



Cholesteric layers with tangential-conical surface anchoring for an electrically controlled polarization rotator

V. S. SUTORMIN,^{1,2,*} M. N. KRAKHALEV,^{1,2} I. V. TIMOFEEV,^{1,2} 
R. G. BIKBAEV,^{1,2}  O. O. PRISHCHEPA,¹
AND V. YA. ZYRYANOV¹

¹Kirensky Institute of Physics, Federal Research Center KSC SB RAS, Krasnoyarsk 660036, Russia

²Siberian Federal University, Krasnoyarsk 660041, Russia

*sutormin@iph.krasn.ru

Abstract: The voltage dependences of polarization characteristics of light passed through the cholesteric layers with tangential-conical boundary conditions have been investigated. It has been shown that such layers allow turning the polarization azimuth more than 70° because of the unique untwisting effect of the cholesteric helix, due to free azimuthal rotation of the director on the substrate with conical anchoring. Optical cells under study have some advantages (low control voltage, smooth variation of polarization azimuth, simplicity of design) and can be used as an electrically controlled polarization rotator of white light.

© 2021 Optical Society of America under the terms of the [OSA Open Access Publishing Agreement](#)

1. Introduction

Liquid crystals (LCs) are unique functional materials owing to their high sensitivity to external factors. The preferred orientation of the long axes of LC molecules is characterized by the unit vector called a director. The director configuration determines the macroscopic optical properties of LC, for instance, the orientation of optical axis which coincides with the director in nematic LC. The director configuration can be changed by an electric field. In practical applications, LC is in contact with the bounding surface which affects the director orientation at the interface and, in turn, assigns an orientational structure in LC bulk. There are several types of surface anchoring: tangential (the tilt angle of director θ_s measured from the interface is equal to 0°), normal ($\theta_s = 90^\circ$), and tilted ($0^\circ < \theta_s < 90^\circ$). For conical surface anchoring, the director is also tilted to the interface, but its azimuthal direction is degenerate [1]. Nowadays, LC systems with a strong surface anchoring are widely used. In these systems, the director at interface does not undergo change at the LC reorientation in the bulk. A weak surface anchoring attracts attention because of the lower operating voltage [2] and the realization of bistability [3] or multistability [4].

LC cells are known to be used in optical components such as polarization rotators [5–8] and wavelength plates [9–11]. One of the widely used polarization rotators is 90° twisted nematic (TN) cell [5]. The polarization parameters of light passed through the 90° TN cell depend on the wavelength of incident light [12]. However the approximation of adiabatic following of the light polarization is often employed for such a cell. If the wavelength of incident light satisfies Mauguin's condition $\varphi_{dir} \ll 2\pi\Delta nd/\lambda$ (φ_{dir} is total twist angle of director, Δn is optical anisotropy, d is LC layer thickness, λ is light wavelength) [13], then the TN cell rotates the linear polarization of the transmitted light by 90°. The electric field applied to the cell causes its switching to the state in which there is no polarization rotation. In this case, the polarization rotation angle cannot be smoothly varied because the total twist angle of the director remains unchanged owing to a strong surface anchoring. To realize the tunable angle of polarization rotation, the alignment layer with weak azimuthal surface anchoring on one of the LC cell

substrates was used [14]. The applied in-plane electric field caused the director reorientation on the substrate with weak surface anchoring that induced the transition from a homogeneous director configuration to the twisted one. The twist angle of LC orientational structure was tuned by the direction of the in-plane electric field. This polarization rotator requires the complex electrode configuration to be formed. Furthermore, the switching of the polarization rotation angle was performed only between several values. The described device can operate as the electrically controlled achromatic polarization rotator if the Mauguin's condition is valid for all light wavelengths of the desired range. Another approach to realize the LC-based achromatic polarization rotator was proposed in [7,15,16] where the LC cells with continuous variation of director twist over the sample area along one direction were formed. The angle of polarization rotation is tuned by changing of beam position.

Cholesteric liquid crystals (CLCs) have a helical configuration of director in a free state. The orientational structures of CLCs in the LC cells with tangential-conical boundary conditions have been recently investigated [17,18]. It has been shown that in the twisted structure of CLC the azimuthal angle of the director on the substrate with conical surface anchoring depends on the ratio between the thickness of LC layer d and the cholesteric pitch p (the distance over which the director rotates by 2π). Moreover, this director azimuthal angle changes by the electric field applied perpendicular to the cell substrates [18]. For this reason, the angle of polarization rotation can be controlled in such a LC system. This paper considers the voltage dependences of the polarization characteristics of light passed through the cholesteric layer with tangential-conical boundary conditions.

