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OPTICAL PHYSICS

# Light interference in a hybrid-aligned nematic layer with nonordered surface disclination lines

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**The propagation of a laser beam through a hybrid-aligned nematic layer with a surface disclination line has been investigated. A model of the light interference has been developed to consider the scattering by the structural inhomogeneities. The analytical expression that includes the factor characterizing an exponential decrease in the light scattering has been obtained. The dependence of the intensity of light transmitted through the layer on the magnetic field has been measured. The dependence has been accompanied by the interference oscillations. The theoretical expression is consistent with the experiment, which confirms the correctness of the model concepts.** 

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## 1. INTRODUCTION

The optical characteristics and application of nematic liquid crystals (LCs) are usually investigated using a cell with a plane-parallel surface on which a uniform director alignment is specified. As a rule, the orientational conditions on both surfaces of such cells are symmetric [1–3]. Depending on the boundary conditions on the nematic surface, a uniform planar, homeotropic, or oblique alignment occurs in the bulk of an LC. Placing the cell between two polarizers and applying an electric or magnetic field perpendicular to its plane, one can observe the variation in the intensity of transmitted light accompanied by the interference minima and maxima. The effect is caused by the birefringence of the plane-polarized light in the nematic and the occurrence of a phase difference between the ordinary and extraordinary waves interfering in the analyzer [3]. In addition, there exist hybrid aligned nematic (HAN) cells [4,5], in which the director is aligned in the plane on one surface and homeotropically on the other. The HAN cells exhibit the interference light transmission due to a change in the orientational structure of an LC and its birefringence under the action of an external field. Later on, the structures with a degenerate planar orientation on one cell surface and a homeotropic orientation on the other, which were called hybrid aligned nematics with degenerate boundary conditions (HAND), were obtained [6], and their topological features were studied [7]. Recently, we have synthesized and examined the structures containing ensembles of domains with a radial LC director configuration on a polycarbonate (PC) film, which gradually transform to a uniform orientation determined by the surface disclination line (SDL) and the opposite surface in the bulk of the cell [8–12]. These structures, following the logic of the review, are hereinafter referred to as hybrid-aligned nematics with the surface disclination line (HANL). The proposed model of the interference of light propagating through such a structure is based on the superposition of the ordinary and extraordinary waves but without using the polarizers [12]. The model was confidently confirmed via experiment but only for an ensemble of domains with consistently aligned SDLs. In this case, the light transmittance in the minima and maxima successively occurring in the optical response to the increasing electric field remained unchanged. However, in many experiments carried out without using special SDL alignment techniques for preparing the HANL, the amplitude of oscillations in the interference response pattern increased with the field [9,11].

The aim of this study is to develop an interference model [12] with allowance for the scattering on inhomogeneities of the orientational structure of the HANL layer with the nonordered SDLs. To avoid the necessity for considering the HANL dielectric inhomogeneity, the magnetooptical methods are used.

## 2. MODEL

Let us consider the features of the orientational structure of a nematic molecular ensemble and its transformation in a HANL under the action of a magnetic field. Figure 1 shows an individual domain in the scheme with the light propagation and magnetic field directions perpendicular to the cell plane.

