁰² The simplest bulk plasmon description can be obtained in ¹⁰³ terms of simple electrostatics and assuming that the electron is a ¹⁰⁴ classical particle having mass *m* and coordinate *x* as a function of ¹⁰⁵ time $\vec{x}(t)$,

$$m\ddot{\vec{x}}(t) = e\vec{E} = -4\pi ne^2 \vec{x}(t).$$
⁽¹⁾

¹⁰⁷ Such an approach, however, cannot be used for the here suggested NPs-COM systems because the nature of the restoring force acting on the electron will be different and because a COM has a 109 few conducting channels where electrons in the vicinity of the Fermi 110 level EFermi will conduct current. Thus, quantum effects of free car-111 rier motion are vital in COMs, which calls for a strict treatment by 112 wave function time evolution $\psi(r, t)^{36}$ in the frame of the real-time 113 propagation approaches, such as real-time time-dependent density 114 functional theory (TDDFT).³⁰ Unfortunately, to avoid the errors 115 accumulating in this technique, the chosen time step needs to be 116 very small (~0.001 fs), which currently restricts the system size to 117 ~250 noble (Au, Ag) atoms, while, as it was mentioned above, an 118 observation of SPRs is possible only in the case of nanoparticles 119 consisting of hundreds of atoms. Therefore, it is vital to develop a 120 methodology to calculate the plasmon frequencies in NPs-COM sys-121 122 tems where the NP size is larger than a nanometer, while at the same time, the electron current through the COM narrow bridge should 123 be treated by a quantum-mechanical approach. The methodology 124 should make it possible to calculate the longitudinal plasmonic fre-125 quencies in dependence of the nanoparticle diameter and also the 126 conductive properties of the organic molecule connecting the NPs. 127 As a first step in this methodology, it is necessary to study the elec-128 tronic conductivity in the systems. The Landauer method is usually 129 used for this purpose,⁴¹ according to which the effective current I130 at zero temperature between two electrodes possessing μ_1 and μ_2 131 electrochemical potentials, respectively, can be estimated as a dif-132 ference of currents from one electrode to the other one, $I = I^+ - I^-$, 133 where 134

$$I^{+} = \frac{e}{L} \sum_{k,n} \frac{\partial E(k)}{\hbar \partial k} f(E,\mu_1), \qquad (2) \qquad ^{135}$$

$$I^{-} = \frac{e}{L} \sum_{k,n} \frac{\partial E(k)}{\hbar \partial k} f(E, \mu_2).$$
(3) 136

Herein, $f(E, \mu_1) = 1/(\exp[(E - \mu_1)/(k_bT)] + 1)$ is the Fermi function of one electrode with potential μ_1 and n and k are the indices of each transverse mode and wave number of electron state $\{n, k\}$ moving along the narrow bridge simultaneously.

Usually in the range $E(k) \in [\mu_1 \dots \mu_2]$, a fixed number M(E) of the channel conducting electrons throughout the bridge is constant $M(E) = \sum_n f(E, \varepsilon_n)$. So, applying (1) and replacing the summation over *k* by integration over *E*, the total current *I* can be written as

$$I = I^{+} - I^{-} = \frac{2e^{2}}{h}M\frac{(\mu_{1} - \mu_{2})}{e}$$
¹⁴⁵

$$\Rightarrow G = \frac{2e^2}{h}M,\tag{4}$$

where *G* is the system conductivity and $M = M(\mu_1) - M(\mu_2)$. If we take into account that an electron has some reflection rate during its motion from one electrode to the other at low bias, Eq. (4) would be transformed into the well-known Landauer⁴² formula, $G = (2 \times 10^2)/h \times TM$, where T = T(E) is the transmission coefficient at any energy $E \in [\mu_1 \dots \mu_2]$. If the bias $[\mu_1 \dots \mu_2]$ is sufficiently large so that T(E) and M(E) can be varied within the range, this formula can be substituted by an integral⁴³

