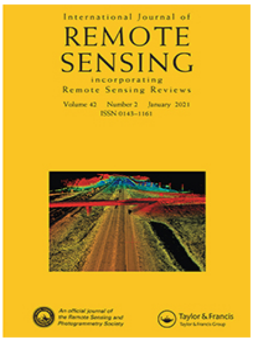

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Retrieving soil moisture profiles based on multifrequency polarimetric radar backscattering observations. Theoretical case study

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ABSTRACT
In this theoretical work, a dual-frequency polarimetric method is proposed for measuring moisture profiles in the topsoil up to 0.30 m thick. A case of measuring soil moisture profiles, which monotonically changes with depth, during 37 days after irrigation is considered. Original values of backscattering coefficients are calculated by the Oh model and by the small perturbation method at frequencies of 5.4 GHz and 435 MHz, respectively. In these calculations, we used measured moisture profiles and spectroscopic refractive mixing dielectric model of non-saline mineral soil with a clay fraction of 9.1%. Soil moisture profiles are retrieved by solving the inverse problem, the cost function of which is constructed based on the co- and cross-polarized ratios, calculated at two frequencies for the measured and modelled soil moisture profiles. An exponential function is used as a modelled soil moisture profile. It is shown that the standard deviation between the retrieved and measured soil moisture values in the surface layer 0.30 m thick appears to be ≤0.02 m^3 m^{-3} (theoretical limit), and the determination coefficient is 0.881. The study shows a promising path towards developing multi-frequency radar systems for remote sensing of soil moisture profiles using satellites-based and unmanned aerial vehicles air-based platforms.

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1. Introduction

The moisture of the surface soil layer is 1 of the 50 essential climatic variables recommended by the World Meteorological Organization for ground-based and satellite-based observations (GCOS 2015). Nowadays (Soil Moisture Active Passive) SMAP and (Soil Moisture and Ocean Salinity) SMOS L-band satellite radiometers operating at a frequency of 1.4 GHz (Monsiváis-Huertero et al. 2020; Wu, Nie, and Shu 2019; Wigneron et al. 2017; Entekhabi et al. 2014), (Global Change Observation Mission) GCOM-W1 X- and Ka-band satellite with (Advanced Microwave Scanning Radiometer) AMSR2 operating at frequencies of 10 GHz and 36 GHz (Fujii, Koike, and Imaoka 2009), and (Meteorological operational satellite) Metop C-band satellite with (Advanced SCATterometer) ASCAT operating at a frequency of 5.3 GHz

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(Brocca et al. 2017) are used to monitor the moisture of the subsurface layer up to 0.025 m thick (Choudhury et al. 1979; Escorihuela et al. 2010). It is impossible to measure directly the root zone (0 m to 1 m) soil moisture using the existing satellite constellation. However, the data of remote sensing (of the soil surface moisture) in combination with hydrological models describing the dynamics of coupled heat and moisture transport (Liou and England 1998) make it possible to retrieve the soil moisture distribution with depth (Hoeben and Troch 2000; Tian et al. 2019). The root-zone soil moisture profiles can also be estimated based on established correlation dependences between the actual soil moisture profile and the field capacity profile the value of which is an agro-hydrological constant for given soil type (Vasilev et al. 1983). As a result, a method for estimating the total soil moisture profile in the root-zone 0 m to 1 m can be suggested (Kondrat et al. 1976) based on an a priori known profile of the field capacity for given soil type and direct remote sensing of the soil surface moisture.

