UDC 539.22
Doi: 10.31772/2587-6066-2020-21-3-433-440

For citation: Krakhalev M. N., Shabanov V. F., Zyryanov V. Ya. Point defects in nematic liquid crystal materials with conical anchoring at the interface. *Siberian Journal of Science and Technology.* 2020, Vol. 21, No. 3, P. 433–440. Doi: 10.31772/2587-6066-2020-21-3-433-440

Для цитирования: Крахалев М. Н., Шабанов В. Ф., Зырянов В. Я. Точечные дефекты в нематических жидкокристаллических материалах с коническим сцеплением на границе раздела // Сибирский журнал науки и технологий. 2020. Т. 21, № 3. С. 433–440. Doi: 10.31772/2587-6066-2020-21-3-433-440

POINT DEFECTS IN NEMATIC LIQUID CRYSTAL MATERIALS WITH CONICAL ANCHORING AT THE INTERFACE

M. N. Krakhalev[1,2], V. F. Shabanov[3], V. Ya. Zyryanov[1*]

[1]Kirensky Institute of Physics, Federal Research Center KSC SB RAS
50/38, Akademgorodok, Krasnoyarsk, 660036, Russian Federation
[2]Institute of Engineering Physics and Radio Electronics, Siberian Federal University
79, Svobodny Av., Krasnoyarsk, 660041, Russian Federation
[3]Federal Research Center “Krasnoyarsk Science Center of the Siberian Branch of the Russian Academy of Sciences”
50, Akademgorodok St., Krasnoyarsk, 660036, Russian Federation
E-mail address: zyr@iph.krasn.ru

The topological point defects in nematic liquid crystal materials have been studied. The method of oblique light incidence has been proposed to determine an azimuthal director angle of an achiral nematic as well as a chiral nematic (cholesteric). The idea of the method is based on the dependence of the optical phase difference between ordinary and extraordinary light beams on the azimuthal director angle at the layer center at oblique incidence of light on a structure in which the polar director angle of a nematic liquid crystal is not equal to 0° or 90° (conical boundary conditions). It has been shown that the phase difference reaches a maximum at a zero azimuthal angle at the center of the layer regardless of the total twist angle of the director. The developed method has been used to analyze topological defects formed in the nematic and cholesteric layers under conical boundary conditions at the interface. The director field distributions of nematic and cholesteric near the surface point defects (boojums) with topological charges m = +1 and m = –1 have been drawn based on the experimental data. The proposed method of oblique light incidence can be used to analyze a wide class of the achiral and chiral liquid crystal media of various types: smectics, nematics, and cholesterics with tilted or hybrid boundary conditions.

Keywords: topological defect, orientational structure, nematic liquid crystal, optical phase difference.

ТОЧЕЧНЫЕ ДЕФЕКТЫ В НЕМАТИЧЕСКИХ ЖИДКОКРИСТАЛЛИЧЕСКИХ МАТЕРИАЛАХ С КОНИЧЕСКИМ СЦЕПЛЕНИЕМ НА ГРАНИЦЕ РАЗДЕЛА

М. Н. Крахалев[1,2], В. Ф. Шабанов[3], В. Я. Зырянов[1*]

[1] Институт физики им. Л. В. Киренского Сибирского отделения Российской академии наук – обособленное подразделение ФИЦ КНЦ СО РАН
Российская Федерация, 660036, г. Красноярск, Академгородок, 50/38
[2]Сибирский федеральный университет, Институт инженерной физики и радиоэлектроники
Российская Федерация, 660041, г. Красноярск, просп. Свободный, 79
[3]Федеральный исследовательский центр «Красноярский научный центр СО РАН»
Российская Федерация, 660036, г. Красноярск, Академгородок, 50
E-mail address: zyr@iph.krasn.ru

Исследованы топологические точечные дефекты в нематических жидкокристаллических материалах. Предложен метод наклонного падения света, позволяющий определять азимутальный угол директора ахирального нематика, а также закрученного нематика (холестерика). Суть метода состоит в том, что при наклонном падении света на структуру с отличным от 0° и 90° полярным углом директора нематического жидкого кристалла (конические граничные условия) оптическая разность фаз, возникающая между обыкновенным и необыкновенным лучами, определяется величиной азимутального угла директора в центре слоя. Показано, что максимальное значение разности фаз достигается при нулевом азимутальном угле в центре слоя независимо от полного угла закрутки директора. Разработанный метод был использован для анализа топологических дефектов, формирующихся в слоях нематика и холестерика с коническими граничными усло-

виями на межфазной границе. На основании полученных экспериментальных данных были построены распределения поля директора нематика и холестерика вблизи поверхностных точечных дефектов (буджумов) с топологическими зарядами m = +1 и m = –1. Полученные результаты интересны для исследований структурированных материалов, анализа оптическими методами дефектов структур, а предложенный метод наклонного падения света может использоваться для анализа широкого класса ахиральных и хиральных жидкокристаллических сред различного типа: смектиков, нематиков и холестериков с наклонными или гибридными граничными условиями.

Ключевые слова: топологический дефект, ориентационная структура, нематический жидкий кристалл, оптическая разность фаз.

