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Analytical calculation of dielectric permittivity tensor from magneto-optical ellipsometry measurements

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Abstract

Magneto-optical ellipsometry combines ellipsometry and magneto-optical Kerr effect measurements which are two powerful techniques. The main difficulty is usually in data processing as a number of parameters should be extracted from measured ellipsometric (ψ, Δ) and magneto-ellipsometric $(\delta\psi, \delta\Delta)$ parameters. Standard procedure of solving magneto-ellipsometry equations involves numerical calculations. In this paper we show that it is possible to find out all elements of dielectric permittivity tensor without numerical calculation methods. It means that the inverse problem of magneto-optical ellipsometry can be solved analytically in the case of expansion of magneto-ellipsometric parameters $\delta\psi$ and $\delta\Delta$ with respect to two small parameters. We present a full set of mathematical expressions that enable us to calculate complex refraction index and complex magneto-optical parameter of a sample from magneto-optical ellipsometry measurements, thereby obtaining diagonal and off-diagonal complex elements of dielectric permittivity tensor. This analytical approach can be used in case of the contribution from magnetism into reflection coefficients being small.

Keywords: ellipsometry, dielectric permittivity tensor, ferromagnetics, transverse magneto-optical Kerr effect

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1. Introduction

When creating new materials with specified functional properties, both the technology of their synthesis and the methods of non-destructive high-precision and high-speed control of their physical properties play an important role. In particular, when creating modern microwave, nanoelectronics, and spintronics devices, magnetic nanoparticles for biomedical applications, there are technological problems in setting and controlling the imperfection of the crystal structure of ferromagnetic materials (FM) [1]. The imperfection of a nanostructured FM actually determines its electronic structure, affects the electronic conductivity and the spin polarization of conduction electrons, plays role in the formation of the domain structure, determines the diffusion and thermodynamic properties of the material. For example, the appearance of magnetocrystalline anisotropy in a nanostructured FM can lead to a significant increase in the specific magnetization [2, 3].

One of important characteristics of a magnetized isotropic FM metal is the dielectric permittivity tensor which looks as follows [4–6]:

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ \varepsilon_{21} & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} = \begin{bmatrix} \varepsilon'_{11} - \mathrm{i}\varepsilon''_{11} & -\mathrm{i}(\varepsilon'_{11} - \mathrm{i}\varepsilon''_{11})(Q_1 - \mathrm{i}Q_2) & 0 \\ \mathrm{i}(\varepsilon'_{11} - \mathrm{i}\varepsilon''_{11})(Q_1 - \mathrm{i}Q_2) & \varepsilon'_{11} - \mathrm{i}\varepsilon''_{11} & 0 \\ 0 & 0 & \varepsilon'_{11} - \mathrm{i}\varepsilon''_{11} \end{bmatrix}, \quad (1)$$

where $\varepsilon'_{11} = {n_1}^2 - {k_1}^2$ and $\varepsilon''_{11} = 2n_1k_1$ are the real and imaginary parts of the dielectric permittivity of the medium, respectively, $Q = Q_1 - \mathrm{i}Q_2$ is the magneto-optical parameter proportional to magnetization, Q_1 is the real part of the magneto-optical parameter, Q_2 is the imaginary part of the magneto-optical parameter, $N_1 = n_1 - \mathrm{i}k_1$ is the complex refractive index of the sample, n_1 is the refractive index, k_1 is the extinction coefficient. The magnetic permeability μ is supposed to trend towards 1 in the visible spectral range [4]. Without loss of generality, we can assume that the magnetization vector is directed along the Z axis, so that YZ is a boundary plane and YX is a plane of incidence. All planes passing through this distinguished Z direction are equivalent to each other.