Direct remote sensing of the root-zone soil moisture requires the use of P- or (very high frequency) VHF-bands. Indeed, theoretical investigations in the approximation of a linear soil moisture profile (Bogorodskij and Kozlov 1985) demonstrated that the moisture gradient could not be estimated at wavelengths shorter than 0.43 m if the soil surface moisture exceeds the field capacity or if the relative dielectric constant of the soil surface is greater than 15. At the same time, a principal possibility of estimating the vertical distribution of soil moisture of agricultural fields to depths of 2.5 m using the multi-frequency (140 MHz, 430 MHz, 1.3 GHz, and 8.6 GHz) 'IMARK' synthetic aperture radar developed at the JSC Radio Engineering Concern 'VEGA' (Kutuza et al. 2004) was experimentally confirmed in (Kutuza 2015). In 2021, the European Space Agency will launch the (Biomass monitoring mission for Carbon Assessment) BIOMASS satellite equipped with a P-band synthetic aperture radar with a working frequency of 435 MHz (Carreiras et al. 2017), thereby for the first time creating a technical possibility for the development of new algorithms of permanent monitoring of the root-zone soil moisture on the global scale (Alemohammad et al. 2018). In this regard, a principal possibility for quantitative estimating the root-zone soil moisture distribution with depth from single-frequency polarimetric observations of P-band radar backscattering at a frequency of 435 MHz is intensively studied (Sadeghi et al. 2017; Tabatabaeenejad et al. 2015; Konings et al. 2014; Moghaddam et al. 2007). Unlike the L-band waves for which the estimated sensing depth is the order of 0.025 m (Escorihuela et al. 2010; Choudhury et al. 1979), the sensing depth for the P-band waves depends on the soil moisture profile and cannot be assigned by universal value. As a result, for different sets of moisture profiles before and after irrigation, it is impossible to determine the average soil moisture in layers of the predetermined thickness (0.05 m, 0.15 m, 0.30 m, and 0.50 m) that would correspond to the effective soil moisture retrieved from remote measurements in the homogeneous half-space approximation (Tabatabaeenejad et al. 2015; Konings et al. 2014). In this regard, in the literature, the problem of retrieval of the soil moisture profile in the P-band is reduced to the finding of the parameters of model function profile describing the root-zone moisture distribution, represented by a set of layers of finite thicknesses (Konings et al. 2014; Moghaddam et al. 2007), by a second or third-degree polynomial (Sadeghi et al. 2017; Tabatabaeenejad et al. 2015), or by Epstein's profile (Fung et al. 1996; Walker, Troch, and Mancini et al. 1997). Owing to a limited number of radar observations at a frequency of 435 MHz (HH, VV, and HV, where H and V stand for horizontal and vertical polarizations, respectively) that allow one to reconstruct simultaneously up to three variables, it is impossible to solve completely the problem on moisture variations with depth taking into account a soil surface roughness and a vegetation cover. However, as shown in (Kalinkevich et al. 2018; Kutuza 2015; Kwon et al. 2002; Zerdev and Kulemin 1993), polarimetric multifrequency radar observations allow one to estimate the distribution of soil moisture with depth taking into account the soil surface roughness. Since 2021, two satellites intended for polarimetric radar survey of the Earth surface on the global scale at frequencies of 435 MHz (BIOMASS) and 5.4 GHz (Sentinel-1) will be available thus offering prospects for the development of new dual-frequency algorithms for remote sensing the topsoil moisture based on observations of P- and C-band radar backscattering. In the present work, a method of measuring topsoil moisture profiles based on observations of C- and P-band backscattering coefficients at two frequencies is suggested.

2. Statement of the problem

In this paper, the bare soil without canopy will be considered. For C-band waves, we consider that standard deviation of roughness heights and slopes of soil surface does not exceed about λ and 0.485, respectively. For P-band waves, we consider that standard deviation of slopes and roughness heights of soil surface do not exceed 1 and about $\lambda/20$, respectively. Then for the C- and P-band waves, the incoherent component of backscattering coefficient can be estimated by the Oh model (Oh, Sarabandi, and Ulaby 1992) and by the small perturbation method (Bass and Fuks 1979; Ulaby and Long 2015), respectively. The ratio of the backscattering coefficients for horizontal–horizontal, σ_{HH}, and for vertical-vertical, σ_{VV}, polarizations, which are calculated by the small perturbations method, $P_1(\theta)$, are independent on the function of the spectral density of roughness heights of the soil surface and can be expressed in the form (Ceraldi et al. 2005; Komarov, Mironov, and Li 2002; Komarov and Yakushev 1998):

$$P_1(\theta) = \frac{\mathrm{HH}}{\mathrm{VV}} = \frac{|1 + R_H(\theta, \varepsilon(z))|^4}{\left|\cos^2[1 + R_V(\theta, \varepsilon(z))]^2 + \frac{\sin^2}{\varepsilon(0)}[1 - R_V(\theta, \varepsilon(z))]^2\right|^2} \tag{1}$$

where ε(0) is the complex permittivity of soil surface at the point with vertical coordinate $z = 0$ m, (z) is the profile of soil complex permittivity, θ is the angle of observation, and $R_H(\theta, \varepsilon(z))$ and $R_V(\theta, \varepsilon(z))$ are the complex reflection coefficients for horizontal and vertical polarizations, respectively. The Oh model gives estimation for co-polarized, $P_2(\theta)$, and cross-polarized, $P_3(\theta)$, ratios in the form (Oh, Sarabandi, and Ulaby 1992)

$$P_2(\theta) = \frac{\mathrm{HH}}{\mathrm{VV}} = \left[1 - \left(\frac{2\theta}{\pi}\right)^{\left[\frac{1}{3R_0}\right]} \exp(-k_0 s)\right]^2 \tag{2}$$