Introduction. Liquid crystals (LC) are anisotropic liquids with long-range orientational order of molecules. The orientation of LC molecules is characterized by the director ***n***, which is a unit vector oriented along the preferred orientation of the long axes of the molecules [1]. A rich variety of forming configurations of the director's field is possible in an LC, depending on the boundary conditions (preferred orientation of the director at the interface), material parameters of the LC (elasticity constants, helix pitch), external electric or magnetic fields [2]. In this case, defects can be formed in the system, where significant distortions of the director field are observed. And vice versa, upon induction of strong distortions in the volume, for example, by magnetic (electric) fields or introduction of microparticles into the LC, topological defects are formed. The resulting distortions of the director field facilitate interaction between defects and particles which can be found in the effects of self-ordering of defects [3–5], colloidal [6; 7] and magnetic [8] particles, makes it possible to control the position of microparticles [9], and determines the group motion of defects in external electric field [10]. Thus, topological defects are an important feature that significantly affects the main characteristics of liquid crystal materials (optical, electro-optical, dynamic) which determine their prospects for various applications in modern information technologies.

Point bulk (hedgehogs) [11; 12] and surface (boojums) [13] defects, linear defects [14] and two-dimensional defects (walls) [2] can be formed in nematic LC. In the case of twisted nematic with the ratio $d / p \approx 1$ of the layer thickness d and the helix pitch p, soliton-like structures [15] or linear defects [16] can be formed in the system. The possibility of formation and stability of various types of defects is determined by the boundary conditions specified on the substrates. Thus, under degenerate tangential (planar) boundary conditions (the angle between the director and the normal to the surface is 90°), a schlieren texture is formed in the LC cell which consists of point and linear defects, the strength of which depends on the ratio of the LC elastic constants [17]. In the case of homeotropic anchoring (the angle between the director and the normal to the surface is 0°), there are initially no defects in the system, but the application of an electric field or a combination of electric and magnetic fields promote the formation of an ordered set of point defects [3], two-dimensional defects [2], or soliton-like structures in twisted nematic [15]. Under homeo-planar boundary conditions, the formation of point defects on a substrate with planar anchoring or a structure of domains with closed linear defects is possible in the system.

Conical boundary conditions (the director is oriented to the surface normal at an angle other than 0° and 90°) are more suitable for the occurrence of defects. For example, in a cell with a conical anchoring, point, linear, and two-dimensional defects are formed even when there are no external fields [18]. In the case of a twisted nematic with planar-conical anchoring, linear defects are formed [19] or domains bounded by a closed linear defect with a pair of point singularities [20]. At the same time, point defects in LC systems with conical anchoring have not been practically investigated to this date.

This work presents for the first time the results of studying the structures of point defects formed in nematic and twisted nematic under conical boundary conditions on both substrates.

Materials and methods. The studies were carried out for a nematic LN-396 mixture (Belarusian State Technological University) and LN-396 mixture doped with a left-handed chiral addition of cholesteryl acetate (Sigma Aldrich). The concentration of cholesteryl acetate was 0.2 % which determines the pitch of the cholesteric helix with a value of $p = 72.5$ μm [19]. The studies were carried out in LC cells consisting of two glass substrates assembled in such a way that a gap is formed between them which is filled with a liquid crystal (fig. 1, *a*). Glass substrates were coated by centrifugation with a poly(isobutyl methacrylate) film (Sigma Aldrich) which sets conical boundary conditions for the used nematic LN-396 with a polar angle $\theta_{d/2} = 40°$ [21]. The thickness d of the LC layer was set with glass balls and was measured by the interference method until the cell was filled with a liquid crystal. LC cells were filled at room temperature and kept for at least 24 hours before to the study. The studies were carried out using a polarizing microscope (POM) AxioImager.M1m (CarlZeiss) with a 20x / 0.22 long-focus objective in monochromatic light with a wavelength of $\lambda = 546$ nm. The refractive indices of LC LN-396 $n_\perp = 1.528$, $n_\parallel = 1.741$ ($\lambda = 546$ nm).

Oblique incidence light method. It is convenient to describe the distribution of the director field in the cell using the polar θ (x, y, z) and azimuthal φ (x, y, z) angles (fig. 1, *b*). In this case, the ***n***-director is described by the following equation:

$$\begin{cases} n_x = \sin\left(\theta[x,y,z]\right)\cos\left(\varphi[x,y,z]\right), \\ n_y = \sin\left(\theta[x,y,z]\right)\sin\left(\varphi[x,y,z]\right), \\ n_z = \cos\left(\theta[x,y,z]\right). \end{cases} \quad (1)$$

Generally, the angles θ and φ can vary both over the thickness of the LC layer and in the plane of substrates.

This is more typical for the LC areas near defects, and the nature of the change in the angles is determined by the type of defect. Conversely, the nature of the change in the angles in the vicinity of the defect can be used to determine its parameters. In the case of point defects in nematics, the method of fluorescence microscopy is used [22]. In the case of an tilted orientation of the director in the bulk which is realized under conical boundary conditions, it is convenient to use the method of oblique incidence of light proposed and described below in order to characterize defects.