2. Materials and methods

The sandwich-like cells consisted of two ITO-coated glass substrates covered with alignment polymer films and the CLC layer between them. One of the cell's substrates was covered with poly(vinyl alcohol) (PVA) (Sigma Aldrich), and another one was coated with poly(isobutyl methacrylate) (PiBMA) (Sigma Aldrich). The polymer films were formed on the substrates by the spin coating method. The PVA film was unidirectionally rubbed while the PiBMA film remained untreated after being formed. The nematic mixture LN-396 (Belarusian State Technological University) doped with the chiral additive cholesterylacetate (Sigma Aldrich) was used as the cholesteric liquid crystal with left-handed helix. The main part of LN-396 composition is alkyl-cyanobiphenyl and alkyloxy-cyanobiphenyls [19]. For LN-396 the PVA specifies the tangential boundary conditions and PiBMA imposes the conical surface anchoring with the tilt angle 50° [17]. Therefore, the PiBMA film was used without additional treatment in the samples under study. The cells were filled with the CLC in the mesophase at room temperature. The LC layer thicknesses were 13.9, 21.6, and 35.3 μm assigned by the glass microspheres (Duke Scientific) or teflon spacers. The ratio of the cholesteric layer thickness d to the helix pitch p was 0.61 for all samples under study. Depending on the ratio d/p , the defect-free twisted structure or twisted structure with defect loops and defect lines could be formed in the CLC cell [17,18]. In the present work, the sample prepared contained a small number of defects in the CLC cells with d equal to 13.9 and 21.6 μm . The latter were eliminated by the voltage application which resulted in the defect-free twisted structure after switching the voltage off [18].

The polarization of light passed through the CLC cell is characterized by the polarization azimuth ψ and ellipticity angle χ (Fig. 1(a)). The polarization azimuth is the angle between the semi-major axis a of the polarization ellipse and the x -axis. The ellipticity angle is an arctangent of the ratio of semi-minor axis b to the semi-major one a of the polarization ellipse. The angles ψ and χ are restricted to the following intervals: $0^\circ \leq \psi \leq 180^\circ$, $-45^\circ \leq \chi \leq +45^\circ$. The positive and negative values of χ correspond to right-handed and left-handed polarizations, respectively.

Polarization characteristics of light passed through the LC cell were measured using the quarter-wave plate Q and the linear analyzer A (Fig. 1(b)) [20]. The light source was the He-Ne

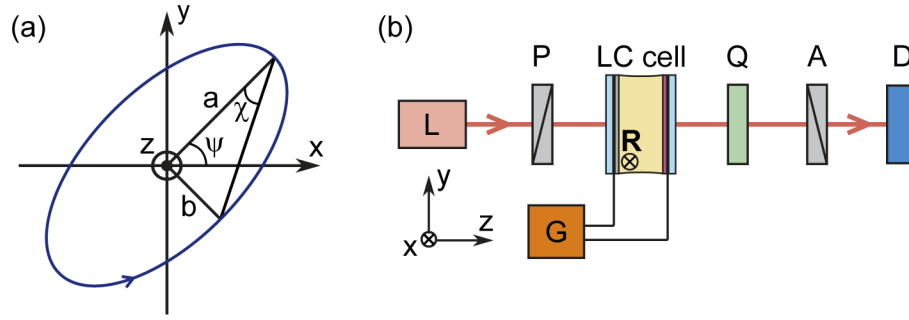


Fig. 1. Polarization ellipse (a) and scheme of the experimental setup (b). a , b are semi-major and semi-minor axes of the polarization ellipse; ψ , χ are polarization azimuth and ellipticity angle. L is He-Ne laser, P is polarizer, Q is quarter-wave plate, A is analyzer, D is photodetector, G is generator, R is rubbing direction of PVA film. The light propagates along the z -axis. The indicated polarization is left-handed.

laser with wavelength $\lambda = 632.8$ nm. The LC cell was placed so that the substrate covered by the PVA film was an entrance substrate and the rubbing direction R of the PVA film was parallel to the x -axis. The linear polarization of incident light was parallel to the y -axis. To investigate the voltage dependences of ψ and χ angles, the 1 kHz AC voltage was applied to the LC cell using the function generator AHP-3122 (AKTAKOM).

To simulate the polarization characteristics of monochromatic light with wavelength $\lambda = 632.8$ nm passed through the cell, the orientational structure of CLC was calculated by means of the free energy minimization method [21]. After that, the Berreman 4×4 matrix method [22] was applied to calculate the polarization of the transmitted light. The following parameters were used for the simulation: the elastic constants of the splay $k_{11} = 11.1$ pN, twist $k_{22} = 7.6$ pN, and bend $k_{33} = 17.1$ pN; the refractive indices for light polarized parallel and perpendicular to the director $n_{\parallel} = 1.720$ and $n_{\perp} = 1.520$ ($\lambda = 632.8$ nm); the dielectric constants parallel and perpendicular to the director $\epsilon_{\parallel} = 19.5$ and $\epsilon_{\perp} = 5.2$ (1 kHz).