$$\frac{e}{h} \int_{-\infty}^{+\infty} T(E) [f(E,\mu_1) - f(E,\mu_2)] dE,$$
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which transforms at temperature T = 0 into

$$\frac{e}{h}\int_{\mu_2}^{\mu_1}T(E)dE.$$

158 Unfortunately, the Landauer approach in the original form is not 159 profitable for a description of the proposed plasmon model because 160 this approach does not account for the influence of the electromagnetic field (EMF) on the electron acceleration during the motion 161 162 inside a narrow conductor. It proclaims the change of total elec-163 tron energy instead, or, in other words, the change of electrochem-164 ical potential in the electrode. However, a change of the electronic 165 state in the COM region due to the influence of an EMF is essen-166 tial for the plasmon vibration description. Another reason why the 167 Landauer approach is not profitable for our model is the assumed 168 constant bias, while in the case of plasmon oscillations, the bias and 169 current are varied according to the harmonic law. In that context, 170 we note that, recently, some progress has been accomplished for AC calculations inside narrow channels.^{48–50} For example, biased molec-171 172 ular junctions subjected to external time-dependent electromagnetic 173 fields have been addressed by solutions of coupled Green's func-174 tions of the quantum system and Maxwell's equations for electric 175 and magnetic fields.⁵⁰ Unfortunately, this method is too compli-176 cated to use for real systems, due to the nature of the self-consistent 177 differential equations for Green functions for electron state calcu-178 lations. In addition, solutions of the simultaneous Maxwell's equations obtained by sampling in time and space regions using the 179 180 finite-difference-time-domain (FDTD) approach are also expensive. 181 Due to these reasons, this method has been used for model systems 182 only, where the conductive molecule is represented by a bridge in two-level systems.50 183

In another example,⁴⁸ the equation of motion for density oper-184 185 ator $\hat{\rho}(t)$, projected onto the subspace of molecular degrees of free-186 dom, was formulated in the weak molecule-leads coupling limit, 187 where this junction has a time-periodic modulation of the chemi-188 cal potential of the leads. For periodic time-dependent terms, the 189 charge transport kinetics were mapped onto a static problem of a 190 molecule interacting with multiple leads. The time-averaged cur-191 rent reproduces the results of scattering approaches to transport 192 through conductance channels with time-dependent perturbations, as derived using the Floquet theory.⁵¹ Unfortunately, this method is 193 194 also too complicated to be used for real systems and is restricted to model systems only. 195

196 Time-dependent density functional theory (TDDFT) quantum 197 method has been successfully used for the description of quantum effects related to plasmons.²⁸⁻³⁰ It has been shown, for example, 198 199 that Ag-Au nanoparticles are very sensitive to the chemical con-200 figuration, and in some cases, the position of the atomic species outweigh the effect of changing composition.³¹ Density Functional 201 202 based Tight Binding (DFTB) methods have been used, for example, 203 for the study of optical properties and the electronic structure of Ag 204 nanorods and nanorod dimers³⁴ and for the description of influence of quantum tunneling on the efficiency of excitation energy trans-205 206 fer in plasmonic Ag NPs chain waveguides.⁵⁵ TDDFT and DFTB, 207 being quantum methods, are, however, very time-consuming meth-208 ods and the possibility of their practical use strongly depends on 209 the size of the NPs, i.e., the number of atoms and electrons.^{32,33,37} 2103 Therefore, the applications are limited to the consideration of small

particles, consisting, in most cases, of a maximum of a few hundred atoms.^{31,34,35}

III. MODELING OF CHARGE-TRANSFER IN A PAIR OF METAL NANOPARTICLES BOUND BY A CONDUCTING MOLECULE

To construct the model, we have considered a system consisting of two gold nanoparticles of 147 atoms connected by a polyacetylene molecule C_8H_8 , having conjugated chemical bonds, where the junctions with the NPs are implemented by the sulfur atoms to ensure the electric conductivity of the full NP-COM-NP system. Nanoparticles of such size exhibit metallic properties due to thermal excitation [see the solid electronic density of states (DOS) near E_{Fermi} in Fig. 2] where the Gaussian broadening 0.03 eV was used, which is comparable with the thermal broadening at temperature 300 K. The NP metallic properties are essential for our model, where carriers move between the two nanoparticles. Herein, it must be noted that the use of sulfur atoms is a well-known practice for organic linker molecules to be attached to gold nanoparticles.