Consider a twisted nematic away from defects. In this case, we can assume that the polar angle of the director is constant and equal to the tilt angle of the director on substrates $\theta(x, y, z) = \theta_{d/2} = \theta_{-d/2}$, and the azimuthal angle $\varphi(x, y)$ depends on the coordinate z linearly:

$$\varphi(x,y,z)=\varphi_0(x,y)+\frac{\varphi_{\mathrm{TOTAL}}}{d}z, \qquad (2)$$

where $\varphi_0(x, y)$ – azimuthal angle of the director at the center of the layer (at $z = 0$), φ_{TOTAL} is the difference between the azimuthal angles of the director on the upper ($z = +d/2$) and lower ($z = -d/2$) substrates (the total azimuthal angle of rotation of the director on the layer d-thickness). Consider the oblique incidence of the light beam on the LC layer in the xOz plane (fig. 1, b). The direction of the beam will be characterized by the unit vector $\boldsymbol{k}^0(\pm\sin\alpha, 0, \cos\alpha)$, where the plus sign corresponds to the zero azimuthal angle for the vector $\boldsymbol{k}^0$, the minus sign corresponds to the value of the azimuthal angle of the vector $\boldsymbol{k}^0$ equal to π. When Mauguin condition [23] is fulfilled, we can assume that two linear waves propagate in such a twisted structure, the phase difference between which in the case of oblique light incidence at an angle α is determined as:

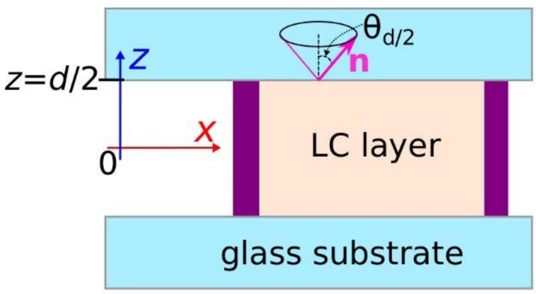
$$\delta=\frac{2\pi d}{\lambda\cos\alpha}\left(\int_{-1/2}^{1/2}\frac{n_\perp n_\parallel}{\sqrt{n_\perp^2+\left(n_\parallel^2-n_\perp^2\right)\cos^2(\theta')}}dz'-n_\perp\right), \qquad (3)$$

where $z' = z/d$, $\cos(\theta')$ equals:

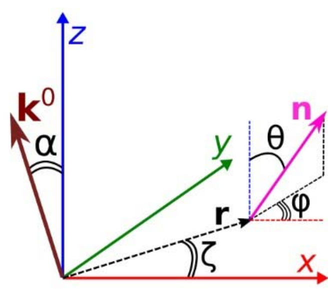
$$\cos(\theta')=\boldsymbol{nk^0}=\pm\sin\alpha\sin\left(\theta_{d/2}\right)\cos\left(\varphi_0(x,y)+\varphi_{\mathrm{TOTAL}}z'\right)+$$
$$+\cos\alpha\cos\left(\theta_{d/2}\right). \qquad (4)$$

Fig. 2 shows the dependences on the angle $\varphi_0(x, y)$ of the δ/d values calculated by equation (3) for several values of the total twist angle φ_{TOTAL} at an oblique incidence at an angle $\alpha = 5.33°$ (for the case shown in fig. 1, b), and polar angle $\theta_{d/2} = 40°$. It can be seen that one maximum is observed in the dependences only at the angle $\varphi_0(x, y) = 0°$, and an increase in the angle φ_{TOTAL} leads to a slight decrease in the difference $(\delta_{max} - \delta_{min})/d$ from a value of approximately 0.43 rad/μm to 0.27 rad/μm for angles φ_{TOTAL} 0° and 180 °, respectively. Such a difference in the phase differences observed for different azimuthal angles $\varphi_0(x, y)$ can be easily fixed for the commonly used LC cell thicknesses of about 10 μm. At the same time, the angle α used for calculation makes it easy to carry out observations with a polarizing microscope using a long-focus lens. The data presented in fig. 2 were obtained for positive angles φ_{TOTAL} (right-handed cholesteric), while due to the symmetry of the structure relative to the center of the layer, identical results are obtained for negative angles φ_{TOTAL} (left-handed cholesteric).

The above results were obtained from the assumption that the polar angle of the θ-director is constant and there is a simple linear dependence of the azimuthal angle φ on the coordinate z. In general, these conditions may not be met. For example, in a twisted nematic structure with significant pretilt angles of the director on substrates, the polar angle θ begins to depend on the z coordinate, while the dependence of the azimuthal angle of the director on the z coordinate deviates from the linear law [24]. Nevertheless, the results obtained for simple cases show that the maximum value of δ is reached at $\varphi_0(x, y) = 0°$ regardless of the values of the angle $\theta_{d/2}$ and φ_{TOTAL}.



a



b

Fig. 1. Scheme of LC cell (a) and the coordinate system used to calculate the optical phase difference at oblique light incidence and the director field around the boojums (b)

Рис. 1. Схематическое представление ЖК ячейки (a) и система координат, используемая для расчетов оптической разности фаз в случае наклонного падения света и поля директора вокруг буджумов (b)