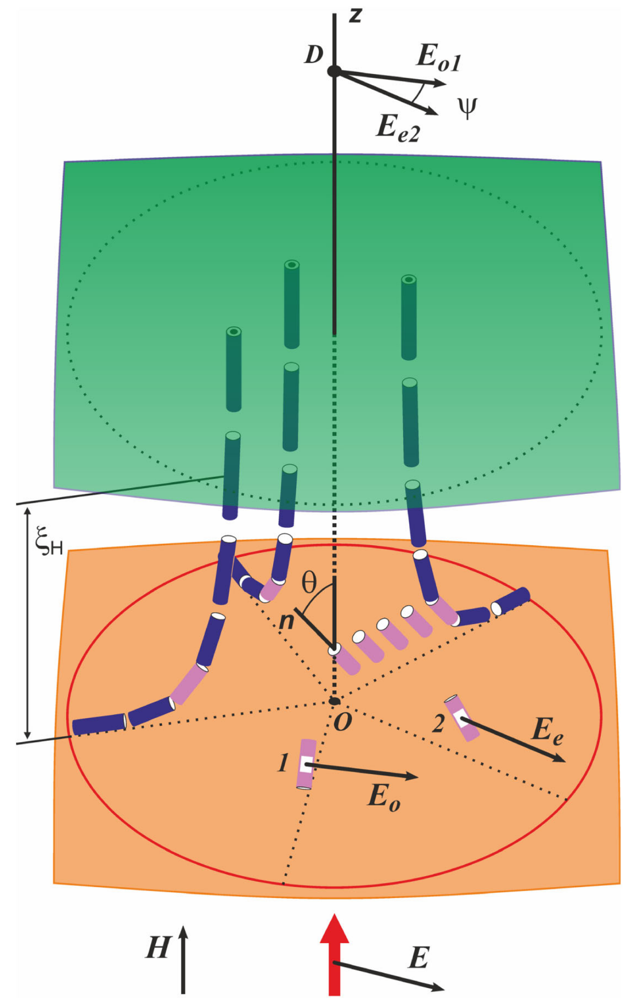


**Fig. 1.** HANL domain in magnetic field $H$. The light propagates along the $z$ axis. The SDL is not shown in the scheme.

We write the nematic free energy in the domain volume $V$ in the form [3]

$$F = \frac{1}{2}\int \left\{K\left[(\mathrm{div}\boldsymbol{n})^2 + (\mathrm{rot}\boldsymbol{n})^2\right] - \Delta\chi H^2\cos^2\theta\right\}\mathrm{d}V, \quad (1)$$

where the elasticity constant $K = (K_{11} + K_{22} + K_{33})/3$ is the average value of the splay, twist, and bent distortions, and the magnetic energy depends on the anisotropy of the magnetic susceptibility $\Delta\chi$ and angle $\theta$ between the nematic director $\boldsymbol{n}$ and field $H$. Presenting div $\boldsymbol{n}$ and rot $\boldsymbol{n}$ in the cylindrical coordinates $(\rho, \varphi, z)$ with the director components $n_\rho = -\sin\theta$, $n_\varphi = 0$, and $n_z = \cos\theta$ and using the Euler–Ostrogradsky equation for determining the minimum of free energy $F$, we obtain

$$\nabla^2\theta = \left(\frac{1}{\rho^2} + \frac{1}{\xi_{\mathrm{H}}^2}\right)\sin\theta\cos\theta, \quad (2)$$

where $\xi_H$ is the magnetic coherence length [3] and $\nabla^2\theta = (1/\rho)\partial/\partial\rho(\rho\partial\theta/\partial\rho) + (1/\rho^2)(\partial^2\theta/\partial\varphi^2) + \partial^2\theta/\partial z^2$ is the Laplacian. At $\partial\theta/\partial\rho = 0$, $(\partial^2\theta/\partial\varphi^2) = 0$ [13] and a fixed value of $\rho = r$, where $r$ is the domain radius, the solution of differential Eq. (2) yields

$$\left(\frac{1}{r} + \frac{1}{\xi_{\mathrm{H}}^2}\right)^{-1/2}\left(\frac{d\theta}{dz}\right)^2 = \sin^2\theta + C. \quad (3)$$

The integration constant $C$ is determined as follows: At any $H$ value, on the domain surface $z = 0$ [Fig. 2(a)], the strict planar boundary conditions $\theta = \pi/2$ are satisfied for a nematic with the azimuthally degenerate radial director alignment. This is due to the physicochemical interaction of LC molecules with polymer macromolecules during domain growth in the presence of a solvent [8,10,13]. In the zero external field, the director configuration in the bulk of a domain is determined by the competing effects of the radial structure and the SDL. On the coherence length $\xi_i$, the planar orientational structure gradually passes from radial to uniaxial aligned perpendicular to the SDL [Fig. 2(b)]. This configuration corresponds to a minimum of free energy [8]. If the LC cell thickness is sufficiently large [9–12], then, on the length $\xi_H$, the planar alignment induced by a magnetic field transforms to homeotropic one with $\theta = 0$ and $\partial\theta/\partial z = 0$ [Figs. 2(c) and 2(d)]. Hence, we have $C = 0$.