It is known that the elements of the dielectric tensor can be found by means of magneto-optical measurements, e.g. by spectroscopic generalized magneto-optical ellipsometry. Numerical algorithms of data processing are proposed in a number of works, e.g. [6–13]. In the general case of anisotropic systems, the problem can be solved using 4×4 Mueller matrices [7–15]. Some of these authors [9–12] decided to combine magneto-ellipsometry measurements with additional *ex situ* experiments, e.g. they used SQUID magnetometry to find out sample magnetisation, that is why they used another definition of dielectric tensor elements as well as of magneto-optical parameter. They defined off-diagonal components of the tensor as $\varepsilon_{xy} = -\mathrm{i}Q_zM_z$ where Q is called a thickness-independent complex magneto-optical coupling constant [9–12] which is assumed to be also magnetisation-independent. It is important not to confuse their definitions of the physical quantities with the conventional definition (1) which is used in this paper.

However, it is obvious that analytical calculations are also of interest, and in the limit of isotropic systems the problem is simplified and can be solved analytically as shown below. Thus, this paper is devoted to the analytical way of magneto-optical ellipsometry data analysis in order to obtain all elements of the dielectric tensor (1).

2. Magneto-optical ellipsometry measurements

Conventional ellipsometry measurements [16, 17] involve measurements of so called ellipsometric parameters or in other words ellipsometric angles ψ and Δ. The researcher has to solve an inverse problem to find out some physical parameters from the experimental data. For example, the basic ellipsometry equation [17] helps find complex reflection coefficients R_p and R_s

$$\tan\,\psi\,\mathrm{e}^{\mathrm{i}\Delta} = \frac{R_p}{R_s}, \tag{2}$$

where subindex p-corresponds to p-polarisation of the incident light and s-to s-polarisation. Using optical models diagonal elements of the dielectric tensor can be found from these reflection coefficients. In case of the interest in off-diagonal elements one needs to add investigations with magnetic field present.

To do magneto-optical ellipsometry research we use a setup [18] which allows to perform both spectral ellipsometry and spectral magneto-optical ellipsometry measurements inside the ultra-high vacuum chamber. Magneto-optical measurements in the setup are held in geometry of the transverse magneto-optical Kerr effect in the visible spectral range.

Since there is a problem of residual magnetization during measurements, the magnetization reversal of the sample in the field from $-$H to $+$H is carried out as follows: (a) by applying a current to the electromagnet on the sample, an external magnetic field is set sufficient for the ferromagnetic film saturation; (b) spectral ellipsometry measurements of the angles ψ and Δ are performed over four optical zones [19] with averaging; (c) magnetization reversal of the sample is carried out with saturation, by setting the magnetic field of the same amplitude as at step a, but the opposite in direction; (d) spectral ellipsometry measurements are repeated over four optical zones with averaging. Finally, the magnetic contribution to the measured ellipsometric angles is calculated as the differences $\delta\psi = \psi(+\mathrm{H}) - \psi(-\mathrm{H})$, $\delta\Delta = \Delta(+\mathrm{H}) - \Delta(-\mathrm{H})$. The arithmetical means of the measured angles ψ and Δ are also calculated. This algorithm is repeated more than 50 times to average the measurements data and calculate the root mean square.

3. Calculation of the dielectric permittivity tensor

In this section we present the algorithm of analytical calculation of dielectric permittivity tensor from magneto-ellipsometry data, which is in general valid for any magnitude of the magnetic field. However, it makes sense to provide saturation for ferromagnetic samples in order to reliably interpret the nature of the ferromagnetism from absorption spectra.

To process magneto-optical ellipsometry data from the setup [18] we suggest considering magneto-optical response in the basic ellipsometry equation (2) and presenting field-free and field-dependent components in it:

$$\tan(\psi_0 + \delta\psi)\mathrm{e}^{\mathrm{i}(\Delta_0+\delta\Delta)} = \frac{R_p}{R_s} = \frac{R_{p0} + R_{p1}}{R_{s0}}, \tag{3}$$

where '0' is a subindex for measurements without external magnetic field, subindex '1' is used when external magnetic field is applied. In the following expressions the real parts are indicated by $'$ (R'_p and R'_s), the imaginary parts by $''$ (R''_p and R''_s). In case of applying magnetic field reflection coefficient R_p can be written as

$$R_p = R_{p0} + R_{p1} = R'_{p0} - \mathrm{i}R''_{p0} + R'_{p1} - \mathrm{i}R''_{p1}, \tag{4}$$

while R_s does not change in comparison to field-free measurements because of the transverse geometry [4].