It is assumed that the conductive bridge is actually a onedimensional conductor in which electrons or holes move in a ballistic mode, i.e., the mean free path of the carriers exceeds the length of the bridge.

The key point of our model is the consideration of electron dynamics, which is described in the language of the wave function. Under the applied electrostatic field, the conduction electrons would accelerate, which leads to a change in their quasimomentum \vec{k} and the band energy $E(\vec{k})$. Changes in these quantities can easily be calculated from the knowledge of only the band structure and effective electron mass m^* near E_{Fermi} . On the basis of so-called effective mass theorem, equation $\hbar \vec{k} = \vec{p} = e\vec{E}$ is proved,³⁶ which is basic when considering the dynamics of an electron and establishing the exact relationship between quantum and classical quantities—a quasimomentum, momentum, and external force. Therefore, the method



FIG. 1. Geometry and electron density of the conductive band for $k \cong k_f$ of the periodic –[Au₁₄₇–S–C₈H₈–S]– system.

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that uses the exact relationship between quantum and classical quan-249 250 tities is significantly easier compared to known methods based on 251 nonequilibrium quantum Green functions⁵⁰ or quantum real-time 252 propagation of wave functions in the frame of time-dependent density functional theory (real-time TDDFT) approaches.^{28–30} These 253 methods can only be used for sufficiently small systems or model 254 255 systems. An important assumption of the proposed model is 256 also that the conductivity of nonperiodical NP-COM-NP sys-257 tems is similar to that of the corresponding periodic (NP-COM) 258 systems.

This assumption can be justified by the fact that the local dynamics of an electron inside the COM between two NPs is determined by the local electric field and does not depend strongly on the contacts of these NPs with other COMs in periodic systems. This allows us to simplify the calculations of the electron dynamics in NP-COM-NP systems and calculate the effective electron mass near E_{Fermi} only in the corresponding periodical system.

In this work, calculations of the system geometry, electronic
 properties, and band structure were carried out within density func tional theory with gradient decomposition (DFT-GGA), using the
 VASP 5.4 software.^{39,40} The results of these calculations indicate



FIG. 3. Geometry of the NP-COM-NP system for the transmission coefficient calculation.

that the system has metallic properties and can conduct DC or AC current (see Figs. 1 and 2).

To confirm the NP-COM-NP system static conductivity, the transmission coefficient T(E), i.e., the probability that an electron with energy E injected at one electrode will be transmitted to the other one, was calculated in the frame of Landauer methodology using the method of Non-Equilibrium Green Functions (NEGF), which was first proposed in 1960s by Keldysh and others^{44,45} and is widely used now.⁴⁶ The T(E) function of the polyacetylene chain $C_8H_8S_2$ connected to two gold electrodes via the sulfur atoms was calculated (see Fig. 3) using the OpenMX package.⁴⁷ The length of the $C_8H_8S_2$ chain was equal to 11.6 Å. Semi-infinite one-dimensional gold nanorods with a pentagonal section of ~5 Å radius were considered as electrodes. T(E) was calculated with zero potential bias at the electrodes (see Fig. 4). The resulting T(E) function at a low voltage is obtained as \simeq 1, which confirms that the electron can easily transfer from one NP to the other.