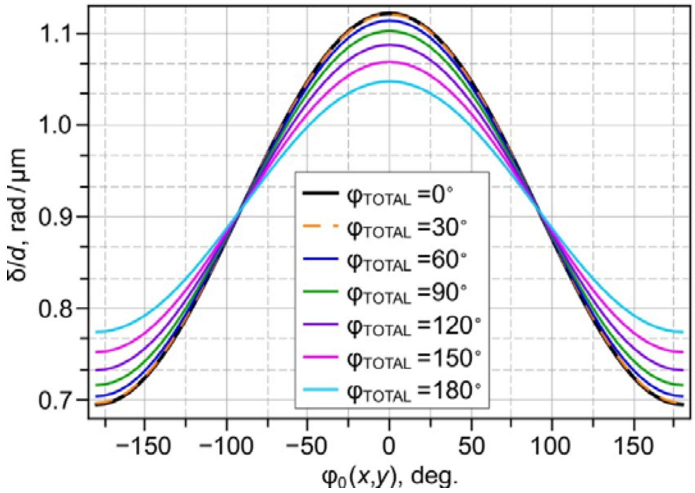


Fig. 2. Dependences of the ratio δ/d on the angle $\varphi_0(x, y)$ calculated for some twist angles φ_{TOTAL}. The data have been calculated for the angles $\alpha = 5.33°$, $\theta(x, y, z) = \theta_{d/2} = 40°$, the wavelength $\lambda = 546$ nm and the refractive indices corresponding to LC LN-396

Рис. 2. Зависимости отношения δ/d от угла $\varphi_0(x, y)$, рассчитанные для нескольких углов закрутки φ_{TOTAL}. Расчет сделан для углов $\alpha = 5{,}33°$, $\theta(x, y, z) = \theta_{d/2} = 40°$, длины волны света $\lambda = 546$ нм и показателей преломления, соответствующих ЖК ЛН-396

Thus, this conclusion can be extended to the case of an inhomogeneous slope and an uneven twist angle in the case when the director distribution in the bulk is symmetric about the center of the layer. It is this situation that is realized in LC cells under the same boundary conditions in the absence of external influences (for example, an electric field). Further, this method will be applied to analyze boojums that form in nematic and twisted nematic under conical boundary conditions.

Boojums in nematic. In the absence of a given orientation of the director on the substrates (degenerate tangential or conical boundary conditions), point defects on the surface (boojums) are formed in the system. Near boojums, the orientation of the director on the surface (plane $z = d/2$) can be described as [25]

$$\begin{cases} n_x = \sin\left(\theta_{d/2}\right)\cos\left(m\zeta + \xi\right), \\ n_y = \sin\left(\theta_{d/2}\right)\sin\left(m\zeta + \xi\right), \\ n_z = \cos\left(\theta_{d/2}\right). \end{cases} \tag{5}$$

where ζ is the angle in the polar coordinate system (fig. 1, *b*), m is the surface topological charge (strength) of the defect, $-\pi \le \xi < \pi$ is the twist angle of the boojum. The strength of the defect m and the angle ξ are used to classify boojums (as well as topological defects of other types), and, accordingly, the determination of these two parameters is sufficient to characterize the properties of topological defects. The LC cell has two substrates and the appearance of a boojum on one of the substrates (for example, the upper one at $z = d/2$) should be accompanied by the formation of a point defect on the second (lower one at $z = -d/2$) substrate. The identical distribution of the director near the boojums on the upper and lower substrates (escaped state) will correspond to the minimum of elastic energy. To determine the charge m, the sample is observed in the geometry of crossed linear polarizers (LP). In this case, extinction lines emerge from the defect, the number of which N is related to the charge of the defect by a simple relationship: $N = |m|\cdot 4$ (fig. 3, *a*). The m sign can be determined by rotating the polarizers relative to the sample or by observing in circular polarizers (CP). In the case of boojums with $m = +1$, the interference pattern observed in the geometry of circular polarizers near defects will have radial symmetry, while for boojums with $m = -1$, due to the inequality of splay, band, and twist LC elastic constants, the interference pattern will have lower symmetry (fig. 3, *b*). In the case of a nematic, the total twist angle of the director is $\varphi_{TOTAL} = 0$; accordingly, the azimuthal angle of the director turns out to be independent of the coordinate z and is equal to $\varphi_0(x, y)$. Since in the vicinity of boojums the azimuthal angle of the director varies in the range $-180° \le \varphi(x, y) = m\zeta + \xi \le 180°$, the twist angle ξ can be determined using the method of oblique incidence of light. In this case, when implementing the scheme shown in fig. 2 δ_{max} is observed under the condition $(m\zeta + \xi) = 0$, and δ_{min} corresponds to the condition $(m\zeta + \xi) = \pi$ $(-\pi)$. It is convenient to use circular polarizers for this method.

The fig. 3 shows boojums with $|m| = 1$ which can be seen from observations in the geometry of crossed linear polarizers. When observed in the geometry of crossed circular polarizers, the observed interference pattern is determined only by the phase difference between the ordinary and extraordinary waves. It can be seen that the interference pattern in the vicinity of the boojums corresponds to the case when the phase difference increases from zero to $\delta > 4\pi$ as we move from the boojum to the periphery, which fits well with the parameters of the LC cell with the LC layer thickness d = 14.6 μm and the director tilt angle $\theta_{d/2}$ = 40°, for which the value δ = 4.17π is obtained from equation (3) at α = 0° and φ_{TOTAL} = 0. This corresponds to the situation described above, when the boojums on the upper and lower substrates are located opposite one another (escaped state). In the geometry of crossed linear polarizers from the boojum with m = +1 (fig. 3, the first row) four extinction bands emerge parallel to the polarizers. When observed in circular polarizers, the interference pattern is symmetric in the case of normal incidence of light (α = 0°) (fig. 3, *b*, the first row). In the case of oblique incidence of light at an angle α = 5.33°, the symmetry of the interference pattern is broken (fig. 3, *c*, the first row). Based on the change in the brightness of the observed interference pattern, it can be concluded that the maximum phase difference δ_{max} corresponds to the extinction line oriented at an angle ζ = –90°, and δ_{min} corresponds to the value of the angle ζ = +90°. This means that the twist angle of the boojum on the upper substrate ξ = 90° (fig. 3, *d*, the first row). Similarly, we can determine the twist angle of a boojum with a charge m = –1 (fig. 3, the second row). In the geometry of crossed linear polarizers (at α = 0°), the extinction bands form a straight cross oriented at an angle of 15° to the direction of the linear polarizer LP. From the interference pattern observed at an oblique (α = 5.33°) incidence of light in the geometry of circular polarizers, it can be seen that δ_{max} corresponds to an extinction band oriented at an angle ζ = –75° which corresponds to a boojum twist angle on the upper substrate ξ = –75° (fig. 3, *d*, the second row).