$$P_3(\theta) = \frac{\mathrm{HV}}{\mathrm{VV}} = 0.23\sqrt{R_0}[1 - \exp(-k_0 s)] \tag{3}$$

where R_0 is the Fresnel reflectivity of the flat surface at nadir, k_0 is the wavenumber of free space, s is the root-mean-square deviation of the heights of soil surface roughness. If the values of $P_2^m(\theta)$ and $P_3^m(\theta)$ are measured, then R_0 can be determined from the system of nonlinear Equations (2) and (3), eliminating the effect of soil surface roughness in the same way as in the article (Oh, Sarabandi, and Ulaby 1992, see Equation (11)):

$$\left(\frac{2\theta}{\pi}\right)^{\frac{1}{3R_0}}\left[1-\frac{P_3^{\mathrm{m}}(\theta)}{0.23\sqrt{R_0}}\right]+\sqrt{P_2^{\mathrm{m}}(\theta)}-1=0 \tag{4}$$

Equation (1) derived in (Komarov, Mironov, and Li 2002; Komarov and Yakushev 1998) is a strict solution of the problem on wave scattering by a rough boundary of a heterogeneous-stratified dielectric half-space. Equations (2) and (3) derived for a dielectrically homogeneous half-space, but it will be assumed that they are applicable to heterogeneous-stratified soil, if R_0 is replaced by $R_0((z))$, and the Brekhovskikh iterative method (Brekhovskikh 1960) for the calculation of the reflection coefficients $R_0((z))$ will be implemented.

According to the classical mean value theorem (Gradshteyn and Ryzhik 2000), for any arbitrary moisture profile, there should exist mean soil moisture of a layer with finite thickness. Indeed, in (Fung, Boisvert, and Brisco 1997; Gorrab et al. 2014) it was shown that for a reflection coefficient, calculated for heterogeneous-stratified dielectric half-space, can be juxtaposed the same value of reflection coefficient, calculated for homogeneous half-space with some effective value of dielectric constant. Below we call effective these values of the soil permittivity or moisture. Measurements of $P_1(\theta)$, and $P_2(\theta)$, $P_3(\theta)$ at frequencies of 435 MHz (P-band) and 5.4 GHz (C-band), respectively, allow two values of effective soil moisture is determined. In the section below, the method is suggested that allows the soil moisture at any arbitrary depth in the topsoil with thickness up to 0.30 m to be retrieved based on two obtained values of the effective soil moisture and a parametric moisture profile.

3. Method of measuring topsoil moisture profiles

To investigate the possibilities of remote sensing of topsoil moisture distribution, we take advantage of the reference moisture profiles (Schmugge, Wilheit, and Gloersen 1976) observed during 37 days since 2 March 1971, in the middle of the day in one of the agricultural fields by employees of the American Water Testing Labs in Phoenix, see Figure 1. Profile 1 was not presented in (Schmugge, Wilheit, and Gloersen 1976); we derived it using two-dimensional interpolation (over depth and time) of the moisture profiles between 3 and 37 days after irrigation. By virtue of the limited number of radar observations, to describe the experimental moisture profiles (see Figure 1), we take advantage of the simplified solution of the equation of vertical soil moisture transfer (Sadeghi et al. 2017):

$$m(z)=m_\infty+(m_0-m_\infty)\exp(z/) \tag{5}$$

where m_0 is the surface soil moisture, m_∞ is the moisture outside of topsoil for $z\to-\infty$, and α is the effective thickness of the capillary edge. We reduce the problem of retrieval of the soil moisture profile to calculations of two of the three unknown parameters in Equation (5). For this purpose, we calculated the average soil moisture $\bar{m}$ of the layer of thickness d:

$$\bar{m}=\frac{1}{d}\int_0^d m(z)\mathrm{d}z=m_\infty+\frac{(m_0-m_\infty)}{\gamma}(1-\exp(-\gamma) \tag{6}$$

where $\gamma=d/\alpha$. Then, taking into account Equations (5) and (6), the soil moisture profile can be written in the form

