Sequences of selective rotation operators for three group clustering on qutrits by means quantum annealing

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Quantum computers in compare with classical ones will allow achieve higher performance when solving artificial intelligent problems such as clustering [1] and associative memory [2]. Clustering is partitioning a set data points into subset in according to proximity of their properties (in our case according to distances between two points on Cartesian plate). This problem has already been solved on two-level quantum elements (qubits) in work [1] with quantum annealing technique. As we showed earlier [3] the transformation to three-level elements (qutrits, the spins $S=1$) will allow solving same problem on less amount of quantum elements. In this report we used sequences of selective rotation operators for realization this algorithm. In the case of clustering by the quantum annealing technique, at initial moment of time system is prepared in the ground state of the initial Hamiltonian including interaction with a transverse magnetic field. Then the Hamiltonian adiabatically changed in time to the final Hamiltonian, in the ground state of which we encode the solution of problems. We take the final Hamiltonian for clustering into three groups in the following form [3]:

$$H_f = \sum_{i,j} H_{ij} \qquad H_{ij} = R_{ij}\left(2\left[|1,1\rangle\langle 1,1|_{i,j} + |0,0\rangle\langle 0,0|_{i,j} + |-1,-1\rangle\langle -1,-1|_{i,j}\right]-1\right) \quad (1)$$

where R_{ij} is distance between data points i and j. The Hamiltonian (1) has written with projectors on calculation basis of eigenvectors of projections operators S_i^z on axis Z with projections 1, 0, -1. If we express projectors in Hamiltonian (1) with spins operators [2], then it takes form:

$$H_{ij} = R_{ij}\left(S_i^z S_j^z + 3S_i^z S_i^z S_j^z S_j^z - 2S_i^z S_i^z - 2S_j^z S_j^z + 1\right) \quad (2)$$

For realization algorithms on real physical system the interaction in the Hamiltonian (2) should be expressed with dipole-dipole and Zeyman interactions. For this purpose, we divide the complete evolution operator into a product of individual operators, to each of which we apply the following transformations of the evolution operator using selective operators of rotations around the axes Y and Z [4]:

$$\exp\left[-3iJ_{ij}S_i^z S_j^z S_j^z\right] = \exp\left[-2iJ_{ij}S_i^z\right]\{-\pi\}_{y,j}^{2\leftrightarrow 3}\exp\left[-iJ_{ij}S_i^z S_j^z\right]\{-\pi\}_{y,j}^{1\leftrightarrow 2}\exp\left[-iJ_{ij}S_i^z S_j^z\right]\{\pi\}_{y,j}^{1\leftrightarrow 2}\{\pi\}_{y,j}^{2\leftrightarrow 3}$$

$$\exp\left[-i\theta S_i^z\right] = \{2\theta\}_{z,i}^{2\leftrightarrow 3}\{2\theta\}_{z,i}^{1\leftrightarrow 2} \qquad \exp\left[-i3\varphi S_i^z S_i^z\right] = \{2\varphi\}_{z,i}^{1\leftrightarrow 2}\{2\varphi\}_{z,i}^{2\leftrightarrow 3}\exp\left[-2i\varphi I\right]$$

All unnecessary interactions in each evolution operator will be “turned off” using the inversion operators. Thus, we have obtained the desired sequence of selective rotation operators and evolution operators with the dipole-dipole interactions. After applying the found sequence to the fife qutrit system, the set of six points is grouped into three clusters according to the proximity of their coordinates.

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