It should be noted that the sample contains boojums with the charge m = +1 and the twisting angle ξ close to ± 90°. Thus, the boojum with m = +1 and |ξ| = 90° is energetically most favorable for LC LN-396 in the case of the director tilt angle $\theta_{d/2}$ = 40°. For boojums with a charge m = –1, different twist angles ξ are observed. This is a consequence of the fact that the distribution of the director field near the boojum with m = –1 at different angles ξ differs only in the orientation of the symmetry axes, i. e., the boojum with the angle ξ can be obtained by rotating the entire structure (coordinate system) of the director field with ξ_0 = 0 around the Oz axis by an angle ξ/2 [25]. Thus, the total free energy of the nematic director field near the boojums with m = –1, in contrast to the boojums with m = +1, does not depend on the twisting angle ξ.

Boojums in twisted nematic (cholesteric). The cholesteric structures were studied for a cell with a thickness of d = 12.7 μm which corresponds to the ratio d/p = 0.18. In this case, extinction bands are observed for an angle of 58° between the analyzer and the polarizer which corresponds to the director twisting angle φ_{TOTAL} = –32 ° (fig. 4, *a* and 5, *a*).

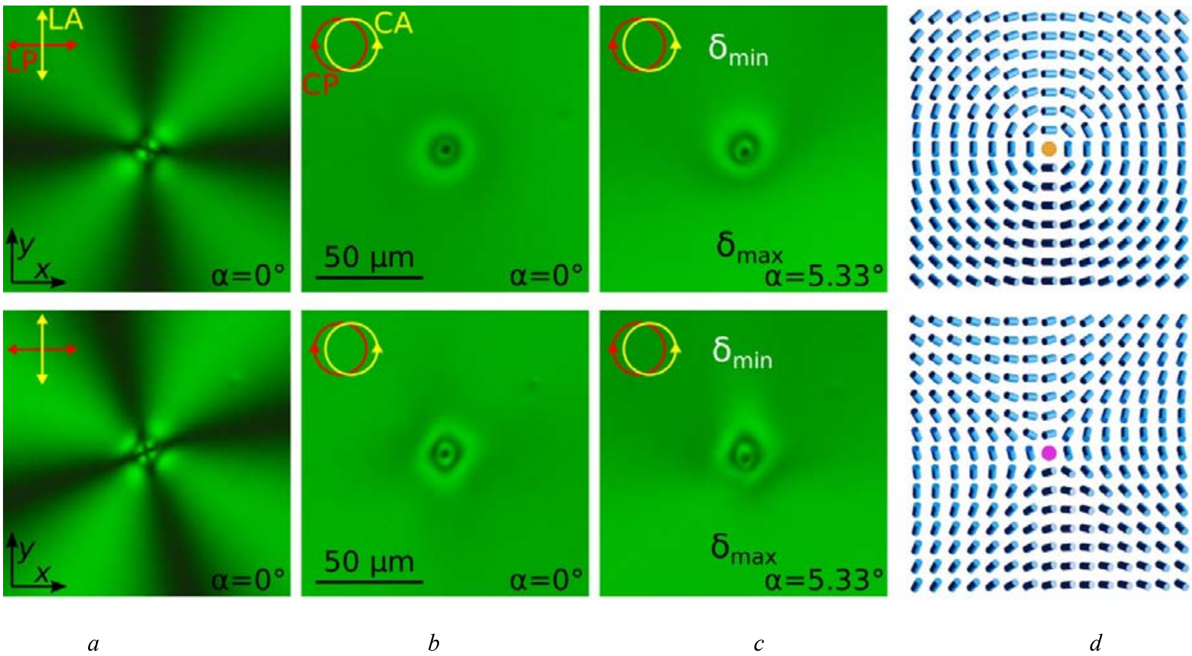


Fig. 3. POM photos of the sample areas of nematic with boojums of m = +1 strength and the twisting angle ξ = 90° (the first row), m = –1 and ξ = –75° (the second row). The photos were taken in the monochromatic light (λ = 546 nm) for the crossed linear polarizer (LP) and analyzer (LA) at the polarizer orientation LP 0° (*a*), for the crossed circular polarizer (CP) and analyzer (CA) at the light incidence angle α = 0° (*b*) and α = 5.33° (*c*). The corresponding director field distributions on the top substrate calculated by eq. (5) (*d*)