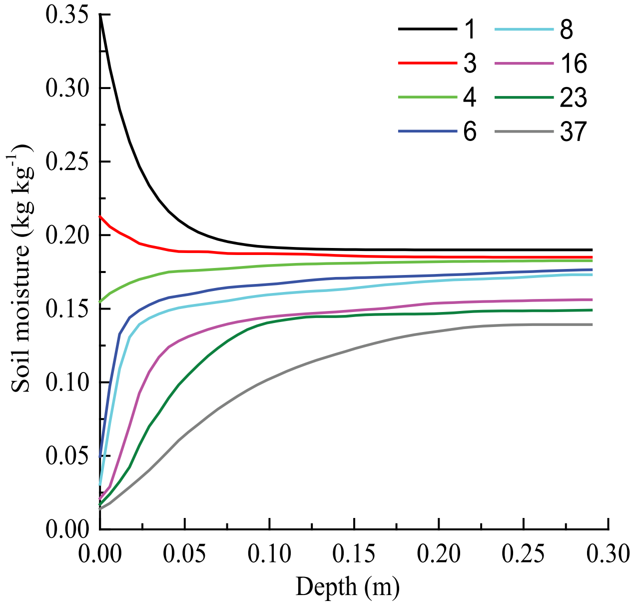

Figure 1. Topsoil moisture profiles between 1 and 37 days after irrigation.

$$m(z) = m_\infty + (m_0 - m_\infty)\exp\left(-\frac{z}{d}\right), m_\infty = \frac{-m_0\frac{1-\exp(-\gamma)}{\gamma}}{1-\frac{1-\exp(-\gamma)}{\gamma}}. \quad (7)$$

As a result, the problem of retrieval of the moisture profile is reduced to the finding of the parameters d and γ for the known soil surface moisture m_0 and the average moisture $\bar{m}$ of the layer of thickness d. The average moisture in the layer of thickness d and the soil surface moisture can be found from the ratio of the radar backscattering coefficients P_1 measured at a frequency of 435 MHz, and from Equation (4) at a frequency of 5.4 GHz under the assumption that the medium being sensed represents a homogeneous dielectric half-space $(z) = (m^*)$. Because the indirect measurements are performed in the homogeneous half-space approximation, we call the obtained moisture, m^*, effective. For original (measured) values of the ratio of radar backscattering coefficients P_1^m and P_2^m, P_3^m we take values calculated from Equations (1)–(3) for a heterogeneous-stratified soil, the moisture profiles for which are shown in Figure 1. In this case, the reflection coefficients in Equations (1)–(3) are calculated by the iteration method (Brekhovskikh 1960). The heterogeneous-stratified half-space was subdivided into 1500 layers for $0 > z > -0.3$ m. The dielectric half-space for $z < -0.3$ m was considered homogeneous with complex permittivity $(z) = (z = -0.3)$. The effective soil moisture, m^*, at a frequency of 435 MHz is retrieved by minimization of the residual norm between P_1^m and P_1 values using the Levenberg–Marquardt algorithm (Gill and Murray 1978). At a frequency of 5.4 GHz, the effective soil moisture, m^*, calculated from the reflection coefficient $R_0((m^*))$, determined by the nonlinear Equation (4). For definiteness, the complex permittivity of the dielectric half-space will be calculated for non-saline sandy loam soil, a sample of which was used to create the dielectric model (Mironov et al. 2020, soil No. 1). For this soil, the content of sand, silt, clay, and organic matter was 41.4%, 49.5%, 9.1%, and 0.9%; other soil characteristics can be found in (Mironov et al. 2017). The dry soil density is 10^3 kg m^{-3}, which corresponds to often cultivated topsoil. The sensing angle is set equal to 40°.

The retrieved values of effective soil moisture are shown in Figure 2. From the data are shown in Figure 2, it can be seen that the effective soil moisture at a frequency of 5.4 GHz (see curve 3) with a standard deviation of less than 0.003 m^3 m^{-3} corresponds to the soil surface moisture (at $z = 0$ m) shown by the dotted curve. Until the 4^{th} day after irrigation (when the soil surface moisture exceeds 0.15 m^3 m^{-3}, see Figures 1 and 2), the soil moistures retrieved at frequencies of 5.4 GHz and 435 MHz practically coincide, and their deviations do not exceed 0.025 m^3 m^{-3}. As expected, the effective soil moisture retrieved at a frequency of 435 MHz becomes more informative about the moisture distribution with depth starting from the 4th day after irrigation. It seems likely that for the considered type of the soil when the surface moisture exceeded 0.15 m^3 m^{-3}, the possibility of obtaining information on the moisture distribution with depth would be provided by the application of frequencies lower than 435 MHz. At an intermediate frequency of 1.4 GHz (the working frequency of SMAP and SMOS radiometers and the average L1 + L2 frequencies of (the Global Positioning System) GPS and (GLObal NAvigation Satellite System) GLONASS), the retrieved effective soil moisture differed from its surface value only between 4 and 22 days after irrigation when the soil surface moisture was lower than 0.15 m^3 m^{-3} and higher than about 0.02 m^3 m^{-3}.