IV. PLASMON CALCULATION MODEL

To build the plasmon model, it is desirable to construct a differential equation of harmonic oscillations, such as in the model for bulk plasmons (1). However, while in the latter case the source of the 292 292 292



FIG. 4. Transmission coefficient spec-	29
trum (in units of 2 \times 10 ² /h) of the	29
NP-COM-NP system. The energy is	29
measured with respect to the Fermi level	29
of the leads.	29

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restoring force is the homogeneous polarization and surface charge 298 299 density, in NP-COM systems, the restoring force arises due to the 300 electric field between the two nanoparticles having opposite charges 301 and due to the electrochemical potential μ difference of the nanopar-302 ticles. In the proposed hybrid model, it is desirable to use the results 303 of calculations of conductive properties of the NP-COM-NP sys-304 tems on the basis of the parameters derived from quantum-chemical 305 calculations. Furthermore, the existence of a few number of quan-306 tum conductive channels in COM and a huge number of elec-307 tronic states in the vicinity of E_{Fermi} of NPs must be taken into 308 account.

The model is based on the assumption that the total energy of the NP-COM-NP system is constant so that

$$\frac{dE_{tot}}{dt} = \frac{d(E_{tot_1} + E_{tot_2} + E_{tot_3})}{dt} = 0,$$
 (5)

where E_{tot_1} , E_{tot_2} , E_{tot_3} are the total energies of the first and the second nanoparticles and the bridge molecule between them, respectively. These energies can be changed with charge variation in time E_{tot_i} $= E_{tot_i}(Q_i(t))$, where $i = 1 \dots 3$. We also assume that the charge distribution inside both nanoparticles correspond to the ground states, so from (5) follows

$$\frac{dE_{tot_i}}{dt} = \frac{dE_{tot_i}}{dQ_i}\frac{dQ_i}{dt} \ i = 1\dots 2.$$
(6)

Geometry optimization end calculations of electronic prop-320 erties of a family consisting of six icosahedron shaped simi-321 lar gold nanoparticles consisting of 55, 147, 309, 561, 923, and 322 1415 atoms (see Fig. 5) were carried out by the DFTB method 323 with use of a parameter set that is appropriate for the descrip-324 tion of bulk gold clusters and bulk material, as well as Au_nSCH₃ 325 clusters.⁵³ A calculation of the band structure for the periodical 326 structure -[Au147SC8H8S]- was also made. Furthermore, for using 327 in (6) the total energies E_{tot} of the isolated gold nanoparticles hav-328 ing different total charges, $Q(e) \in \{-2, -1, 0, 1, 2\}$ were calculated 329 (Table I).

It was found that the total energy of the all considered nanoparticles is well approximated by the quadratic function of the charge (Fig. 6), i.e.,

$$E_{tot} = aQ^2 + bQ + c. (7)$$

The coefficients of determination R^2 in all cases were higher than 0.9999. In these quadratic functions, the coefficient *c* is the total energy, and the coefficient *b* is the opposite value of the Fermi energy of the corresponding neutral nanoparticle correspondingly (see Tables I and II). According to the well-known formula of the charged sphere electrostatic energy $[E = Q^2/(2C)]$, the coefficient *a*



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FIG. 5. Geometry of the Au_{147} -S- C_8H_8 -S- Au_{147} system.

	Au ₅₅		Au_{147}		Au ₃₀₉	
Q	E _{tot}	E_{f}	E _{tot}	E_{f}	E _{tot}	E_{f}
2	-156.810	-0.384	-420.710	-0.313	-885.377	-0.293
1	-157.149	-0.294	-420.988	-0.249	-885.644	-0.243
0	-157.399	-0.204	-421.201	-0.184	-885.861	-0.193
-1	-157.559	-0.113	-421.351	-0.120	-886.029	-0.143
-2	-157.630	-0.022	-421.436	-0.056	-886.147	-0.093
	Au ₅₆₁		Au ₉₂₃		Au ₁₄₁₅	
Q	E _{tot}	E_{f}	E _{tot}	E_f	E _{tot}	E_f
2	-1608.594	-0.276	-2647.436	-0.260	-4059.262	-0.252
1	-1608.851	-0.235	-2647.677	-0.225	-4059.498	-0.222
0	-1609.066	-0.194	-2647.884	-0.191	-4059.705	-0.192
-1	-1609.241	-0.153	-2648.056	-0.156	-4059.883	-0.163
-2	-1609.374	-0.113	-2648.194	-0.122	-4060.030	-0.133

makes it possible to determine the capacities *C* of the considered Au clusters as C = 1/(2a).