The optical properties of the CLC structure for white light were examined by Finite-Difference Time-Domain (FDTD) method implemented in commercial Lumerical package. The LN-396 liquid crystal had the following refractive indices n_{\parallel} and n_{\perp} depending on the wavelength λ :

$$n_{\parallel} = 1.786 - \frac{0.046(\lambda^{-2} - 0.4513^{-2})}{0.5493^{-2} - 0.4513^{-2}} - 0.02 \left(\frac{\lambda^{-4} - 0.4513^{-4}}{0.5493^{-4} - 0.4513^{-4}} - \frac{\lambda^{-2} - 0.4513^{-2}}{0.5493^{-2} - 0.4513^{-2}} \right), \quad (1)$$

$$n_{\perp} = 1.5445 - \frac{0.0165(\lambda^{-2} - 0.4513^{-2})}{0.5493^{-2} - 0.4513^{-2}}. \quad (2)$$

CLC structure was illuminated by the plane wave with normal incidence along the z -axis and linear polarization along the y -axis. The white light source was modeled by choosing a pulse duration that allowed an accurate span of wavelength range from 400 to 700 nm. The accuracy of the simulation was ensured by using 1001 points per wavelength. The periodic boundary conditions were applied at the lateral boundaries of the simulation box (along the x and y -axis), while the perfectly matched layers were used on the remaining top and bottom sides. As a result, x and y electrical components of light wave at the output of the CLC structure were determined. The projections of E_x and E_y on the axis of analyzer allowed us to calculate the resulting electric field E along its direction. Using the value E the light transmission of system consisting of the CLC structure and analyzer was found. Such calculations were carried out for different angles of the analyzer to obtain the dependence of light transmission on the analyzer rotation angle.

3. Results and discussion

The orientational structure with simultaneously varying the director tilt and azimuthal angles is formed in the CLC cells with tangential-conical boundary conditions. Figure 2 shows the calculated orientational structures of CLC in the initial state and when the voltage $U = 2$ V is applied perpendicular to the cell. The values of tilt θ and azimuthal φ angles of the director profile are presented in the same figure. In the initial state, θ changes nonlinearly from 0° (substrate with tangential anchoring) to 50° (substrate with conical anchoring), and φ varies from 0° to -200° along z -axis (Fig. 2(a)). The negative value of the azimuthal angle of director means that the cholesteric helix is left-handed. In our case, the value of $\varphi = -200^\circ$ on the substrate with conical anchoring corresponds to the total twist angle of the director φ_{dir} in the CLC cell. The voltage applied perpendicular to the substrates induces the change in θ angle and, at the same time, decreases the absolute value of the total twist angle of the director. The Fig. 2(b) shows that $\varphi = -117^\circ$ on the substrate with conical surface anchoring for $U = 2$ V.

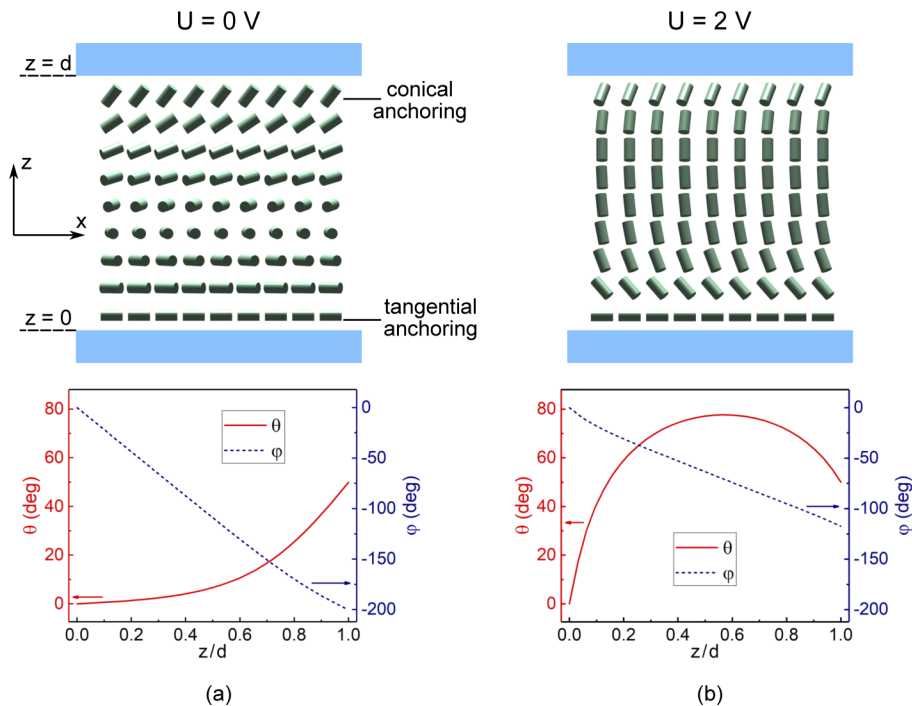


Fig. 2. Calculated orientational structures of the CLC layer with tangential-conical boundary conditions (top row); the tilt θ and azimuthal φ angle of the director vs z/d (bottom row). (a) Initial state; (b) voltage $U = 2$ V is applied perpendicular to the CLC layer. $z/d = 0$ and $z/d = 1$ correspond to the substrates with tangential and conical surface anchoring, respectively. Ratio $d/p = 0.61$.