Using the left-hand side of Equation (6), we now estimate the thickness d of the soil layer in which the average moisture $\bar{m}$ is equal to the effective soil moisture retrieved for each day after irrigation (see Figure 2). We call the thickness d the effective thickness or the sensing depth. The results of the calculation are shown in Figure 3. Indeed, as can be seen from Figure 3, the depth of sensing at a frequency of 5.4 GHz does not exceed

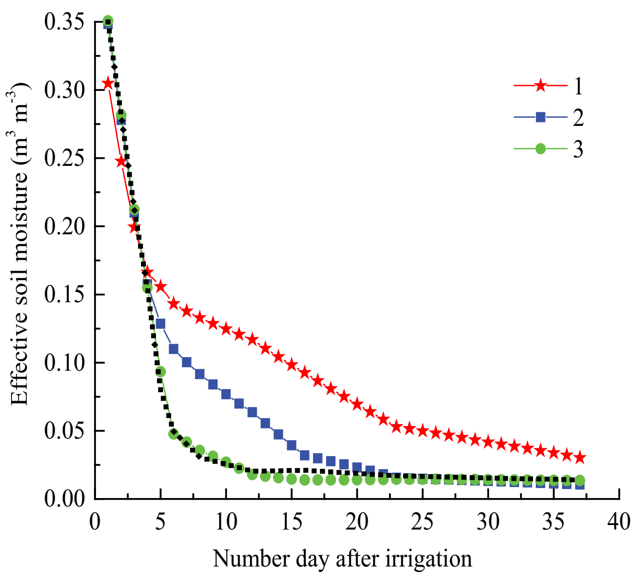

Figure 2. Retrieved effective soil moisture versus the number of days after irrigation. Here curve 1 is for 435 MHz, curve 2 is for 1.4 GHz, and curve 3 is for 5.4 GHz. The dotted curve shows the soil surface moisture measured at the depth of $z = 0$ m. Here the volumetric soil moisture defined as a product of the weight moisture (see Figure 1) by the soil dry bulk density equal to 10^3 kg m^{-3} is plotted on the ordinate.

0.002 m (see curve 3 in Figure 3), and the retrieved effective moisture practically coincides with the surface soil moisture (see the dotted curve and curve 3 in Figure 2). The maximal sensing depth at a frequency of 435 MHz reached about 0.07 m (see curve 1 in Figure 3). The depth of sensing at a frequency of 1.4 GHz did not exceed 0.02 m (see curve 2 in Figure 3), which is in good agreement with the literature data (Escorihuela et al. 2010; Choudhury et al. 1979). From Figure 3, it can be seen that the sensing depth depends not only on the frequency but also on the shape of the moisture profile. The sensing depth reached a maximum between 6 to 14 days after irrigation when the moisture profiles (or which is the same, the refractive index) had a maximal gradient in the layer of the thickness d (see Figure 1). Indeed, as shown in (Brekhovskikh 1960), for a linear model of the refractive index profile versus depth, the wave reflection coefficient is directly proportional to the refractive index gradient at the soil surface. As a result, the increase of the sensing depth between 6 and 14 days after irrigation is most likely due to the need to take into account noticeable contributions of the amplitudes of waves re-reflected in the zone of maximal refractive index gradients to the formation of the total reflection coefficient of the heterogeneous-stratified dielectric half-space.

In accordance with the foregoing analysis and the suggested method, the soil surface moisture m_0 in Equation (7) was set equal to effective soil moisture value, found from Equation (4) based on measurements of P_2^m and P_3^m at a frequency of 5.4 GHz. The mean soil moisture $\bar{m}$ in Equation (7) was set equal to effective soil moisture value, retrieved from the P_1^m measured at a frequency of 435 MHz. As a result, the desired soil moisture profile described by Equation (7) will be a function of only two variables: d and γ. The variables d and γ can be determined by minimization of the residual norm between P_1^m and the corresponding values calculated from Equation (1) using the model profile Equation (7) in which the parameters m_0 and $\bar{m}$ have been above determined. The formulated inverse

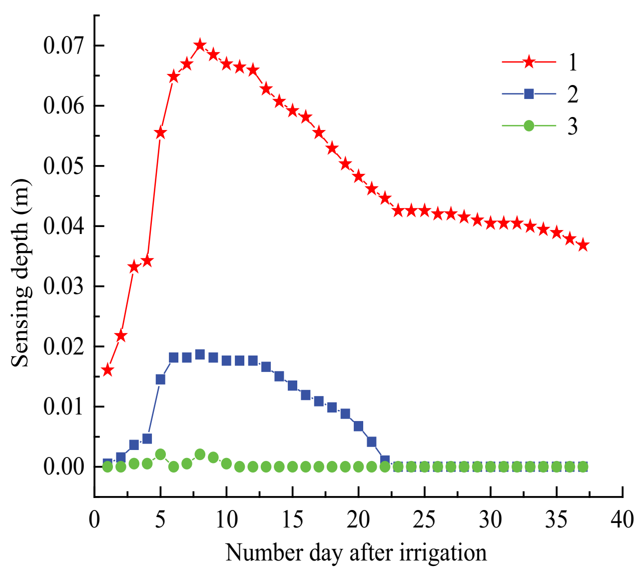

Figure 3. Thickness of the soil layer the average moisture of which (see data in Figure 1) is equal to the retrieved effective soil moisture (see Figure 2). Here curve 1 is for 435 MHz, curve 2 is for 1.4 GHz, and curve 3 is for 5.4 GHz.