As we discussed previously [12], in any LC local region with director $\boldsymbol{n}$, the light is split into the mutually polarized ordinary and extraordinary waves with strengths $\boldsymbol{E_o}$ and $\boldsymbol{E_e}$ [14]. In a

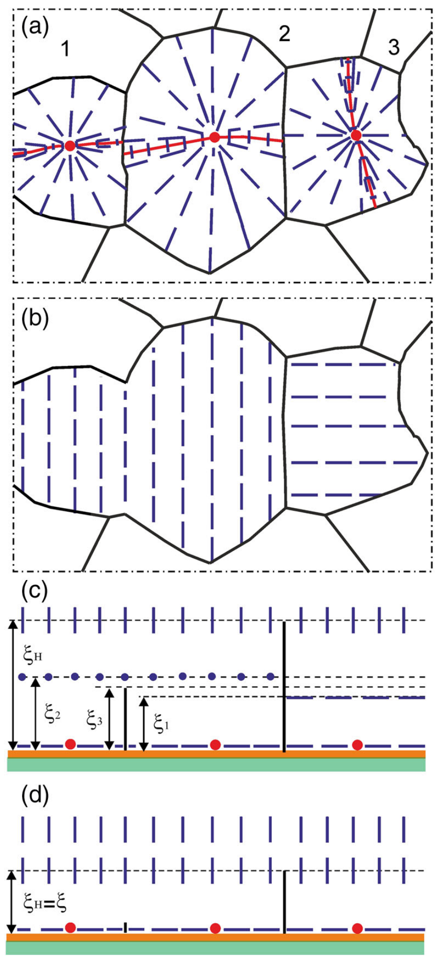


**Fig. 2.** HANL domain structure (a) on the PC surface; (b) in the cross section at the average coherence length $\xi = (\xi_1 + \xi_2 + \xi_3)/3$ for an ensemble of domains 1, 2, 3; (c), (d) in the cross section perpendicular to the PC film through the central defects at the magnetic coherence lengths $\xi_H > \xi$ and $\xi_H = \xi$, respectively.

HANL domain with the radial director structure (Fig. 1), the vectors $\boldsymbol{E}_{o1}$ and $\boldsymbol{E}_{e2}$ corresponding to areas 1 and 2 chosen in the plane $z = \text{const}$ make angle $\psi$. For the amplitudes of waves forming the superposition $\boldsymbol{E}_{12} = \boldsymbol{E}_{o1} + \boldsymbol{E}_{e2}$ in some remote point $D$, taking into account the phase difference $\delta_z$, we can write

$$E_{12}^2 = E_{o1}^2 + E_{e2}^2 + 2E_{o1}E_{e2}\cos\psi\cos\delta_z. \quad (4)$$

The propagation of the monochromatic light through the entire domain results in the occurrence of the phase difference $\delta$ between the waves. The equation for $\delta$ at the length $\xi_H$ can be obtained by the integration of the difference between the effective extraordinary refractive index $n_{\text{eff}}$ changing along $z$ and constant ordinary refractive index $n_o$ over $dz$:

$$\delta = \frac{2\pi}{\lambda}\int_0^{\xi_H}[n_{\text{eff}}(z) - n_o]\mathrm{d}z, \quad (5)$$

where $\lambda$ is the wavelength in vacuum. The index $n_{\text{eff}}(z)$ is independent of the director alignment in the plane perpendicular to the $z$ axis and only determined by the slope angle $\theta$

$$n_{\text{eff}} = n_o(1 - \nu\sin^2\theta)^{-1/2}, \quad (6)$$

where $\nu = (n_e^2 - n_o^2)/n_e^2$.

