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Method of Anisotropic Metasurface Unit Cell Surface Impedance Calculation

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***Abstract*—High gain antennas based on metasurfaces with modulated surface impedance are a new class of antennas that have appeared in the last decade. They characterized by a simple design, but a relatively complex synthesis procedure. The paper briefly describes the modulated metasurface antennas principle of operation, and gives a specific method for calculating the anisotropic cell impedance tensor based on the calculation of cell eigenmodes in the infinite surface approximation. The method is intended for use at the first stage of the metasurface antenna synthesis, which consists in creating a database of cell tensorial impedances.**

Keywords—metasurface antenna, anisotropic metasurface, surface impedance, holographic beamforming

I. INTRODUCTION

Metasurfaces are two-dimensional analogs of metamaterials. In the last decade, they have been founding more and more applications in microwave technique. One of such applications is high gain antennas based on metasurfaces with modulated surface impedance. Propagation of TM and TE waves along surfaces with sinusoidally modulated along the propagation direction scalar impedance was first considered in [1]. Excitation of the impedance plane with periodic lumped inhomogeneities was later considered in [2]. Nowadays, numerical modeling capabilities have made it possible to calculate two-dimensional surfaces with a tensorial impedance and create efficient antennas based on them. For the first time such an antenna was proposed in [3], then the most advanced results were presented in [4-7]. High efficiency (up to 70%) of such antennas was achieved and the possibility of creating antennas with special beam shapes was shown. The modulated metasurface antennas principle of operation significantly differs from the traditional antennas principles. Modulated metasurface antennas can be considered as phase holograms or as leaky wave antennas [8]. They have a simple design, but require relatively complex synthesis method.

Despite of quite detailed description of the metasurface antennas synthesis methods based on the cells with known impedance (see, for example, [4, 7]), the methods of cells tensorial surface impedance calculation are not described in details in the literature. In this paper, Section II briefly describes modulated metasurface antennas principle of operation, and Section III provides an original method for calculating the cell tonsorial surface impedance based on the numerical calculation of cell eigenmodes.

II. MODULATED METASURFACE ANTENNAS

The modulated metasurface antenna is a dielectric substrate with a ground plane on one side and subwavelength patches on the other side (Fig. 1). The patches are arranged in a rectangular grid with a spacing of about 1/8 wavelength. They can have different shapes, such as ellipses ("grain of rice" cell), circles with a slit in the middle ("coffee bean" cell), or any other form with several degrees of freedom.

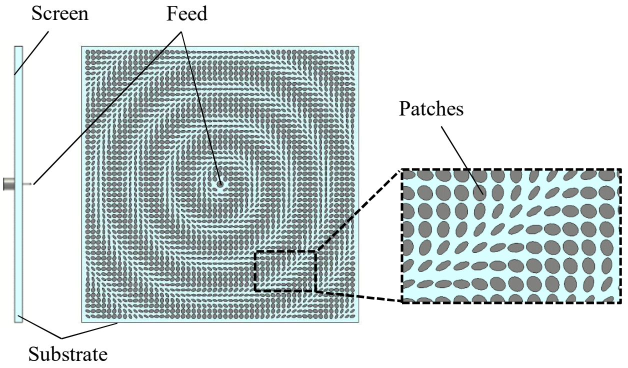

Fig. 1. Metasurface antenna example.

A feed with a special design excites cylindrical TM surface wave on the surface, which are then radiated by converting the TM surface wave into a leaky wave when propagating from the center to the edge of the structure. It is possible to control the radiation to form a free space wave of the desired polarization and direction by correctly selecting the shape of each patch.

In general, the metasurface-based antenna synthesis procedure consists of three main steps: (1) calculation of the cell impedance database, (2) calculation of the required impedance distribution on the surface, (3) antenna construction (Fig. 2). In the first step, a database is calculated, putting a certain value of the impedance tensor in accordance with each cell shape. Section III describes in detail the method by which such a calculation can be carried out. The second step consists in calculating the distribution of the impedance tensor components on the surface, which is necessary for the scattering of a cylindrical TM wave into the

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desired free space wave. This distribution can be calculated, for example, treating the metasurface as a phase hologram [3]:

$$H = \frac{\Psi_{obj}\Psi^{*}_{ref}}{\left|\Psi_{ref}\right|^{2}}, \tag{1}$$

where Ψ_{obj} is the wave to be formed in free space, and Ψ_{ref} is the metasurface excitation wave.

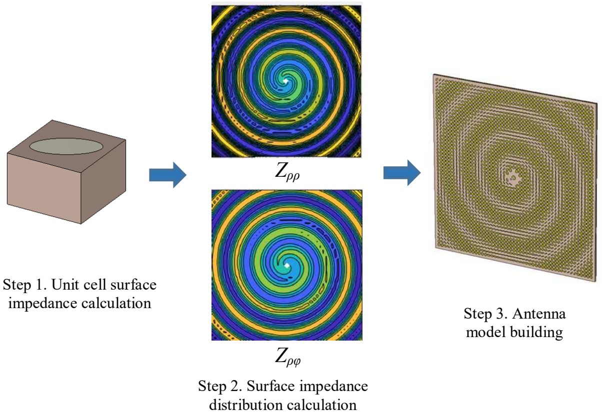

Fig. 2. Procedure of metasurface antenna design.