Figure 3 shows experimental dependences of the polarization azimuth ψ and ellipticity angle χ versus the applied voltage U . The parameters ψ_{in} and χ_{in} of linearly polarized incident light are equal to 90° and 0° , respectively, since the linear polarization is parallel to the y -axis. In the absence of applied voltage, $\psi = 84.5^\circ$ and $\chi = -2.8^\circ$ for the light passed through the LC cell with $d = 13.9 \mu\text{m}$ (Fig. 3(a)). The significant changes in the polarization parameters are not observed in the range of voltages from 0 to 0.7 V. Further increasing the voltage leads to the complicated changes in ψ and oscillations of χ (Fig. 3(a)). One can observe the increase in polarization azimuth from 84.5° to 144.5° with one oscillation in the range of voltages 0.7–1.18 V. The

rise of ψ is accompanied by χ oscillations which are characterized by approximately constant peak-to-peak value equal to 14° . The nonmonotonic decrease in ψ and significant increase in the absolute value of χ occur when applied voltage $U > 1.18$ V.

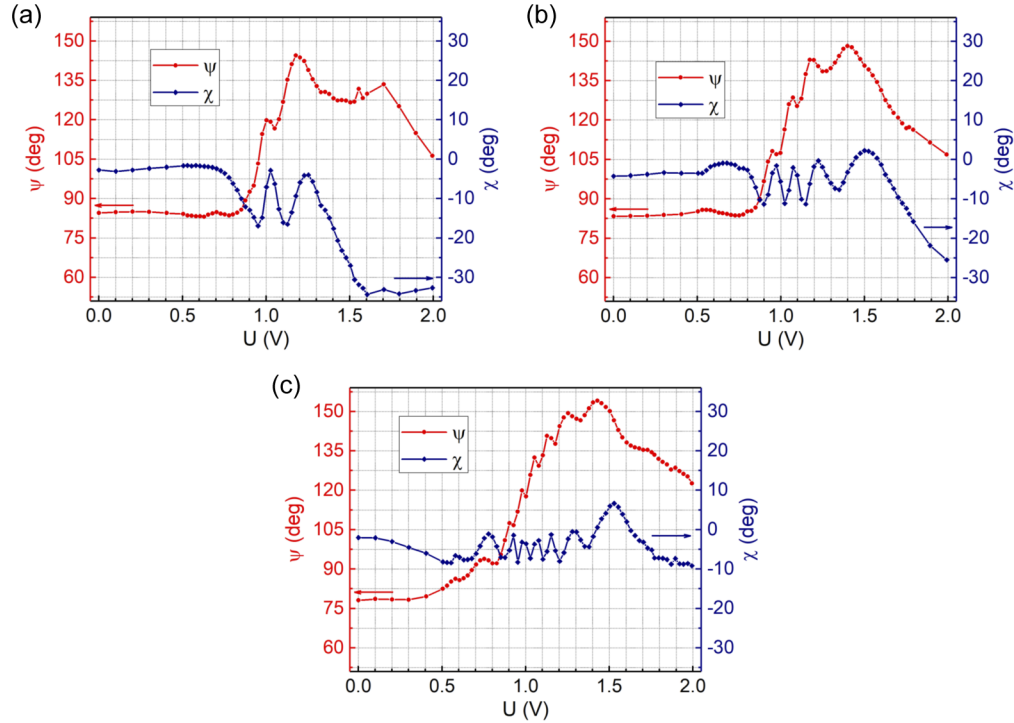


Fig. 3. Experimental dependences of the polarization azimuth ψ and ellipticity angle χ of the transmitted light from the applied voltage U . (a) $d = 13.9 \mu\text{m}$; (b) $d = 21.6 \mu\text{m}$; (c) $d = 35.3 \mu\text{m}$. The ratio d/p is equal to 0.61 for all the samples.

The similar dependences $\psi(U)$ and $\chi(U)$ are observed for the samples with $d = 21.6 \mu\text{m}$ and $d = 35.3 \mu\text{m}$ (Figs. 3(b) and 3(c)). However, the number of ψ and χ oscillations increases for the cells with thicker CLC layers. At the same time, the peak-to-peak values of these oscillations decrease: for example, the peak-to-peak values of χ oscillations are approximately equal to 10° and 7° for the samples with $d = 21.6 \mu\text{m}$ and $d = 35.3 \mu\text{m}$, respectively. Moreover, one can observe voltage ranges where the peak-to-peak values are approximately constant similarly to LC cell with $d = 13.9 \mu\text{m}$. The increase in CLC layer thickness leads to the rise of ψ variation from the initial value. The polarization azimuth changes from 83.3° to 148.2° in the range of voltages 0–1.40 V (Fig. 3(b)) for the LC cell with $d = 21.6 \mu\text{m}$. For the sample with $d = 35.3 \mu\text{m}$, ψ increases from 78.1° to 154.1° in the range of voltages 0–1.43 V (Fig. 3(c)).