It is worth noting that Eq. (3), relating the orientational structure of a nematic with the domain morphology, is strictly valid only for the model with a cylindrical object. In such a model, the general expression for the phase difference was derived considering the average domain radius [12]. In practice, the experimental samples we investigated are ensembles of domains based on a polygonal grid formed spontaneously on the PC surface [Fig. 2(a)]. The object can be described more correctly if the model uses the coherence length $\xi_i$ of the radial structure in each separate domain, rather than the radius. Considering the spread of the morphological parameters over the ensemble, we introduce the average coherence length $\xi = \langle\xi_i\rangle$. Then, substituting $dz$ and $n_{\text{eff}}$ from Eqs. (3) and (6) into Eq. (5), we arrive at the equation for the phase difference

$$\delta = \frac{2\pi n_o}{\lambda(A^2/\xi^2 + 1/\xi_{\text{H}}{}^2)^{1/2}}\int_0^{\pi/2}\left[(1 - \nu\sin^2\theta)^{-1/2} - 1\right]\frac{\mathrm{d}\theta}{\sin\theta}, \quad (7)$$

where $A = [(\pi^3/12 + \pi^2/4 + 1)/2\pi\ln(l/b)]^{1/2}$ was obtained from the equation derived for $\xi$ [13]. Here, $l = \langle 2r\rangle$ is the average SDL length and $b$ is the width.

Let us consider the contribution of the effects caused by the mutual SDL orientations in the ensemble to the overall picture of the optical transmittance. Figure 2 illustrates the transformations of the director field in the hybrid structured layers with the identical (1 and 2) and different (2 and 3) SDL directions. In the first case, in the zero external field, in the bulk of the HANL layer, at the distance $\xi$, a uniform planar structure is formed with the director aligned perpendicular to the SDL preferred direction. The transformation of the director in such a layer to the predominantly homeotropic orientation under the action of the field should not be accompanied by the occurrence of defects that cause the strong scattering of the transmitted light. The expression we obtained in [12] for the intensity of light transmitted through the HANL corresponds exactly to this case. The scattering by stationary defects and surface roughness, as well as the loss to the reflection at the boundaries of cell elements, was considered by simply introducing a correction term and a linear coefficient into the calculation formula. In the second case, a picture should be different. The isolated character of the orientational transformations in each domain inevitably leads to the optical inhomogeneity of the layer. The efficiency of light scattering caused by this inhomogeneity should exponentially decrease with decreasing $\xi_H$ value as the effective field increases. The scattering is caused by the spatial inhomogeneity of the LC optical anisotropy $\Delta n = n_e - n_o$ [1]. The validity of our consideration of this scattering type was based on the results of experimental observations of the growth of a domain ensemble on a PC film, which was accompanied by the elimination of the schlieren or threadlike texture (8–10, 12, 14). Therefore, in the expression for the transmitted light intensity $I = \sum\langle E_{12}^2\rangle$, the interference term $J_{12} = 2\sum\langle\boldsymbol{E}_{o1}\cdot\boldsymbol{E}_{e2}\rangle$ [14] should contain the factor $\exp(-\xi_H/\xi)$. After averaging Eq. (4), we obtain

$$I = \frac{1}{2}I_0\left[1 + \frac{2}{\pi}\exp(-\xi_{\text{H}}/\xi)\cos\delta\right], \quad (8)$$

where $I_0$ is the intensity of the light transmitted through the cell with a uniform homeotropic nematic layer, which is lower than the intensity of the incident light. This approach allows us to ignore the stationary effects that are nonessential in the proposed optical transmission model. The light scattering dependent on the energy of LC anchoring to the surface [15,16] is not considered due to the strict boundary conditions for a nematic on the PC film [13]. The molecular scattering characterized by the mean free path of a photon through an LC layer [17] is considered to be small as compared with the scattering by structural defects [1].