We use the introduced notations in four steps of data analysis (3.1–3.4) which yields all components of dielectric permittivity tensor (1).

3.1. Dealing with field-free ellipsometric angles

We rewrite the right side of the basic ellipsometry equation (3) for a nonmagnetic state:

$$\begin{aligned}\tan\psi_0\,\mathrm{e}^{\mathrm{i}\Delta_0} &= \frac{R_{p0}}{R_{s0}} = \frac{R'_{p0} - \mathrm{i}R''_{p0}}{R'_{s0} - \mathrm{i}R''_{s0}} \\ &= \frac{(R'_{p0}R'_{s0} + R''_{p0}R''_{s0}) - \mathrm{i}(R''_{p0}R'_{s0} - R''_{s0}R'_{p0})}{(R'_{s0})^2 + (R''_{s0})^2}\end{aligned} \tag{5}$$

Then we use the definition of a complex number z

$$\begin{aligned}&x + \mathrm{i}y = z \\ &|z| = \sqrt{x^2 + y^2} \\ &\tan\arg z = y/x = \tan\,\theta \\ &z = |z|\,\mathrm{e}^{\mathrm{i}\theta}\end{aligned} \tag{6}$$

to write that

$$\tan\,\psi_0\,\mathrm{e}^{\mathrm{i}\Delta_0} = \frac{R_{p0}}{R_{s0}} = \frac{\sqrt{(R'_{p0}R'_{s0} + R''_{p0}R''_{s0})^2 + (R''_{s0}R'_{p0} - R''_{p0}R'_{s0})^2}}{(R'_{s0})^2 + (R''_{s0})^2}\,\mathrm{e}^{\mathrm{i}\theta}, \tag{7}$$

where

$$\tan\,\theta = \frac{R''_{s0}R'_{p0} - R'_{s0}R''_{p0}}{R'_{p0}R'_{s0} + R''_{p0}R''_{s0}}, \tag{8}$$

$$\tan\,\psi_0 = \frac{\sqrt{(R'_{p0}R'_{s0} + R''_{p0}R''_{s0})^2 + (R''_{s0}R'_{p0} - R''_{p0}R'_{s0})^2}}{(R'_{s0})^2 + (R''_{s0})^2}, \tag{9}$$

$$\mathrm{e}^{\mathrm{i}\Delta_0} = \mathrm{e}^{\mathrm{i}\theta}, \tag{10}$$

$$\Delta_0 = \theta = \arctan\,\frac{R''_{s0}R'_{p0} - R'_{s0}R''_{p0}}{R'_{p0}R'_{s0} + R''_{p0}R''_{s0}}. \tag{11}$$

3.2. Dealing with field-dependent ellipsometric angles

Similar to step 3.1, we rewrite the basic ellipsometry equation in the case of applying magnetic field to the sample:

$$\begin{aligned}\tan(\psi_0 + \delta\psi)\mathrm{e}^{\mathrm{i}(\Delta_0+\delta\Delta)} &= \frac{R'_{p0} + R'_{p1} - \mathrm{i}(R''_{p0} + R''_{p1})}{R'_{s0} - \mathrm{i}R''_{s0}} \\ &= \frac{(R'_{p0} + R'_{p1})R'_{s0} + (R''_{p0} + R''_{p1})R''_{s0}}{(R'_{s0})^2 + (R''_{s0})^2} \\ &\quad + \mathrm{i}\frac{(R''_{s0}(R'_{p0} + R'_{p1}) - R'_{s0}(R''_{p0} + R''_{p1}))}{(R'_{s0})^2 + (R''_{s0})^2}.\end{aligned} \tag{12}$$