Рис. 3. POM фотографии участков образца нематика с буджумами, имеющими силу m = +1 и угол закрутки ξ = 90º (первый ряд), m = –1 и угол закрутки ξ = –75º (второй ряд). Фотографии сделаны в монохроматическом свете (λ = 546 нм) в геометрии скрещенных линейных поляризатора (LP) и анализатора (LA) для ориентации поляризатора LP 0º (*a*), скрещенных циркулярных поляризатора (CP) и анализатора (CA) для углов падения света α = 0º (*b*) и α = 5,33º (*c*). Рассчитанные по уравнению (5) соответствующие распределения поля директора на верхней подложке (*d*)

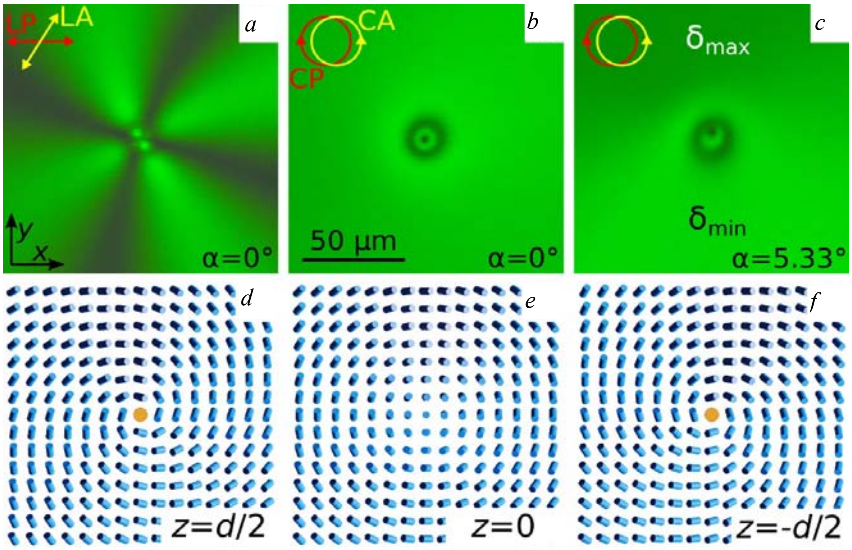


Fig. 4. POM photos of the sample area of cholesteric with boojum of $m = +1$ strength and the twisting angle on the top substrate $\xi = -106^{\circ}$ taken in the monochromatic light ($\lambda = 546$ nm) for the crossed at 58° angle linear polarizers at the polarizer orientation LP 0º (*a*), for the crossed circular polarizers at the light incidence angle $\alpha = 0^{\circ}$ (*b*) and $\alpha = 5.33^{\circ}$ (*c*). The corresponding director field distributions on the top substrate (*d*), at the layer center (*e*) and on the bottom substrate (*f*) calculated by equation (5) considering the conditions (2)

Рис. 4. POM фотографии участка образца холестерика с буджумом, имеющими силу $m = +1$ и угол закрутки на верхней подложке $\xi = -106^{\circ}$, сделанные в монохроматическом свете ($\lambda = 546$ нм) в геометрии скрещенных под углом 58° линейных поляризаторов для ориентации поляризатора LP 0º (*a*), скрещенных циркулярных поляризаторов для углов падения света $\alpha = 0^{\circ}$ (*b*) и $\alpha = 5{,}33^{\circ}$ (*c*). Рассчитанные по уравнению (5) с учетом условия (2) соответствующие распределения поля директора на верхней подложке (*d*), в центре слоя (*e*) и на нижней подложке (*f*)

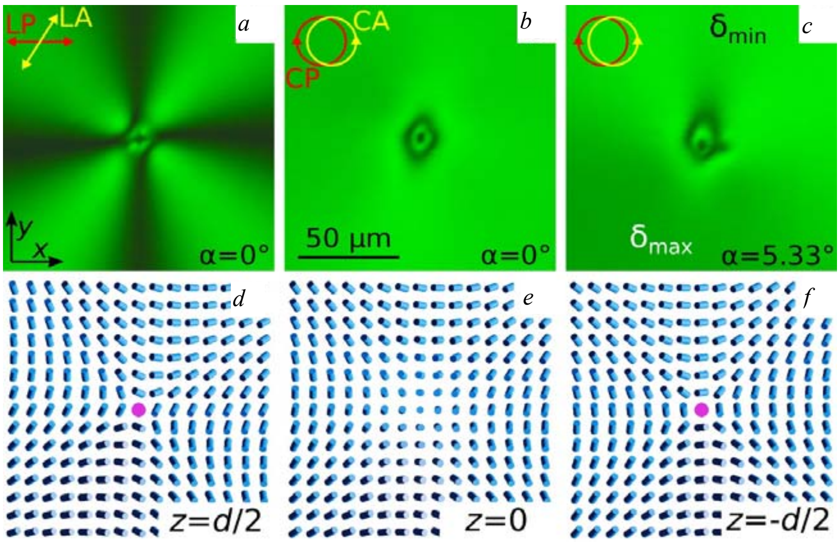


Fig. 5. POM photos of the sample area of cholesteric with boojum of $m = -1$ strength and the twisting angle on the top substrate $\xi = -116^{\circ}$ taken in the monochromatic light ($\lambda = 546$ nm) for the crossed at 58° angle linear polarizers at the polarizer orientation LP 0º (*a*), for the crossed circular polarizers at the light incidence angle $\alpha = 0^{\circ}$ (*b*) and $\alpha = 5.33^{\circ}$ (*c*). The corresponding director field distributions on the top substrate (*d*), at the layer center (*e*) and on the bottom substrate (*f*) calculated by equation (5) considering the conditions (2)