In Fig. 7, the dependence of the capacitance C for gold nanoparticles on their radius is shown. It is obvious that this dependence is linear. Since it is not clear how to define the radius of a nanoparticle, two dependencies are presented—on the maximal r_{max} and on the minimal r_{\min} radius (Table II). The first one is taken as a distance to the atom at the cluster vertex; the second is a distance from the cluster central atom to the cluster faces. The coefficients of R^2 were in both cases were higher than 0.9998. The linear coefficients of the straight lines (1.13 and 0.84) in Fig. 7 show in both cases that for some average radius $R_{aver} \in [r_{\min} \dots r_{\max}]$ of the particle, its capacitance will coincide with the results of the classical electrostatics C = R, i.e., the capacitance of the nanoparticle is equivalent to that of a conducting sphere and equal to the average nanoparticle radius R_{aver} . We speculate that the existence of free terms in both linear dependencies, which do not change much, is related to the bulk distribution of the extra charge inside the particle. In any case, this contribution is significantly smaller than the surface



FIG. 6. Dependence of the Au₁₄₇ nanoparticle total energy on the charge.

84	TABLE II . Coefficients of the quadratic dependence $E = f(Q)$ and nanoparticles radii
85	(everything is in a u)

Cluster	а	b	С	$r_{\rm max}$	r_{\min}
Au ₅₅	0.0447	0.205	-157.399	7.55	10.17
Au ₁₄₇	0.0320	0.182	-421.201	11.41	15.49
Au ₃₀₉	0.0249	0.193	-885.861	15.33	20.81
Au ₅₆₁	0.0206	0.195	-1609.066	19.18	26.12
Au ₉₂₃	0.0172	0.190	-2647.884	23.42	31.43
Au ₁₄₁₅	0.0149	0.192	-4059.705	27.36	36.78

contribution, which suggests that additional charge is almost com pletely distributed over the nanoparticle surface.

Equation (7) also assumes that extra-charge in the particle is 398 distributed according the ground state. Actually, the relaxation of 399 nonequilibrium electrons occurs with a frequency $v \approx \tau^{-1}$, where 400 τ^{-1} is the reciprocal time between electron collisions. For small 401 nanoparticles, this time is even reduced due to the scattering at the 402 nanoparticle boundaries. For gold, $\tau \sim 29$ fs⁵⁶ and we assume that 403 the frequency of the plasmons under consideration is much less than 404 this inverse value; hence, the nanoparticles having a varying charge 405 can be considered to be in the ground state under the plasmon 406 oscillations. 407

The dependency of the total energy of the two isolated nanopar-408 ticles with index $i E_{tot_i}(Q_i)$ was calculated as described in Sec. III [see 409 (7)], where b_i denote the nanoparticle chemical potentials. Using 410 the fact that the volumes of both nanoparticles are much larger 411 than the volume of the conductive molecule, most likely the elec-412 tronic density of the states (DOS) at the Fermi level E_f for the two 413 nanoparticles $DOS(E_f)_{NP_1}$ and $DOS(E_f)_{NP_2} \gg DOS(E_f)_{COM}$. It 414 means that with the change of E_f , the change of cluster charges fol-415 low $\Delta Q_{\rm NP_1}$ and $\Delta Q_{\rm NP_2} \gg \Delta Q_{\rm COM}$, and that during the plasmon 416 oscillations, the charges $Q_i(t)$ of both nanoparticles are opposite, and the conducting molecule would stay neutral. Following this situation and assuming that the two nanoparticles are identical, one 417 gets

 $\frac{\frac{418}{419}}{\frac{420}{420}} \frac{d(E_{tot_1} + E_{tot_2})}{dt} = [(b_1 - b_2) + (a_1 + a_2)2Q_1]\frac{dQ_1}{dt}$



(8)

FIG. 7. Dependence of the Au nanoparticles capacitance on their radius.