The calculated dependences of light polarization parameters ($\lambda = 632.8 \text{ nm}$) on the applied voltage for the LC cell with tangential-conical boundary conditions are presented in Fig. 4. The calculations were performed for the sample with $d = 21.6 \mu\text{m}$ by the Berreman 4×4 matrix method. The simulation results are in agreement with the experiment. The small quantitative discrepancy could result from using in calculations of the elastic constants and dielectric anisotropy values of nematic mixture E7. In Fig. 4(a) the calculated voltage dependence of φ_{dir} is also shown. The applied electric field causes the smooth variation of φ_{dir} from -200° to -117° in the range of voltages 0–2 V. The observed dependences $\psi(U)$ and $\chi(U)$ result from the change in the director tilt and azimuthal angles induced by the applied voltage (Fig. 2). The decrease in total twist angle of the director leads to significant variation of the polarization

azimuth. The change in the director tilt angle causes the decrease in effective birefringence of LC which results in ψ and χ oscillations. For comparison, Fig. 4 also shows the calculation data for the CLC cell with the same parameters, but the surface anchoring is tangential on one substrate and tilted without azimuthal degeneration on another one. The initial director orientation for two presented CLC cells is the same. It can be seen that for the CLC cell with tangential-tilted boundary conditions the increase in the applied voltage leads only to the ψ oscillations and there is almost no increment in ψ angle (Fig. 4(a)). Moreover, the peak-to-peak values of χ oscillations are slightly higher for this cell in comparison with the cell in which the tangential-conical boundary conditions are specified (Fig. 4(b)). Such a great difference in behavior of polarization parameters in CLC cell with tangential-tilted boundary conditions is connected with the fact that in this case the applied voltage does not change the total twist angle of director (Fig. 4(a)).

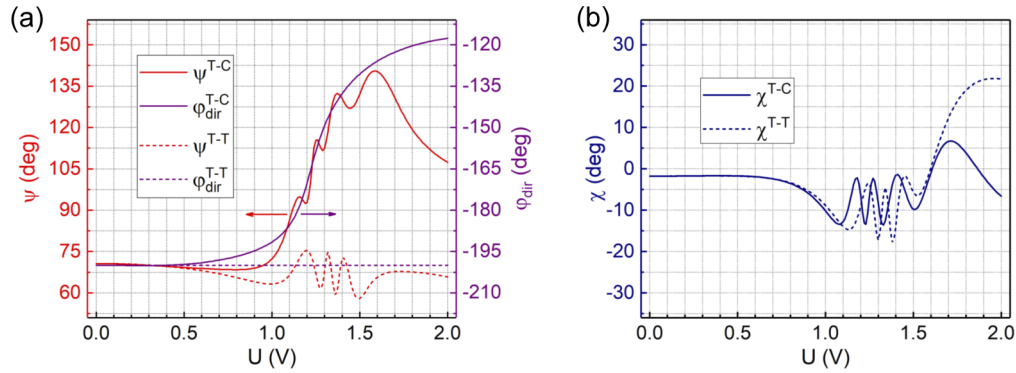


Fig. 4. Calculated dependences of ψ , φ_{dir} (a) and χ (b) from the applied voltage U for light ($\lambda = 632.8$ nm) passed through the CLC cell with tangential-conical (T-C) and tangential-tilted (T-T) boundary conditions. The polarization characteristics of light were obtained by the Berreman 4×4 matrix method. The CLC layer thickness $d = 21.6 \mu\text{m}$. The ratio $d/p = 0.61$.

In general, the linearly polarized light passed through the twisted director configuration becomes elliptically polarized. As mentioned above, the analytical expressions of the light polarization parameters for TN cell with tangential surface anchoring on both substrates were given in [12]. The general expressions for TN cell with the equal tilt angles of the director on both substrates were obtained in [23]. To the best of our knowledge, there are no analytical expressions for light polarization parameters for TN cell with different director tilt angles on substrates. Nevertheless, the observed voltage dependences of ψ and χ angles can be qualitatively explained using the simplified model. In this model the structure with inhomogeneous director tilt angle and twisted linearly along the z -axis is substituted by the structure with uniform director tilt angle $\bar{\theta} = (1/d) \int_0^d \theta(z) dz$ and linear twist [24]. The value $\bar{\theta} = 13.3^\circ$ was obtained using the simulation data of the director configuration of CLC with tangential-conical boundary conditions at $U = 0$ V (Fig. 2(a)). For the structure with the constant tilt director angle, the effective birefringence of LC is given by [25]:

$$\Delta n_{eff} = n_{\parallel} n_{\perp} / \sqrt{n_{\perp}^2 \cos^2 \bar{\theta} + n_{\parallel}^2 \sin^2 \bar{\theta}} - n_{\perp}. \quad (3)$$