problem is incorrect (incompletely determined). It is solved in the literature, for example, by finding a global minimum using the simulated annealing algorithm (Sadeghi et al. 2017; Tabatabaeenejad et al. 2015; Corana et al. 1987). Unlike (Sadeghi et al. 2017; Tabatabaeenejad et al. 2015), in the present work, we suggest taking advantage of all solutions (local minima of the residual norm) bounded by a certain area in the plane of desired variables (d, γ). If we look at Figure 1, we note that the soil moisture at infinity m_∞ never falls outside the interval 0.14 m^3 m^{-3} to 0.2 m^3 m^{-3}. This interval of m_∞ variations bounds the range of variations of the parameter γ determined by nonlinear Equation (7) by $1 < \gamma < 10$. The d values, according to the data shown in Figure 3, lie in the interval of 0.005 m $< d <$ 0.1 m. As an example, Figure 4 shows the residual norm of minimized functional in the logarithmic scale in coordinates (d, γ) on the 6th day after irrigation. As can be seen from Figure 4, there are 13 local minima with coordinates (d_i, γ_i) in the solution area, where i = 1, . . .,13.

4. Results and discussion

According to the suggested method, the soil moisture profiles were calculated for each i^{th} local minimum (see Figure 4) using Equation (7); then the standard deviations and a 95% confidence intervals for the derived moisture profiles were calculated (on the grid with a depth step of 0.001 m). Similarly, for all days after irrigation, the residual norm (analogous to that shown in Figure 4) was calculated. At points (d_i, γ_i) of the local minima, the moisture

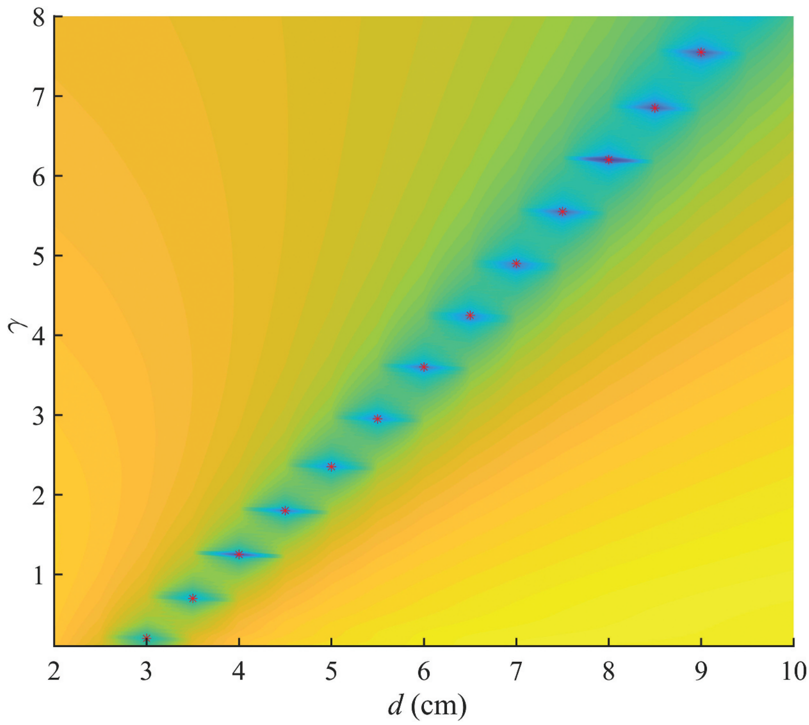

Figure 4. Residual norm to be minimized shown in the logarithmic scale in the coordinate plane (d, γ). The local minima are pointed out by red asterisk.