Рис. 5. POM фотографии участка образца холестерика с буджумом, имеющими силу $m = -1$ и угол закрутки на верхней подложке $\xi = -116^{\circ}$, сделанные в монохроматическом свете ($\lambda = 546$ нм) в геометрии скрещенных под углом 58° линейных поляризаторов для ориентации поляризатора LP 0º (*a*), скрещенных циркулярных поляризаторов для углов падения света $\alpha = 0^{\circ}$ (*b*) и $\alpha = 5{,}33^{\circ}$ (*c*). Рассчитанные по уравнению (5) с учетом условия (2) соответствующие распределения поля директора на верхней подложке (*d*), в центре слоя (*e*) и нижней подложке (*f*)

Fig. 4 shows a boojum with $m = +1$, for which the extinction bands form a straight cross rotated at an angle of –16° with respect to the direction of the polarizer. Observations with oblique incidence of light on the cell (fig. 4, *c*) show that δ_{max} and, consequently, the value $\varphi_0(x, y) = 0$, is observed at an angle $\zeta \approx 90°$. In this case, in the approximation of the linear dependence of the azimuthal angle of the director on the coordinate z (equation (2)), the twisting angle of the boojum on the upper substrate can be found from the relation $(+1\cdot 90° + \xi = \varphi_{TOTAL}/2)$, whence $\xi_{d/2} = -106°$ (fig. 4, *d*), while on the lower substrate $\xi_{-d/2} = -74°$ (fig. 4, *f*). The orientation of the four extinction lines corresponds to the condition when $\varphi_{-d/2}$ equals 0°, ± 90° and –180°, which corresponds to angles 74°, 164°, –16°, –106° and is consistent with the orientation of extinction bands observed in the geometry of crossed polarizers (fig. 4, *a*).

The twisting angle of boojums with $m = -1$ (fig. 5) can be determined in a similar manner. The cross that forms the extinction bands is oriented at an angle of approximately 5° to the polarizer (fig. 5, *a*), while δ_{max} is reached at an angle of $\zeta \approx -100°$ (fig. 5, *c*). This corresponds to the twisting angle of the boojum on the upper substrate which is found from the relation $(-1\cdot(-100°) + \xi = \varphi_{TOTAL}/2)$, whence $\xi_{d/2} = -116°$ (fig. 5, *d*). On the lower substrate, it corresponds to a boojum with $\xi_{-d/2} = -84°$ (fig. 5, *f*).

Conclusion. In this work we propose a method of oblique incidence of light on an LC cell which makes it possible to determine the polar and azimuthal angles of the director orientation in the case of conical boundary conditions. The capabilities of this method were demonstrated by the example of point topological defects-boojums that form in nematic with a director tilt angle at the boundary $\theta_{d/2} = 40°$, for which the defect strength m and twisting angles ξ were first determined. It is shown that in the used nematic liquid crystal LN-396, there is a tendency for the formation of boojums with $m = +1$, having a twisting angle $\xi = \pm 90°$. This method is also applicable for a twisted cholesteric structure. In this case, if the Mauguin regime is fulfilled, the maximum phase difference δ_{max} between the ordinary and extraordinary beams is achieved at the azimuthal orientation of the director at the center of the layer $\varphi_0(x, y) = 0$, regardless of the total angle of rotation of the director φ_{TOTAL}. Using the proposed method, we determined the orientational structure of boojums, formed on the upper and lower substrates specifying conical anchoring for the liquid crystal. The results obtained are of interest for studies of structured materials, analysis of defect structure by optical methods, and the developed method of oblique incidence of light can potentially be used to analyze a wide class of optically anisotropic and chiral media, such as smectics, nematics and cholesterics with tilted or hybrid boundary conditions.

It is necessary to note the applied significance of the research results, since liquid crystal media today remain the most competitive functional materials for use in optoelectronic devices requiring low weight and dimensions, low-voltage control and low power consumption which, for example, is very important for use in space technologies.