The ellipticity angle and orientation of the polarization ellipse of light passed through the TN cell is given by [25]:

$$\chi = \frac{1}{2} \sin^{-1} \left[\frac{\Gamma \varphi_{dir}}{\varphi_{dir}^2 + (\Gamma/2)^2} \sin^2 \left(\sqrt{\varphi_{dir}^2 + (\Gamma/2)^2} \right) \right], \quad (4)$$

$$\psi' = \frac{1}{2} \tan^{-1} \left[\frac{2\varphi_{dir} \sqrt{\varphi_{dir}^2 + (\Gamma/2)^2} \tan \left(\sqrt{\varphi_{dir}^2 + (\Gamma/2)^2} \right)}{\varphi_{dir}^2 + (\Gamma/2)^2 - (\varphi_{dir} - (\Gamma/2)^2) \tan^2 \left(\sqrt{\varphi_{dir}^2 + (\Gamma/2)^2} \right)} \right], \quad (5)$$

where ψ' is the angle of major axis of the polarization ellipse measured from the projection of director on the output substrate, Γ is the phase retardation and φ_{dir} is the total twist angle of director. Γ is given by

$$\Gamma = 2\pi d \Delta n_{eff} / \lambda. \quad (6)$$

Figure 5(a) shows the dependences $\chi(\Gamma)$ calculated by the Eq. (4) for the various twist director angles φ_{dir} . One can see that the decrease in the phase retardation Γ caused, for example, by the action of electric field results in the oscillations of ellipticity angle. If Γ decreases, then the peak-to-peak χ rises. If the director twist angle decreases, then the peak-to-peak χ reduces. Thus, when the applied voltage increases, the variation of peak-to-peak χ connected with Γ can be compensated by the decrease in φ_{dir} . This case is probably realized in the LC system under study (Fig. 3), where the peak-to-peak χ is approximately constant in the certain voltage ranges. In addition, Fig. 5(a) shows that for the thicker CLC layer and, correspondingly, the larger initial phase retardation the peak-to-peak χ is less than for the thinner CLC layer that is in agreement with the experiment. The similar situation is observed for the parameter ψ' . Figure 5(b) shows the dependences $\psi'(\Gamma)$ calculated by the Eq. (5) for the various total twist angles of director φ_{dir} . It can be seen that ψ' value depends on the phase retardation Γ and, correspondingly, on LC layer thickness d . Therefore, ψ values for LC cells with various d should be different at $U = 0$ V. This situation is observed in the experiment (Fig. 3). When the phase retardation changes (for example, by electric field), the major axis of polarization ellipse of transmitted light oscillates relative to the director on the output substrate (Fig. 5(b)). Like the ellipticity angle, the change in the peak-to-peak ψ' can be small due to the simultaneous variation of the phase retardation and total twist angle of director. The appearance of ψ oscillations (Figs. 3 and 4) is related with the behavior of ψ' . Moreover, the peak-to-peak value of the ψ and ψ' oscillations decreases when the thickness of CLC layer increases. The behavior of $\psi(U)$ and $\chi(U)$ changes at certain voltage (Fig. 3). In particular, the major axis of polarization ellipse does not oscillate relative to the director on the output substrate when applied voltage increases (Fig. 4(a)). For samples of thicker LC layers this change occurs at higher control voltages and, consequently, at less total twist angles of the director. For this reason, the maximal value of ψ grows with increasing d (Fig. 3).

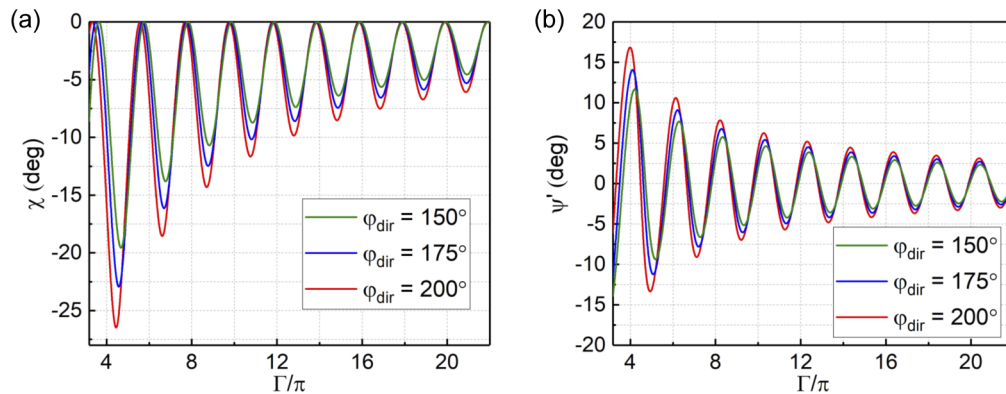


Fig. 5. Calculated dependences of χ and ψ' angles from the phase retardation Γ for the structures with the various total twist angles of the director φ_{dir} .