profiles were retrieved. Then, average moisture profiles were calculated; the standard deviations and a 95% confidence intervals for the retrieved moisture profiles were estimated. As an example, Figure 5 shows the original and retrieved average soil moisture profiles calculated with a depth step of 0.001 m on the 3rd, 6th, 16th, and 37th days after irrigation. A 95% confidence intervals corresponding to the retrieved profiles are indicated by shaded areas in Figure 5. As can be seen from Figure 5, the retrieved profiles of soil moisture (within confidence intervals) are in good agreement with the originally set profiles, while the most significant deviations in the retrieved profiles are observed on days with minimal sensing depths (compare with Figure 3). The dependence of the retrieved soil moisture on the original soil moisture in the 0.30 m thick layer is shown in Figure 6. To draw the plot shown in Figure 6, the soil moisture values were calculated with a step of 0.001 m. The root-mean-square error (RMSE) between the retrieved and original given soil moisture values was appeared to be 0.016 m^3 m^{-3} with a determination coefficient (R^2) of 0.881. The exponential function of the modelled profile (see Equation (7)) adequately describes the experimentally observed soil moisture variations with depth in the 0.30 m thick layer (see Figures 5 and 6) during 37 days after irrigation. The parameters of the retrieved moisture profiles (see Equation 7) for some days after irrigation are presented in Table 1. The retrieved soil moisture at infinite depth m_∞ (see Table 1) is in good agreement with the soil moisture at a depth of 0.30 m for original moisture profiles, see Figure 1. In this case, the standard deviations of the retrieved m_∞ values are in the range from about 0.02 m^3 m^{-3} to 0.16 m^3 m^{-3}. The retrieved d values (see Table 1) for the model profile described by Equation (7) correlated well with the corresponding values estimated in the homogeneous half-space approximation, see Figure 3. The sensing depths d estimated in this work (see Figure 3 and Table 1) are considerably less than those presented in (Moghaddam et al. 2007) where only

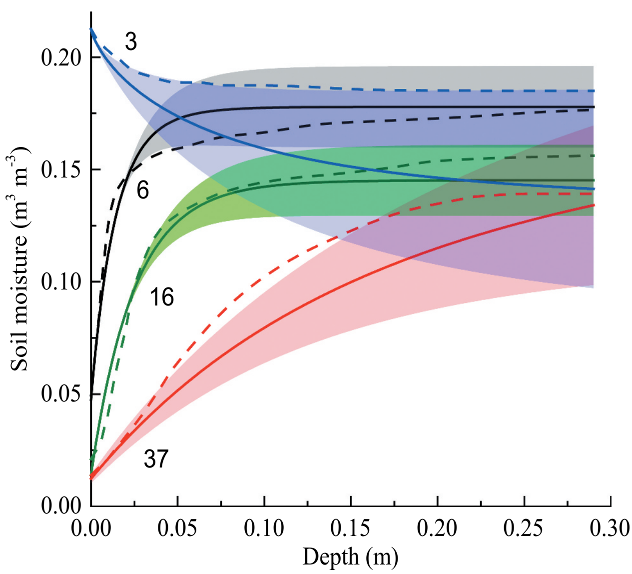

Figure 5. Original soil moisture profiles (dash curves) analogous to those shown in Figure 1 and retrieved soil moisture profiles (solid curves), with a 95% confidence intervals, which are indicated by shaded areas of the corresponding colour.

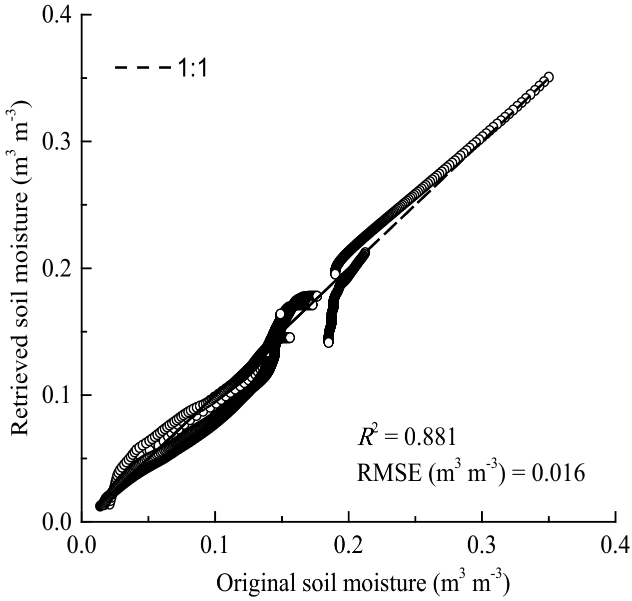

Figure 6. Dependences of the retrieved soil moisture on the original soil moisture of the 0.30 m thick layer.

Table 1. Retrieved parameters of the moisture profiles on the indicated days after irrigation*.