If the necessary surface distribution of the impedance tensor components and the impedance tensor values realized by each cell geometry are known, it is possible to make the third step – the cell geometry selection for each point of the surface and the construction of the whole antenna. For example, the antenna shown in Fig. 1 forms right-handed polarized radiation normally to the antenna surface.

III. METHOD OF UNIT CELL SURFACE IMPEDANCE CALCULATION

Let us consider, for example, a metasurface that consists of "grain of rice" type elementary cells. The cell (Fig. 3) is described by the following parameters: cell side *d*, substrate thickness *h*, substrate permittivity ε_r, major ellipse axis *a*, minor ellipse axis *b*, ellipse rotation angle α. The cell side *d* is 1/6 to 1/10 of the wavelength for which the metasurface is calculated. The thickness of the substrate *h* should be much smaller than the wavelength.

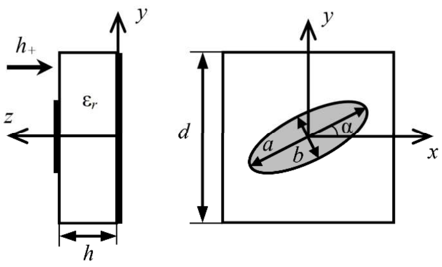

Fig. 3. Geometry of the "grain of rice" unit cell.

Suppose that a metasurface composed of such cells extends infinitely in all directions. The task is to find the surface impedance tensor of such a metasurface and, consequently, of a single cell.

The surface impedance tensor $\hat{Z}$ connects the tangential components of the current on the surface with height h_+, that is on the upper boundary of the patch layer (see Fig. 3), with the tangential components of the electric field on it:

$$\begin{pmatrix} \dot{E}_x \\ \dot{E}_y \end{pmatrix} = \begin{bmatrix} \dot{Z}_{xx} & \dot{Z}_{xy} \\ \dot{Z}_{yx} & \dot{Z}_{yy} \end{bmatrix} \begin{pmatrix} \dot{J}_x \\ \dot{J}_y \end{pmatrix} \tag{2}$$

The current is numerically equal to the tangential component of the magnetic field that is orthogonal to the current $\dot{J} = \left[\vec{n} \times \dot{H}\right]$, where $\vec{n}$ is the unit vector directed along to the surface normal. Therefore, the equation (2) can be written as:

$$\begin{cases} \dot{E}_x = \dot{Z}_{xx}\dot{H}_y + \dot{Z}_{xy}\dot{H}_x \\ \dot{E}_y = \dot{Z}_{yx}\dot{H}_y + \dot{Z}_{yy}\dot{H}_x \end{cases} \tag{3}$$

The fields $\dot{E}_x, \dot{E}_y, \dot{H}_x, \dot{H}_y$ can be found by numerical calculation in some numerical simulation software using Eigenmode Solver. On the cell boundaries x_{min}, x_{max}, y_{min}, y_{max} periodic boundary conditions should be set, and on the boundaries z_{min}, z_{max} – the electric boundary conditions should be set. The upper boundary z_{max} must be high enough above the substrate that the TM wave fields does not touch it. Since the surface wave field exponentially decreases with distance from the surface, it is sufficient to set z_{max} to be around 1.5–2 wavelengths.

To simulate wave propagation along the surface, say, along the *x*-axis, it is necessary to supplement the periodic boundary conditions with the phase difference of oscillations at the boundaries x_{min} and x_{max}. The phase difference can be chosen arbitrarily; it will determine the eigenfrequency of the cell f_e, for which the field components are calculated. The relationship between the eigenfrequency f_e, the phase velocity of the wave v_{ph}, the phase difference at the boundaries Δφ and the cell size *d* have the form:

$$f_e = \frac{2\pi}{\Delta\varphi}\frac{v_{ph}}{d} \tag{4}$$

Obviously, two pairs of independent equations of the type (2) are needed to find the four tensor components. They can be obtained by calculating the fields when the wave propagates along two orthogonal directions, for example, along the *x*-axis and along the *y*-axis. Then, when the wave propagates along the *x*-axis, we obtain tangential fields $\dot{E}_{1x}, \dot{E}_{1y}, \dot{H}_{1x}, \dot{H}_{1y}$, and when the wave propagates along the *y*-axis, we obtain $\dot{E}_{2x}, \dot{E}_{2y}, \dot{H}_{2x}, \dot{H}_{2y}$. Then we can write a system of equations:

$$\begin{cases} \dot{E}_{1x} = \dot{Z}_{xx}\dot{H}_{1y} + \dot{Z}_{xy}\dot{H}_{1x} \\ \dot{E}_{1y} = \dot{Z}_{yx}\dot{H}_{1y} + \dot{Z}_{yy}\dot{H}_{1x} \\ \dot{E}_{2x} = \dot{Z}_{xx}\dot{H}_{2y} + \dot{Z}_{xy}\dot{H}_{2x} \\ \dot{E}_{2y} = \dot{Z}_{yx}\dot{H}_{2y} + \dot{Z}_{yy}\dot{H}_{2x} \end{cases} \tag{5}$$

which has the following solution:

$$\dot{Z}_{xx}=\frac{\dot{E}_{1x}\dot{H}_{2x}-\dot{E}_{2x}\dot{H}_{1x}}{\dot{H}_{1y}\dot{H}_{2x}-\dot{H}_{2y}\dot{H}_{1x}}$$
$$\dot{Z}_{xy}=\frac{-\dot{E}_{1x}\dot{H}_{2y}+\dot{E}_{2x}\dot{H}_{1y}}{\dot{H}_{1y}\dot{H}_{2x}-\dot{H}_{2y}\dot{H}_{1x}}$$
$$\dot{Z}_{yx}=\frac{-\dot{E}_{1y}\dot{H}_{2y}+\dot{E}_{2y}\dot{H}_{1y}}{\dot{H}_{1y}\dot{H}_{2x}-\dot{H}_{2y}\dot{H}_{1x}} \qquad (6)$$
$$\dot{Z}_{yy}=\frac{\dot{E}_{1y}\dot{H}_{2x}-\dot{E}_{2y}\dot{H}_{1x}}{\dot{H}_{1y}\dot{H}_{2x}-\dot{H}_{2y}\dot{H}_{1x}}$$

The impedance calculated in this way is inhomogeneous over the cell surface (fig. 4). Therefore, the obtained values of the tensor components must be averaged over the surface. Such averaging is legitimate, because the cell is significantly smaller than the wavelength, and the wave “perceives” the effective (averaged) impedance of the cell.

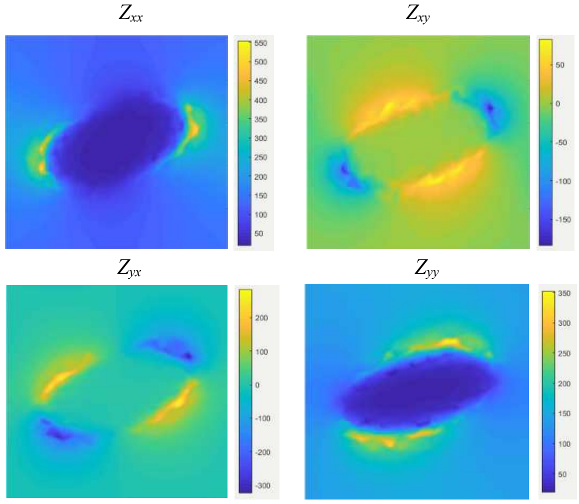

Fig. 4. Example of the impedance tensor components distribution on the unit cell surface (*d*=3 mm, *h*=2 mm, ε_r=4.6, *a*=2.6 mm, *b*=1.3 mm, α=30°).

It should be noted that the impedance calculated in this way is the impedance of an infinite metasurface consisting of cells of the same shape. The real metasurface with modulated impedance is quasiperiodic with the neighboring cells being slightly different from each other (see Fig. 1). When calculating a metasurface based antenna, it is assumed that this difference is insignificant.

For example, the results are given of calculating the averaged impedance tensor for the “grain of rice” cell with the parameters $d = 3$ mm, $h = 2$ mm, $\varepsilon_r = 4.6$, $a = 2.6$ mm, $b = 1.3$ mm, $\alpha = 30°$ at a frequency of $f = 12$ GHz:

$$Z = j\begin{bmatrix} 283 & 43 \\ 45 & 239 \end{bmatrix} \text{ Ohm},$$

and for the cell with the parameters *d*=3 mm, *h*=2 mm, ε_r=4.6, *a*=2.6 mm, *b*=2.6 mm, α=0° at the same frequency:

$$Z = j\begin{bmatrix} 350 & 0 \\ 0 & 349 \end{bmatrix} \text{ Ohm}.$$

The impedance values in these examples are rounded to integers. As can be seen, the impedance is an imaginary positive value, that is, it is inductive in nature. In the first example, the cell is anisotropic, so the tensor diagonal components have different values. The non-diagonal components should be equal to each other, but they turn out to be slightly different due to numerical errors. In the second example, the cell is isotropic ($a = b$), so the diagonal components are equal to each other (within calculation error), and the non-diagonal components are zero. That is, the impedance of an isotropic cell is essentially a scalar.

IV. CONCLUSION

Using the method described in this paper, one can form a database linking the geometric parameters of the metasurface cells with the components of their surface impedance tensors. Then this database can be used in the third stage of the modulated metasurface antenna synthesis, when for each point of the surface the cell geometry is selected, realizing the necessary values of impedance. The described method can be easily implemented using modern numerical electrodynamic modeling software such as CST Microwave Studio or Ansys HFSS.

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