Let us consider the light polarization characteristics of the cell with CLC thickness $d = 35.3 \mu\text{m}$ (Fig. 3(c)). As mentioned above, the polarization azimuth of monochromatic light ($\lambda = 632.8 \text{ nm}$)

changes from 78.1° to 154.1° in the range of voltages 0–1.43 V. The absolute value of ellipticity angle is less than 10° in this range and, consequently, the polarization of transmitted light is close to linear. If the wavelength of incident light were less than 632.8 nm, the applied voltage would lead to less peak-to-peak χ due to the larger initial phase retardation of the LC layer. Since $\lambda = 632.8$ nm is in the long-wave range of the visible light, one can suppose that the polarization of transmitted light would be close to linear for all the wavelengths of the visible light. In this case, the CLC cell with tangential-conical boundary conditions can operate as the electrically controlled polarization rotator for white light. To check the hypothesis, the situation when the linearly polarized white light passes through the CLC cell with tangential-conical boundary conditions and the analyzer has been theoretically considered. Figure 6 shows the calculated voltage dependences of β angle and ratio T_{max}/T_{min} . β is the angle between the analyzer and the x -axis corresponding to the maximum light transmittance T_{max} of the system. T_{min} is the minimum light transmittance reached at the analyzer orientation $\beta \pm 90^\circ$. The applied voltage causes in the range of 0–2.0 V the variation of the polarization direction of white light from 72° to 153° (Fig. 6). The angle β does not change significantly in the range of voltage from 0 to 0.8 V and it varies noticeably at $U > 0.8$ V. Such behavior of the light polarization rotation is explained by the change in φ_{dir} angle. One can observe in Fig. 4(a) that significant variations of φ_{dir} occur at $U > 0.8$ V in spite of the thresholdless reorientation process of director in CLC bulk. The average value of T_{max}/T_{min} is equal to 113 in the range of voltage from 0 to 1.6 V which is evidence of a reasonably high degree of light polarization. The ratio T_{max}/T_{min} decreases sharply when the voltage exceeds 1.60 V and becomes equal to 7 at $U = 2$ V. Thus, one can conclude that the CLC cell operates as an optical rotator for white light in the certain voltage range.

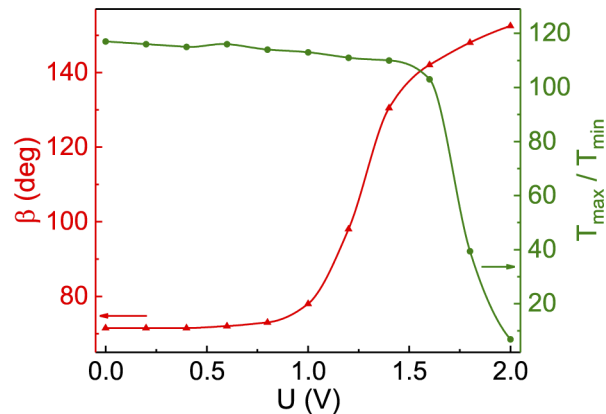


Fig. 6. Calculated dependences of the β angle and ratio T_{max}/T_{min} from the applied voltage U for the linearly polarized white light passed through the CLC cell with tangential-conical boundary conditions and analyzer. The CLC layer thickness $d = 35.3 \mu\text{m}$. The ratio $d/p = 0.61$.

4. Conclusion

The polarization characteristics of linearly polarized light passed through the cholesteric layers with tangential-conical boundary conditions have been investigated. CLC layers of various thicknesses but with the same ratio $d/p = 0.61$ have been examined. The experimental dependences of the polarization azimuth ψ and the ellipticity angle χ of the transmitted light from the voltage applied to the CLC layer have been measured. The electric field applied perpendicular to the layers induces both oscillations of the ellipticity angle and a significant change in polarization azimuth. The oscillations are mainly conditioned by a decrease in the

effective birefringence of LC owing to an increase in the director tilt angle. The polarization azimuth ψ varies more than 70° angle because of the unique untwisting effect of the cholesteric helix under the action of electric field due to the free azimuthal rotation of the director on the substrate with conical anchoring. At that, the curve $\psi(U)$ weakly oscillates in accordance with the oscillations of ellipticity angle χ .

The numerical simulation of polarization characteristics is in agreement with the experiment. It has been shown that the CLC layers with tangential-conical surface anchoring allow realizing significant changes in the polarization azimuth in contrast to the CLC layers with tangential-tilted boundary conditions without azimuthal degeneration. The voltage dependences of the polarization azimuth ψ and ellipticity angle χ have been qualitatively explained using a simplified model with a homogeneous tilted director configuration.

The optical cells based on the cholesteric layers with tangential-conical boundary conditions can be used as an electrically controlled polarization rotator of white light. Such devices have the distinct features such as smooth variation of polarization azimuth by electric field and simplicity of design that provides them an advantage over closest counterparts [7,14–16]. Moreover, some parameters of the material and cell, such as the tilt angle of director on the substrate with conical anchoring, LC birefringence and its dielectric anisotropy, layer thickness, can be optimized to improve the electrooptical characteristics of CLC cells. For example, the variation of d/p and tilt angle of director on the substrate can increase the angle of polarization rotation and decrease the ellipticity angle. The change in dielectric anisotropy, elastic constants, and cholesteric pitch can reduce the control voltages. Thus, the influence of the above-mentioned parameters on the director reorientation and corresponding change in light polarization is an interesting matter for future investigation.

Funding. Russian Science Foundation (Grant No. 18-72-10036).