References

1. Gennes P. G. de, Prost J. The physics of liquid crystals. 2. ed., Reprint. Oxford: Clarendon Press, 1998. 597 p.
2. Oswald P., Pieranski P. Nematic and cholesteric liquid crystals: concepts and physical properties illustrated by experiments. Boca Raton: Taylor & Francis, 2005. 618 p.
3. Sasaki Y. et al. Large-scale self-organization of reconfigurable topological defect networks in nematic liquid crystals. *Nature Communications.* 2016, Vol. 7, No. 1, P. 13238.
4. Kim M., Serra F. Tunable dynamic topological defect pattern formation in nematic liquid crystals. *Advanced Optical Materials.* 2020, Vol. 8, No. 1, P. 1900991.
5. Kim M., Serra F. Topological defect arrays in nematic liquid crystals assisted by polymeric pillar arrays: effect of the geometry of pillars. *Crystals.* 2020, Vol. 10, No. 4, P. 314.
6. Senyuk B. et al. Topological colloids. *Nature*. 2013, Vol. 493, No. 7431, P. 200–205.
7. Nych A. et al. Assembly and control of 3D nematic dipolar colloidal crystals. *Nature Communications*, 2013, Vol. 4, No. 1, P. 1489.
8. Medle Rupnik P. et al. Field-controlled structures in ferromagnetic cholesteric liquid crystals. *Science Advances*. 2017, Vol. 3, No. 10, P. 1701336.
9. Lavrentovich O. D. Transport of particles in liquid crystals. *Soft Matter*. 2014, Vol. 10, No. 9, P. 1264–1283.
10. Sohn H. R. O., Liu C. D., Smalyukh I. I. Schools of skyrmions with electrically tunable elastic interactions. *Nature Communications*. 2019, Vol. 10, No. 1, P. 4744.
11. Sengupta A. Microbial active matter: A topological framework. *Front. Phys. Frontiers*. 2020, Vol. 8, P. 184.
12. Nabarro F. R. N. Singular lines and singular points of ferromagnetic spin systems and of nematic liquid crystals. *Journal de Physique*. 1972, Vol. 33, No. 11–12, P. 1089–1098.
13. Meyer R. B. Point disclinations at a nematic-isotropic liquid interface. *Molecular Crystals and Liquid Crystals*. 1972, Vol. 16, No. 4, P. 355–369.
14. Kleman M., Friedel J. Disclinations, dislocations, and continuous defects: A reappraisal. *Rev. Mod. Phys.* 2008, Vol. 80, No. 1, P. 61–115.
15. Oswald P., Baudry J., Pirkl S. Static and dynamic properties of cholesteric fingers in electric field. *Physics Reports*. 2000, Vol. 337, No. 1, P. 67–96.
16. Smalyukh I. I., Lavrentovich O. D. Three-dimensional director structures of defects in Grandjean-Cano wedges of cholesteric liquid crystals studied by fluorescence confocal polarizing microscopy. *Phys. Rev. E.* 2002, Vol. 66, No. 5, P. 051703.
17. Madhusudana N. V., Pratibha R. Studies on high strength defects in nematic liquid crystals. *Molecular Crystals and Liquid Crystals.* 1983, Vol. 103, No. 1–4, P. 31–47.
18. Ryschenkow G., Kleman M. Surface defects and structural transitions in very low anchoring energy nematic thin films. *Journal of Chemical Physics.* 1976, Vol. 64, No. 1, P. 404–412.

19. Krakhalev M. N. et al. Nematic and cholesteric liquid crystal structures in cells with tangential-conical boundary conditions. Crystals. 2019, Vol. 9, No. 5, P. 249.

20. Krakhalev M. N. Electrically induced transformations of defects in cholesteric layer with tangential-conical boundary conditions. *Scientific reports*. 2020. Vol. 10. P. 4907.

21. Krakhalev M.N. et al. Director configurations in nematic droplets with tilted surface anchoring. *Liquid Crystals*. 2017, Vol. 44, No. 2, P. 355–363.

22. Ohzono T. Uncovering different states of topological defects in schlieren textures of a nematic liquid crystal. *Scientific reports*. 2017, Vol. 7, P. 16814.

23. Yeh P., Gu C. Optics of liquid crystal displays. New York, Wiley, 1999, 438 p.

24. Lien A. The general and simplified Jones matrix representations for the high pretilt twisted nematic cell. *Journal of Applied Physics*. 1990, Vol. 67, No. 6, P. 2853–2856.

25. Harth K., Stannarius R. Topological point defects of liquid crystals in quasi-two-dimensional geometries. *Front. Phys. Frontiers.* 2020, Vol. 8, P. 112.

Krakhalev Mikhail Nikolaevich – Cand. Sc., senior researcher, Laboratory of molecular spectroscopy, Kirensky Institute of Physics, Federal Research Center KSC SB RAS, Associate Professor at the Department of General Physics, Institute of Engineering Physics and Radio Electronics, Siberian Federal University. E-mail: kmn@iph.krasn.ru.

ShabanovVasilyPhilipovich – Dr. Sc., Professor, Academician, Scientific director of Federal Research Center "Krasnoyarsk Science Center of the Siberian Branch of the Russian Academy of Sciences". E-mail: shabanov@ksc.krasn.ru.

Zyryanov Victor Yakovlevich – Dr. Sc., Professor, Head of the scientific direction and Laboratory of molecular spectroscopy, Kirensky Institute of Physics, Federal Research Center KSC SB RAS. E-mail: zyr@iph.krasn.ru.

Крахалев Михаил Николаевич – кандидат физико-математических наук, старший научный сотрудник лаборатории молекулярной спектроскопии, Институт физики им. Л. В. Киренского Сибирского отделения Российской академии наук – обособленное подразделение ФИЦ КНЦ СО РАН; доцент кафедры общей физики, Институт инженерной физики и радиоэлектроники Сибирского федерального университета. E-mail: kmn@iph.krasn.ru.

Шабанов Василий Филиппович – доктор физико-математических наук, профессор, академик РАН, научный руководитель; Федеральный исследовательский центр «Красноярский научный центр СО РАН». E-mail: shabanov@ksc.krasn.ru.

Зырянов Виктор Яковлевич – доктор физико-математических наук, профессор, руководитель научного направления, заведующий лабораторией молекулярной спектроскопии; Институт физики им. Л. В. Киренского Сибирского отделения Российской академии наук – обособленное подразделение ФИЦ КНЦ СО РАН. E-mail: zyr@iph.krasn.ru.