Thus, according to (6)

$$\tan(\psi_0+\delta\psi) = \left((R'_{s0})^2 + (R''_{s0})^2\right)^{-1} \\ * \begin{pmatrix} ((R'_{p0}+R'_{p1})R'_{s0} + (R''_{p0}+R''_{p1})R''_{s0})^2 + \\ +(R''_{s0}(R'_{p0}+R'_{p1}) - R'_{s0}(R''_{p0}+R''_{p1}))^2 \end{pmatrix}^{1/2}, \tag{13}$$

$$\Delta_0 + \delta\Delta = \arctan \frac{R''_{s0}(R'_{p0}+R'_{p1}) - R'_{s0}(R''_{p0}+R''_{p1})}{R'_{s0}(R'_{p0}+R'_{p1}) + R''_{s0}(R''_{p0}+R''_{p1})}. \tag{14}$$

Taking into account expression (11) for Δ_0 for the non-magnetic case, we have

$$\delta\Delta = \arctan \frac{R''_{s0}(R'_{p0}+R'_{p1}) - R'_{s0}(R''_{p0}+R''_{p1})}{R'_{s0}(R'_{p0}+R'_{p1}) + R''_{s0}(R''_{p0}+R''_{p1})} \\ - \arctan \frac{R''_{s0}R'_{p0} - R'_{s0}R''_{p0}}{R'_{p0}R'_{s0} + R''_{p0}R''_{s0}}. \tag{15}$$

Similarly, it is necessary to get an expression for $\delta\psi$. For this purpose we compare (9) and (13). The denominators are equal, which means that we compare the numerators, namely, the radicands. When expanding all brackets, one can notice the following:

$$\tan(\psi_0) = \frac{\sqrt{\gamma}}{(R'_{s0})^2 + (R''_{s0})^2}, \tag{16}$$

$$\tan(\psi_0+\delta\psi) = \frac{\sqrt{\gamma+\chi}}{(R'_{s0})^2 + (R''_{s0})^2} \\ = \frac{\sqrt{\gamma}}{(R'_{s0})^2 + (R''_{s0})^2}\sqrt{1+\frac{\chi}{\gamma}} = \tan(\psi_0)\sqrt{1+\frac{\chi}{\gamma}}, \tag{17}$$

where

$$\frac{\chi}{\gamma} = \frac{({R'_{s0}}^2 + {R''_{s0}}^2)({R'_{p1}}^2 + {R''_{p1}}^2 + 2(R'_{p0}R'_{p1} + R''_{p0}R''_{p1}))}{(R'_{p0}R'_{s0} + R''_{p0}R''_{s0})^2 + (R''_{s0}R'_{p0} - R'_{s0}R''_{p0})^2}. \tag{18}$$

Thus,

$$\delta\psi = \arctan\left(\tan(\psi_0)\sqrt{1+\frac{\chi}{\gamma}}\right) - \psi_0. \tag{19}$$

3.3. Expanding $\delta\psi$ and $\delta\Delta$ into the Maclaurin series

We expand the obtained expressions for $\delta\psi$ and $\delta\Delta$ into the Maclaurin series by introducing small parameters α and β which are the ratios of magnetic to non-magnetic parts of reflection coefficient R_p:

$$\alpha = \frac{R'_{p1}}{R'_{p0}}, \tag{20}$$

$$\beta = \frac{R''_{p1}}{R''_{p0}}, \tag{21}$$

$$f(\alpha,\beta) \approx f(0,0) + \alpha\frac{\partial f(0,0)}{\partial\alpha} + \beta\frac{\partial f(0,0)}{\partial\beta}. \tag{22}$$

We do not take into account higher order power terms to the Maclaurin series as they describe effects which are not linearly proportional to the magnetization. For example, quadratic terms describe the Voight effect [20] which is proportional to the square of the magnetization. We neglect it because the Voight effect is usually 1000 times smaller than the magneto-optical Kerr effects [6]. That is why we leave only the terms proportional to the first power α and β on the grounds that there is a proportionality of the magneto-optical Kerr effect to the first power of the magneto-optical parameter [19]. Accordingly, we have

$$\delta\psi \approx \frac{\tan\,\psi_0}{1+\tan^2\,\psi_0}\frac{\alpha(R'_{p0})^2+\beta(R''_{p0})^2}{(R'_{p0})^2+(R''_{p0})^2}, \tag{23}$$

$$\delta\Delta \approx \frac{(\alpha-\beta)R'_{p0}R''_{p0}}{(R'_{p0})^2+(R''_{p0})^2}. \tag{24}$$

See supplementary 1 (http://stacks.iop.org/JPA/54/295201/mmedia) for details in deducing equations (23) and (24).