Disclosures. The authors declare no conflict of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

References

1. B. Jerome, "Surface effects and anchoring in liquid crystals," *Rep. Prog. Phys.* **54**(3), 391–451 (1991).
2. G. P. Bryan-Brown, E. L. Wood, and I. C. Sage, "Weak surface anchoring of liquid crystals," *Nature* **399**(6734), 338–340 (1999).
3. I. Dozov, M. Nobili, and G. Durand, "Fast bistable nematic display using monostable surface switching," *Appl. Phys. Lett.* **70**(9), 1179–1181 (1997).
4. J. S. Gwag, Y.-K. Kim, C. H. Lee, and J.-H. Kim, "Realization of multi-stable ground states in a nematic liquid crystal by surface and electric field modification," *Sci. Rep.* **5**(1), 11368 (2015).
5. M. Schadt and W. Helfrich, "Voltage-dependent optical activity of a twisted nematic liquid crystal," *Appl. Phys. Lett.* **18**(4), 127–128 (1971).
6. O. Aharon and I. Abdulhalim, "Liquid crystal wavelength-independent continuous polarization rotator," *Opt. Eng.* **49**(3), 034002 (2010).
7. H. Ren and S.-T. Wu, "Liquid-crystal-based linear polarization rotator," *Appl. Phys. Lett.* **90**(12), 121123 (2007).
8. T.-Y. Chung, M.-C. Tsai, C.-K. Liu, J.-H. Li, and K.-T. Cheng, "Achromatic linear polarization rotators by tandem twisted nematic liquid crystal cells," *Sci. Rep.* **8**(1), 13691 (2018).
9. M. J. Abuleil and I. Abdulhalim, "Tunable achromatic liquid crystal waveplates," *Opt. Lett.* **39**(19), 5487–5490 (2014).
10. R. K. Komanduri, K. F. Lawler, and M. J. Escuti, "Multi-twist retarders: broadband retardation control using self-aligning reactive liquid crystal layers," *Opt. Express* **21**(1), 404–420 (2013).
11. M. D. Lavrentovich, T. A. Sergan, and J. R. Kelly, "Switchable broadband achromatic half-wave plate with nematic liquid crystals," *Opt. Lett.* **29**(12), 1411–1413 (2004).
12. C. H. Gooch and H. A. Tarry, "The optical properties of twisted nematic liquid crystal structures with twist angles ≤ 90 degrees," *J. Phys. D: Appl. Phys.* **8**(13), 1575–1584 (1975).
13. L. M. Blinov and V. G. Chigrinov, *Electrooptic Effects in Liquid Crystal Materials* (Springer, 1996).
14. R. Yamaguchi, T. Yamanaka, and S. Sato, "Liquid-crystal optical rotator using weak azimuthal anchoring surface," *Jpn. J. Appl. Phys.* **40**(Part 1, No. 11), 6522–6525 (2001).
15. Y.-D. Chen, K.-T. Cheng, C.-K. Liu, and A. Y.-G. Fuh, "Polarization rotators fabricated by thermally-switched liquid crystal alignments based on rubbed poly(n-vinyl carbazole) films," *Opt. Express* **19**(8), 7553–7558 (2011).

16. C.-Y. Huang, H.-Y. Tsai, Y.-H. Wang, C.-M. Huang, K.-Y. Lo, and C.-R. Lee, "Linear polarization rotators based on dye-doped liquid crystal cells," *Appl. Phys. Lett.* **96**(19), 191103 (2010).
17. M. N. Krakhalev, R. G. Bikbaev, V. S. Sutormin, I. V. Timofeev, and V. Y. Zyryanov, "Nematic and cholesteric liquid crystal structures in cells with tangential-conical boundary conditions," *Crystals* **9**(5), 249 (2019).
18. M. N. Krakhalev, O. O. Prishchepa, V. S. Sutormin, R. G. Bikbaev, I. V. Timofeev, and V. Y. Zyryanov, "Electrically induced transformations of defects in cholesteric layer with tangential-conical boundary conditions," *Sci. Rep.* **10**(1), 4907 (2020).
19. M. N. Krakhalev, O. O. Prishchepa, V. S. Sutormin, and V. Y. Zyryanov, "Director configurations in nematic droplets with tilted surface anchoring," *Liq. Cryst.* **44**(2), 355–363 (2017).
20. R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, 1977).
21. I. V. Timofeev, Y.-T. Lin, V. A. Gunyakov, S. A. Myslivets, V. G. Arkhipkin, S. Y. Vetrov, W. Lee, and V. Y. Zyryanov, "Voltage-induced defect mode coupling in a one-dimensional photonic crystal with a twisted-nematic defect layer," *Phys. Rev. E* **85**(1), 011705 (2012).
22. D. W. Berreman, "Optics in stratified and anisotropic media: 4×4-matrix formulation," *J. Opt. Soc. Am.* **62**(4), 502–510 (1972).
23. H. L. Ong, "Origin and characteristics of the optical properties of general twisted nematic liquid-crystal displays," *J. Appl. Phys.* **64**(2), 614–628 (1988).
24. A. Lien, "The general and simplified Jones matrix representations for the high pretilt twisted nematic cell," *J. Appl. Phys.* **67**(6), 2853–2856 (1990).
25. P. Yeh and C. Gu, *Optics of Liquid Crystal Displays* (Wiley, 1999).