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where $Q_1 = Q_1(t)$ is the charge of the first nanoparticle. Furthermore, by the virtue of the argument stated above, this charge of the nanoparticle will be denoted Q(t), and the derivative $\frac{dQ_1(t)}{dt}$ will be denoted as a current,

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This implies that the electron freely moves from one particle through the conducting molecule to the other particle. In reality, it is necessary to take into account and determine the transmission probability T as the probability for an electron to transmit through the bridge multiplied by the number of transverse modes, each of which transfers electron through the bridge. Using the definition data, the total energy of the two isolated nanoparticles in (8) can be rewritten as

$$\frac{d(E_{tot_1} + E_{tot_2})}{dt} = 4aQ(t)I(t).$$
(10)
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Unfortunately, in (5)-(10), it was assumed the two nanoparticles do not influence each other. In contrast, due to the electrostatic interaction their total energy is rewritten as

$$\frac{dE_{tot_{1-2}}}{dt} = 4aQ(t)I(t) - \frac{d}{dt} \left[F(R,L) \frac{Q(t)^2}{2R+L} \right]$$
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$$= Q(t)I(t)\left(4a - 2\frac{F(R,L)}{2R+L}\right),$$
(11)
⁴³⁸
⁴³⁹

where the second term on the right side equation corresponds to the 440 derivative of the electrostatic interaction of the two opposite charges 441 at a distance 2R + L between the NP centers. A correction function 442 F(R, L) is introduced to take into account the difference between the 443 interaction of the two conducting spheres (nanoparticles) and point 444 charges at the polarized spheres. The function F(R, L) was addressed in Refs. 57 and 58, where it was shown that this function decreases 446 rapidly from 2.0 when two conducted spheres are touching to 1.0 when $\frac{L}{2R} \rightarrow \infty$ and $F(R, L) \simeq 1.07$ at $\frac{L}{2R} = 1.4$.⁵⁸ Next, the force acting 447 448 on the conduction electron (hole) inside the conducting molecule is 449 obtained as

$$F(x,t) = -eE(x,t) - e\left[\vec{v}\vec{B}\right],$$
⁴⁵⁰

$$E(x,t) = -\nabla\phi(x,t) - \frac{\partial A}{\partial t}.$$
(12) 45

We recall that for one-dimensional carriers moving in the COM, the Lorentz magnetic force in the second right term of (12) should be neglected.

Assuming that the characteristic size \tilde{R} of the system is much 455 smaller than the wavelength of the electromagnetic wave or $\tilde{R} \ll cT$, 456 where the *T* is the period of EMF and *c* is the light velocity, the qua-457 458 sistatic approximation (QSA) may be used in the model. It means that EMF delay effects can be neglected. In QSA, the characteris-459 tic speed of electrons are much smaller than c ($v \ll c$), and the bias 460 current is much smaller than the conduction current: $\left|\frac{\partial D}{\partial t}\right| \ll |j|$ 461 in QSA $\Delta A(x, t) = -\frac{4\pi}{c}I(x, t)$, where I(x, t) is the current at coor-462 463 dinate x passing through the COM at the time t. By estimation, taking into account the speed of light in the atomic system of units 464 465 $C \simeq 137$ and the magnitude of the current $I(x,t) \approx \frac{e}{T} v_{Fermi}$, the

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466 second term in the right part of (12) can be neglected and the force 467 acting on the carriers can be defined as the gradient of the elec-468 trostatic potential $\nabla \phi(x, t)$ only. Furthermore, in QSA, the current reads

$$j = \sigma E, \operatorname{div}(j) \approx 0,$$
 (13)

470 and so it is possible to disregard the inhomogeneity of the current 471 inside the conductive molecule and I(x, t) = I(t).