As indicated above, the contribution made by the magnetic field to the reflection coefficients is denoted by R'_{p1} and R''_{p1}, and in expressions (23) and (24) the small parameters α and β are responsible for magnetism. Therefore, it is necessary to express α and β from (23) and (24), then to obtain the expressions for R''_{p1} and R'_{p1} from α and β, and finally find out the desired Q_1 and Q_2 from R''_{p1} and R'_{p1}. We express small parameters α and β in terms of $\delta\psi$ and $\delta\Delta$ which are measured in the experiment:

$$\alpha \approx \frac{\delta\psi(1+\tan^2\,\psi_0)}{\tan\,\psi_0}+\frac{R''_{p0}}{R'_{p0}}\delta\Delta, \tag{25}$$

$$\beta \approx \frac{\delta\psi(1+\tan^2\,\psi_0)}{\tan\,\psi_0}-\frac{R'_{p0}}{R''_{p0}}\delta\Delta. \tag{26}$$

3.4. Getting the complex refractive index and the magneto-optical parameter

To get information about optical properties of a sample one needs well-known expressions [17]

$$\rho_0 = \tan\,\psi_0\,\exp(\mathrm{i}\Delta_0), \tag{27}$$

$$N_1 = n_1 - \mathrm{i}k_1 = N_0\,\sin\,\varphi_0\sqrt{1+\tan^2\varphi_0\left(\frac{1-\rho_0}{1+\rho_0}\right)^2}, \tag{28}$$

$$\cos\,\varphi_1 = \sqrt{1-\frac{{N_0}^2\sin^2\varphi_0}{N_1^2}}, \tag{29}$$

where N_0 is the complex refractive index of the external medium, φ_0 is the angle of light incidence, φ_1 is the angle of light refraction. Expression (28) is sufficient to calculate the complex refractive index of an FM sample from the spectral ellipsometry data (ψ_0 and Δ_0) [17] and is valid for bulk samples only. It means that the following algorithm of solving the inverse problem of magneto-optical ellipsometry corresponds to the optical model of a homogenous semi-infinite medium. That is why it can be used either when it can be assumed that there is material's isotropy in the plane, sharp interfaces between media, the uniformity in depth, or such approximations are enough for particular material's study [16, 17].

As soon as the refractive index (28) is calculated, one has to perform some mathematical operations with reflection coefficients which lead to the desired expressions for the magneto-optical parameter Q.

The formulae of the reflection coefficients which contain Q are obtained from solving Maxwell's equations with the fulfillment of the condition of equality of the tangential components of the electric and magnetic fields at the interface between the non-magnetic dielectric and the ferromagnetic media in the visible spectral range where magnetic permeability $\mu \approx \mu_0 \approx 1$ [4]. It is well-known from experiments that $Q \ll 1$ [4, 21], which is also taken into account while deriving the formulae of the reflection coefficients for the geometry of the transverse magneto-optical Kerr effect [4, 21]. Unlike the reflection at the interface of two non-magnetic media which is described by Fresnel coefficients [17], in the case of the light reflection at the interface between a non-magnetic medium and bulk ferromagnetic medium the symmetry of the reflection coefficient for p-polarization is broken due to the appearance of the second term, proportional to Q, appearing due to magneto-optical measurements in geometry of the transverse magneto-optical Kerr effect [4, 21]:

$$R_p = \frac{N_1 \cos\varphi_0 - N_0 \cos\varphi_1}{N_1 \cos\varphi_0 + N_0 \cos\varphi_1} - \mathrm{i}\frac{2QN_0{}^2 \sin\varphi_0 \cos\varphi_0}{(N_1 \cos\varphi_0 + N_0 \cos\varphi_1)^2}, \tag{30}$$

where φ_0 and φ_1 are considered as complex numbers due to the complex refractive index [16]. However, the transverse magneto-optical Kerr effect does not contribute to the complex reflection coefficient R_s [4, 21]:

$$R_s = \frac{N_0 \cos\varphi_0 - N_1 \cos\varphi_1}{N_0 \cos\varphi_0 + N_1 \cos\varphi_1}. \tag{31}$$