## 3. EXPERIMENTAL

The LC cells for the magnetooptical measurements were assembled from two plane-parallel glass plates with the indium tin oxide (ITO) coating. A PC was deposited onto one of the plates from a 2-% solution in $CH_2Cl_2$ by centrifuging. The other plate was coated with a lecithin film from a 4-% alcohol solution. A gap was formed using 30 µm thick Teflon spacers. A flat capillary of the cell was filled with a 4-n-pentyl-4′-cyanobiphenyl (5CB) LC. Within approximately 20 min, a domain ensemble was formed on the PC film [8,10]. The orientational structure of the nematic in the cell was studied on a polarizing microscope. The region for probing by a laser beam was bounded by a 1 mm diaphragm. The sample was mounted in a thermostatic cuvette between the electromagnet poles so that the field lines were perpendicular to the glass plates. In the same direction, along the holes on the axial line of the electromagnet core, a laser beam with $\lambda = 633$ nm propagated through the sample. The intensity $I$ of the light transmitted in the forward direction was detected as a function of the dc magnetic field $H$. With a slow increase, a maximum value of 20 kOe was obtained. This value is not enough to achieve homeotropic orientation in the LC layer [9]. Therefore, an electric field was specially used to determine the intensity $I_0$. In this case, an ac (1 kHz) electric voltage of 80 V was applied to the ITO electrodes to induce the saturation of optical transmission corresponding to the homeotropic state [12]. All the procedures were performed at a temperature of 23°C.

## 4. RESULT AND DISCUSSION

Figure 3(a) shows a HANL fragment studied in the crossed polarizers. Analysis of the texture did not reveal a preferred direction in the orientation structure of the nematic. Dark lines corresponding to disclinations on the PC surface are randomly oriented in the ensemble. In each domain, two such lines extend from the center of the radial structure to its bound. We take the sum of the measured lengths of both lines as the length of the SLD. The distribution of SLD lengths in the ensemble of 49 domains is shown in the histogram [Fig. 3(b)]. The average SDL length and width were assumed to be $l = 100$ µm and $b = 10$ µm, respectively.

The light transmission characteristics measured and calculated using Eq. (8) are presented in Fig. 4. The calculation of the theoretical curve in comparison with the experimental one was made starting with a threshold value of $H_{th} = 1.57$ kOe at which the magnetic coherence length $\xi_H = (1/H)(K/\Delta\chi)^{1/2} = 16.6$ µm and the coherence length $\xi = l[(\pi^3/12 + \pi^2/4 + 1)/8\pi \ln(l/b)]^{1/2} = 16.6$ µm are equal. The phase difference $\delta$ from Eq. (8), which was calculated by substituting the materials constants into Eq. (7), is shown in the inset. We used literature values of $n_o = 1.5271$, $n_e = 1.7103$ [18], $K = 6.21 \cdot 10^{-7}$ dyn, and $\Delta\chi = 0.97 \cdot 10^{-7}$ [19], which correspond to a temperature of 23°C. The $\delta$ decrease inversely proportional to $H$ corresponds to approaching of the $n_{eff}$ and $n_o$ values upon detuning the orientation structure to the uniformly homeotropic one. At the sequential attaining the $\delta(H)$ values multiple of $\pi$, interference minima and maxima arise in the $I(H)$ curves. The increasing range of the deviation of the extremum light transmittances from $I_0/2 = 0.5$ is caused by an exponential decrease in the light scattering by orientation defects in the HANL. There is good agreement between the theoretical and experimental curves of light transmission despite a wide deviation of $l$ from the average value. We verified that only at $l < 50$ µm a significant shift of the first extremum is observed. That explains using rounded values $l = 100$ µm. It should be noted that Eq. (8) describes a particular case of a domain ensemble. In general, it can be supplemented by a coefficient in the exponential factor that considers the relative SDL orientation.