Day number	1	3	6	8	16	23	37
m_∞ (m^3 m^{-3})	0.238	0.194	0.171	0.164	0.140	0.145	0.156
Δm_∞ (m^3 m^{-3})	0.153	0.140	0.023	0.021	0.019	0.157	0.158
γ	0.82	0.92	5.77	5.13	2.85	0.89	0.31
$\Delta\gamma$	0.48	0.63	2.10	1.83	0.74	0.60	0.19
d (m) 10^{-3}	0.9	1.6	7.7	7.5	6.8	5.1	4.2
Δd (m) 10^{-3}	0.5	0.6	1.6	1.5	0.8	1.2	0.6

*Here Δ denotes the standard deviation of the corresponding retrieved parameter.

the wave attenuation in soil was considered at a frequency of 435 MHz. The sensing depths in that work were about 0.15 m and about 0.25 m for soils with a clay content of 20% and moistures of 25% and 5%, respectively. This can be explained by the fact that in our calculations, not only the attenuation of electromagnetic waves in the heterogeneous-stratified soil but also the change of the amplitudes of waves reflected from all internal boundaries of layers into which the half-space was subdivided was taken into account. Indeed, in (Moghaddam et al. 2007) the sensing depth of moist soil was underestimated in L-band also in comparison with those calculated in (Khankhoje, van Zyl, and Cwik 2013) for heterogeneous soil. Backscattering coefficients in (Khankhoje, van Zyl, and Cwik 2013) were calculated using the finite difference method which also takes into account multiple wave reflection into stratified dielectrically heterogeneous soil.

5. Conclusions

The suggested dual-frequency polarimetric method can be used to reconstruct the topsoil moisture profiles, which are monotonically varying with the depth. The use of the exponential soil moisture profile has allowed retrieving the moisture at various depths in the topsoil 0.30 m thick with a root-mean-square error no more than 0.02 $m^3 m^{-3}$ (theoretical limit). To describe heterogeneous soil moisture distribution with depth having one or several maxima or minima, a more complicated model of the moisture profile with a more significant number of unknown parameters is required. This, in turn, requires a larger number of independent observations of backscattering coefficients. In this regard, to retrieve a more complicated moisture profile or to refine the parameters of the exponential moisture profile, it is most expedient to measure the backscattering coefficients at additional frequencies lying in the range from about 435 MHz (or lower) to 1.4 GHz, rather than to use their angular dependences at the same frequencies (since at observation angles from 25° to 50°, the radar backscattering coefficient does not change significantly (Oh, Sarabandi, and Ulaby 1992) for soil moistures less than 0.29 $m^3 m^{-3}$). The suggested method of retrieval of the soil moisture profiles does not take into account the vegetation cover. Especially large errors can be introduced by vegetation when the soil surface moisture is determined at a frequency of 5.4 GHz due to significant wave scattering by vegetation cover in this frequency range. In (Khankhoje, van Zyl, and Cwik 2013), it was shown that the sensing depth increases with increasing the soil surface roughness. The models used in the present work to calculate the ratio of backscattering coefficients on co- and cross-polarizations by the Oh model or by the small perturbation method did not allow us to establish the effect of the soil surface roughness on sensing depth and the errors of retrieval of the soil moisture profiles. For further research of this point in question, it is necessary to address to an experiment or to use more complicated models of backscattering coefficients, for example, based on integral equations (Fung 1994) or direct numerical modelling (Khankhoje, van Zyl, and Cwik 2013). In addition, the problem of convergence of the method used for solving the inverse problem due to the error in measuring the backscattering coefficients should be investigated. Also, an important issue, that was not considered in this article is the effect of the clay fraction and another soil constants such as soil density, sand, silt, and organic matter content, soil salinity on the sensing depth and therefore on the error of moisture profiles retrieval. At present, it is difficult to carry out such an analysis because of the absence of a generalized dielectric model in the P- and C-bands, which simultaneously takes into account all the soil constants mentioned above for different kind of soils. At the same time, the author believes that the created method based on the dielectric model (Mironov et al. 2020) can be used to moisture profiles retrieval for a wide set of mineral non-saline soils (with different contents of sand (1.6% to 41.4%), silt (39.0% to 57.1%), clay (9.1% to 41.3%), and organic matter (0.9% to 2.3%)), samples of which were used to develop the dielectric model (Mironov et al. 2020). However, the application of the proposed method to retrieve of moisture profiles for soils with different texture and density requires additional investigation and justification.

The developed method can be realized for remote sensing using not only BIOMASS and Sentinel-1 satellites but also an airborne platform. In connection with wide development and availability of small unmanned aerial vehicles (UAV), the suggested method can be implemented in small UAV platform, equipped with compact multifrequency reflectometers manufactured on the basis of the portable spectrum analysers such as (CMT 2020).

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