To find out Q, it is necessary to compare the reflection coefficient R_p in the form (30) with (4). It is obvious that

$$\frac{N_1 \cos\varphi_0 - N_0 \cos\varphi_1}{N_1 \cos\varphi_0 + N_0 \cos\varphi_1} = R'_{p0} - \mathrm{i}R''_{p0}, \tag{32}$$

$$-\mathrm{i}\frac{2QN_0{}^2 \sin\varphi_0 \cos\varphi_0}{(N_1 \cos\varphi_0 + N_0 \cos\varphi_1)^2} = R'_{p1} - \mathrm{i}R''_{p1}. \tag{33}$$

Then small parameters α, β (25) and (26) and the values of the complex reflection coefficient for the non-magnetic case are used to calculate real and imaginary components of the complex reflection coefficient for the magnetic case:

$$R'_{p1} = \alpha R'_{p0}, \tag{34}$$

$$R''_{p1} = \beta R''_{p0}. \tag{35}$$

At this step one should check whether both α and β are small, i.e. $\alpha \ll 1$ and $\beta \ll 1$.

From (33) one gets the following expression for the complex magneto-optical parameter:

$$Q = \frac{\mathrm{i}\left(R'_{p1} - \mathrm{i}R''_{p1}\right)(N_1 \cos\varphi_0 + N_0 \cos\varphi_1)^2}{N_0{}^2 \sin(2\varphi_0)}. \tag{36}$$

Taking into account (34) and (35), expression (36) turns into the expression for the desired complex magneto-optical parameter $Q = Q_1 - \mathrm{i}Q_2$:

$$Q = \frac{\left(\beta R''_{p0} + \mathrm{i}\alpha R'_{p0}\right)}{\sin(2\varphi_0)}\left(\frac{N_1}{N_0}\cos\varphi_0 + \cos\varphi_1\right)^2. \tag{37}$$

After substituting (25) and (26) into (37) and, grouping the multipliers we have

$$Q = \frac{R_{p0}}{\sin(2\varphi_0)}\left(\frac{N_1}{N_0}\cos\varphi_0 + \cos\varphi_1\right)^2\left(\frac{\delta\psi(1+\tan^2\psi_0)}{\tan\psi_0}\mathrm{i} - \delta\Delta\right). \tag{38}$$

Let us take into account expression (32), so that all physical quantities in the final expression for the magneto-optical parameters are measured parameters.

$$Q = \frac{\cos^2\varphi_1 - \left(\frac{N_1}{N_0}\cos\varphi_0\right)^2}{\sin(2\varphi_0)}\left(\delta\Delta - \mathrm{i}\frac{\delta\psi(1+\tan^2\psi_0)}{\tan\psi_0}\right). \tag{39}$$

Now, we have everything for analytical calculation of the desired complex magneto-optical parameter Q. Finally, the dielectric permittivity tensor (1) is calculated. Its diagonal components are a square complex refractive index (28). Off-diagonal components of dielectric tensor are a product of the imaginary unit i, diagonal component of the dielectric tensor and the complex magneto-optical parameter given by expression (39).

From (39) one can see that if the angle of incidence goes to zero or is equal to $\pi/2$, $Q \to \infty$. When complex refraction indices of the ambient and the studied medium are close in meaning, i.e. $(N_1/N_0) \approx 1$, the angle of light incidence becomes almost equal to the angle of light refraction because of expression (29) which turns into $\cos(\varphi_1) \approx \cos(\varphi_0)$. It leads to R_{p0} vanishing which means that the necessary condition $\alpha, \beta \ll 1$ breaks and the proposed algorithm cannot be used for calculating Q. Also parameters alpha and beta are not small when the media are transparent and the incidence angle is close to the Brewster's angle which leads to the value of R_{p0} approaching to zero. That is why it should be checked that the incidence angle is not close to the Brewster's angle which is easy as one of the advantages of the transverse configuration is that the researcher can use different angles of incidence of light with almost no restrictions. For non-zero absorption media it should be checked that the incidence angle is not close to pseudo-Brewster's one. Despite the fact that at this angle R_{p0} does not vanish, R_{p0} can be too small for fulfilling the condition of the smallness of α and β. Thus, the proposed algorithm is recommended to be used to analyse data from absorbing materials (when the value of k, the extinction coefficient, is close to n, the refractive index) with significant difference in complex refractive indices at the interface.