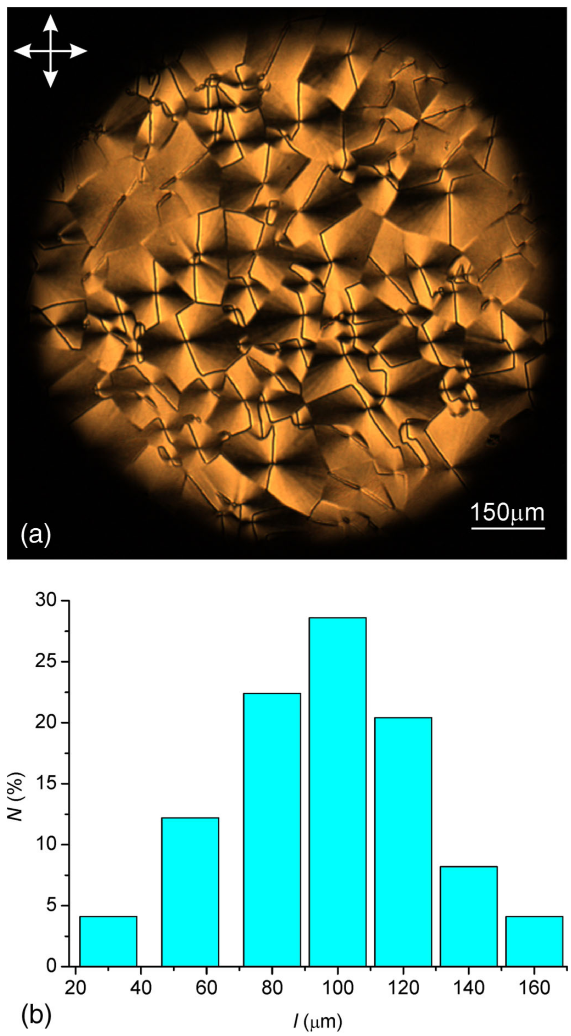


**Fig. 3.** (a) Microphotograph of the HANL fragment in crossed polarizers (shown by arrows) and (b) statistical distribution of the SDL lengths in the ensemble.

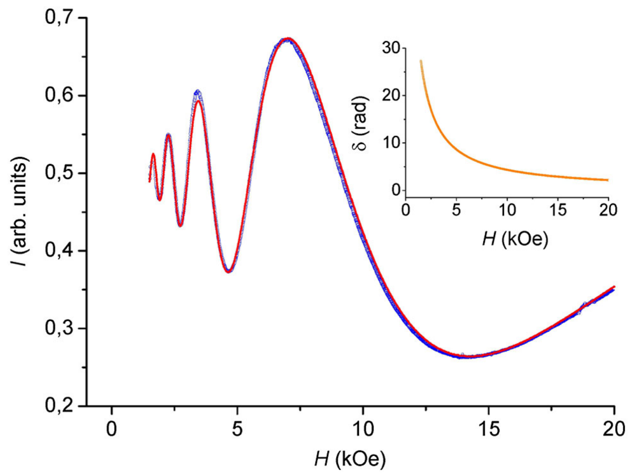


**Fig. 4.** Experimental (circles) and theoretical (solid line) dependences of intensity $I$ of light propagating through the HANL on magnetic field $H$. Inset: Phase difference $\delta$ between the extraordinary and ordinary waves as a function of field $H$.

## 5. CONCLUSIONS

Thus, we investigated the propagation of the laser radiation through a hybrid-aligned nematic with the surface disclination lines, which we called the HANL. It consists of domains with a radial configuration of the nematic director on a PC, which gradually transforms into a uniaxially oriented configuration on the coherence length $\xi$. The previously proposed light interference model based on the superposition of the ordinary and extraordinary waves was extended to the case of the nonordered disclinations. The analytical expression was obtained that relates the intensity $I$ of the transmitted light, depending on the magnetic field $H$, to the structural parameters of the HANL. A factor characterizing the exponential decrease in the light scattering with a decrease in the magnetic coherence length $\xi_H$ relative to $\xi$ was added to the interference term. The scattering is caused by the nonuniformity of the HANL orientational structure and the LC optical anisotropy. The consistent experimental and theoretical curves $I(H)$ confirm the correctness of the model concepts.

**Disclosures.** The authors declare no conflicts of interest.

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