472 Opposite to the Landauer approach, where the free electron 473 or hole dynamics inside the COM of NP-COM-NP system under 474 EMF is not analyzed, we have to do just that for building the plas-475 mon differential equation. As it was mentioned above, it is assumed 476 that this carrier dynamics is similar to the dynamics in periodi-477 cal (NP-COM) systems under the same EMF. Assuming that the 478 EMF is sufficiently small for Zener or avalanche breakdown, the 479 electrons (holes) will be accelerated according to the effective mass 480 tensor near the Fermi level. Due to the 1D nature of the car-481 rier movement, this tensor transforms to the carrier effective mass value

$$m^* = \hbar^2 \left[\frac{d^2 E}{dk^2} \right]_{k=k_f}^{-1}.$$
 (14)

484 In the following, we will build our model in relation to the peri-485 odic -[Au₁₄₇-S-C₈H₈-S]- system. According to Fig. 2, the system is metallic. The calculated effective mass value $m^* = -0.454 m_e$ was cal-486 487 culated for the system using (14). Due to the system symmetry and 488 the symmetry $(k \Rightarrow -k)$, the total current of the free electrons and holes near E_{fermi} is cancelled without any AC electromagnetic field. 489 In other words, the linear dependence of energy E(k) is insignificant 490 and the parabolic dependence 491

⁴⁹²
$$E(k) = \frac{(\hbar k)^2}{2m^*}$$
 (15)

493 should be used for the carriers. We recall that the COM stays neu-494 tral under plasmon generation; hence, the total number of carriers 495 inside the COM does not change under an applied weak EMF. The 496 COM total energy E_{tot_3} can be written via a sum of electrons in the 497 conduction band, having different quasimomentum (15),

 $E_{tot_3} = \sum_i n_i \frac{\hbar^2 k_i^2}{2m^*},$

where n_i -occupation degrees of electrons having quasimomentum 499 k_i .

500 Taking into account that under the influence of a weak electric 501 field, the carriers are excited only near the Fermi level, we can write 502 the energy derivative

$$\frac{dE_{tot_3}}{dt} = n_f \frac{\hbar k}{m^*} \left[\frac{d(\hbar k)}{dt} \right]_{k=k_f},$$
(16)

505 where n_f -occupation degrees of electrons having Fermi quasimomentum $k = k_f$. Due to spin degeneracy and generation of holes 506 507 with the same effective mass upon excitation of electrons near the 508 Fermi level, in the future, we will consider only electrons for which $n_f = 4.$

$$I(t) = \frac{-en_f}{L} \frac{1}{\hbar} \frac{\partial E(k)}{\partial k} = \frac{-en_f}{L} \frac{\hbar k}{m^*}|_{k=k_f},$$
 (17) 511

$$\frac{dI(t)}{dt} = \frac{-en_f}{Lm^*} \frac{d(\hbar k)}{dt} \Big|_{k=k_f},$$
(18) 512

where Lis the length of COM.

Combining (16)–(18), the E_{tot_3} derivative is equal to

$$\frac{dE_{tot_3}}{dt} = I(t)\frac{dI(t)}{dt}\frac{m^*L^2}{ne^2}.$$
 (19) 510

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Using (9), (11), and (19), Eq. (5) is transformed into

$$\frac{dE_{tot}}{dt} = Q(t)I(t)\left(4a - 2\frac{F(R,L)}{2R+L}\right) + I(t)\frac{d^2Q(t)}{dt^2}\frac{m^*L^2}{ne^2} = 0.$$
 (20) 519

Dividing by I(t), one can get a differential equation of the 520 harmonic oscillations having the square of modified plasmonic 521 frequency ω_{pl}^2 , 522

$$\frac{d^2 Q(t)}{dt^2} = -\widetilde{\omega_{pl}^2} Q(t), \qquad (21) \qquad 523$$

$$\widetilde{\omega_{pl}^2} = \left(\frac{1}{C} - \frac{F(R,L)}{2R+L}\right) \frac{2ne^2}{m^*L^2}.$$
(22) 524