In fact, the condition $\alpha, \beta \ll 1$ is very often fulfilled due to Q being often much less than 1 [4, 21]. Thus it means that all components of dielectric permittivity tensor can be analytically obtained from the complex refractive index of the external medium N_0, the angle of incidence of light on the ferromagnetic sample φ_0, the data of spectral ellipsometry (ψ_0 and Δ_0) and magneto-ellipsometry ($\delta\psi$ and $\delta\Delta$).

As for the error of the components of dielectric permittivity tensor (1), it is primarily caused by the hardware error of the magneto-ellipsometry setup. The hardware error affects the values of ψ_0, Δ_0, $\delta\psi$ and $\delta\Delta$. Also there is an error of settling angle of light incidence φ_0 but it is no more than 20', so this contribution to the error of dielectric components is negligible.

For instance, let us consider the hardware error of the magneto-ellipsometry setup which is reported in [18]. It is important to note that the random hardware error in measuring the ellipsometric angles has some spectral dependence. This error is proportional to the ratio of the route-mean-square noise of the analog-to-digital converter (ADC) electronic circuit to the desired signal taken from the photodetectors. The desired signal is the product of the emission spectra of the light source, the optical transmission of all elements of the optical circuit, at a constant power of the incident light. The resulting spectral dependence of the photocurrent at the photodetectors for the spectral ellipsometer with a xenon lamp usually has a wide maximum in the region of 470 nm and several narrow peaks in the near infrared region. The noise of the ADC electronic circuit, in turn, does not depend on the state of the monochromator and is constant over the entire spectral range. Thus, when the integrated power of the radiation incident on the photodetectors is about several mW, the spectral dependence of the random hardware error takes the form of a flat plateau of minimum values in the range from 400 to 800 nm.

An example of calculating Q for a real sample is presented in supplementary 2.

4. Summary

In conclusion, using the presented expressions one can analytically calculate the values of the refractive index (n_1) and extinction coefficient (k_1) of the ferromagnetic sample, as well as the real (Q_1) and imaginary (Q_2) parts of its magneto-optical parameter Q from the data of ellipsometry (ψ_0 and Δ_0) and magneto-ellipsometry ($\delta\psi$ and $\delta\Delta$) measurements. The presented approach may be applied for processing both *in situ* and *ex situ* experimental spectral magneto-optical ellipsometry data. In general this analytical calculation is valid for both saturated and unsaturated cases. For example, it is possible to understand the ionic mechanism of ferromagnetism from the absorption spectra of the sample based on saturation measurements.

As shown in the text of the article, the restrictions for the proposed analytical solution of magneto-ellipsometry inverse problem comprise, firstly, all limits which arise from the use of the model of a homogenious semi-infinite medium, secondly, the transverse magneto-optical Kerr effect geometry, thirdly, magnetic contribution into reflection coefficient R_p should be small enough to introduce small parameters α, β and expand magneto-ellipsometry angles $\delta\psi$ and $\delta\Delta$ into series.

If the condition $\alpha, \beta \ll 1$ is fulfilled, certainly, the values of the complex refractive index N_1 of the ferromagnetic sample, as well as the complex magneto-optical parameter Q can be used to calculate the dielectric permittivity tensor ε, completing by that the magneto-ellipsometry inverse problem solving.

The proposed algorithm primarily can be applied for bulk FM samples characterisation. However this approach may be valuable to get the initial values of fitting parameters such as magneto-optical parameter Q and complex refractive index of an FM layer N_1 for numerical calculations which are necessary for thin and multilayered samples.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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