Here, substitution C = 1/(2a) is used again. From this equation, 525 one can see that the square of modified plasmonic frequency 526 ω_{pl}^2 is similar with expression of conventional plasma frequency 527 $\omega_{pl}^2 = 4\pi n e^2 / (m^* \Omega)$, where *n* denotes the number of electrons occu-528 529 pying the volume Ω . Under the assumption of $R \gg L$, remembering that $C \cong R$ (NP radius) and replacing the expression in parentheses 530 of (22) by 1/R, one gets 531

$$\widetilde{\omega_{pl}^2} \approx \frac{2ne^2}{m^*L^2R} = \frac{4\pi ne^2}{m^*\widetilde{\Omega}},$$
(23)
⁵³²
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where $\widetilde{\Omega} = 2\pi RL^2$ —effective volume per *n* electrons. Comparing $\widetilde{\omega_{pl}^2}$ and ω_{pl}^2 , we can verify that the modified plasma frequency resides in the infrared (IR) region. For example, in the bulk gold, one electron has a volume of 16.85 $Å^3$, and the plasmonic frequency ω_{pl} is \approx 9.1 eV. Using the parameters of the investigated system con-**5**38 taining the nanoparticle of 147 gold atoms ($\langle R \rangle \cong 14.0$ Å, $L \cong 14.3$ Å), the estimation of the modified plasma frequency is $\widetilde{\omega_{pl}} \approx 0.35$ eV.

In order to show the influence of system geometric parameters (R, L) on the plasmon frequency, we plotted $\widetilde{\omega_{pl}}$ as a function of the nanoparticle radius R for three conducting molecules -[S- C_nH_n-S]- (n = 6,7,8), having different length L (see Fig. 8 and legend there). The electron effective mass was here taken again as $m^* = 0.454 m_e$.

One can see that in the proposed NP-COM-NP systems, the 547 plasmonic frequencies are expected to lie in the IR region and are 548 very significantly changed with a change of the system geometric 549 550 parameters.

Following Ref. 43 and Eqs. (2), (3), and (15), the current and its 509 derivative can be expressed as 510

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551 V. CONCLUSION

552 In this work, the possibility of generating specific plasmons 553 due to charge transfer in a couple of metal nanoparticles bridged 554 by conductive molecules is investigated. To describe the properties 555 of such plasmons, an original quantum-classical model was developed based on a description of the time dependence of the ballistic 556 current through the conductive bridge and which includes quan-557 tum effects. For the test system, consisting of two gold nanoparticles 558 559 bridged by the conjugated polyacetylene molecule C8H8 terminated 560 by sulfur atoms, an analytical expression for the frequency of the 561 plasmons was derived. Our approach can be qualified from the fact 562 that although approaches based on calculations of nonequilibrium 563 Green functions or real-time propagation of wave functions in prin-564 ciple can be applied to describe ballistic transport, their complexity 565 disallows the description of real sophisticated systems of the kind 566 considered here. The model proposed in this work resolves this 567 problem since it uses only band structure information that can be 568 easily obtained from the calculations of the corresponding period-569 ical system, whose unit cell consists of the connected nanoparticle 570 and the conductive molecule. It was shown that for the systems 571 under study, the plasmon frequency is determined by expression 572 (23), which is similar to the one for bulk materials. Herewith, the 573 effective density of the conduction electrons becomes, in the present case, significantly lower than the density of conduction electrons in 574 575 the bulk material. It results in shifting the modified plasmon fre-576 quency to the IR region. The strong dependence of the modified 577 plasmon frequency on the system conductivity makes it possible 578 to use such NP-COM-NP systems to build different chemical sen-579 sors which can be based on a change in the conductivity of the 580 conducting molecule during its chemical interaction with external 581 molecules. Such an interaction can thus significantly change the 582 conjugated character of the π -bonding in the molecule and, there-583 fore, its conductivity. The conductivity directly changes the effec-584 tive mass and, according to (22), the plasmon frequency, that can 585 be easily measured. We believe that this new type of plasmon can 586 have a unprecedented impact on the field of deep tissue chemical 587 sensing.

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