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SEQUENCES OF SELECTIVE ROTATION OPERATORS FOR THREE GROUP CLUSTERING ON QUTRITS BY MEANS OF QUANTUM ANNEALING

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Abstract

Recently, it was demonstrated[1] that the clustering considered in this articlec, which consists in partitioning a set of data points into subsets depending on the proximity of some properties, can be performed using quantum annealing on a system of two-level quantum elements - qubits. In previous work[2], we proposed to pass from qubits to qutrits - three-level quantum elements showed the advantages of such a replacement and obtained a time-dependent effective Hamiltonian. In the present paper, we have found a sequence of selective rotation operators that allows one to realize adiabatic evolution with this effective Hamiltonian in the discrete-time approximation. On five qutrits, represented by spins $S=1$, we performed a simulation of clustering a set of six points on a plane into three groups by means of quantum annealing.

Keywords: quantum annealing, clustering, qutrit, selective rotation operators.

1. Introduction

The main part of modern work in the quantum computation area is based on two-level quantum elements – qubits[3, 4]. However, recently, researchers have become more interested in quantum computing on quantum elements with a large number of levels[5]: with three - qutrits or, in general, d levels – qudits. These computations have several advantages. They have greater noise resistance[6, 7]. By using additional qudit levels, it is possible to more efficiently implement gates and algorithms on qubits[8-10]. Finally, when using n qudits instead of qubits, the Hilbert space dimension (computational basis) will grow $\left(d/2\right)^n$ times[5, 11-13].

To date, several experimental implementations of quantum computations have been performed on systems of qutrits: trapped ions[14], NV centers in diamond[15 - 17], transmons in a superconductor[18, 19]. To control the qutrits, pulses of an electromagnetic field (microwave or laser) are applied to the system, resonant to one or another pair of qutrit levels. These pulses cause selective state transformations of the chosen two-level system, which can be represented by selective rotation operators. Sequences of selective rotation operators for solving some problems were proposed in theoretical works[20 - 22].

Recently, we have shown theoretically that on qutrits it is possible to solve artificial intelligence problems: associative memory[23] or data clustering[2], by means of a slow (adiabatic) change in the Hamiltonian in time[24]. To obtain the possibility of experimental implementation of the algorithm, it is necessary to pass from logical operators and evolution operators in theoretical formulas to a sequence of selective rotation operators. In the present work, we found such a sequence for solving the problem of data clustering. A set of six data points is encoded in the states of a system of five spins $S=1$, coupled by spin-spin interactions. After applying the found sequence to the system, the set is grouped into three clusters according to the proximity of properties.

2. Adiabatic clustering algorithm on qutrits.

At present, the quantum adiabatic clustering algorithm has already been realized on qubits[1], while on qutrits it has not yet been considered. In this paper, we consider the implementation of clustering on a system of qutrits represented by spins $S=1$. As the computational basis, we will use the basis $\left|m_1,m_2,...,m_n\right\rangle$ of the eigenfunctions of the operators S_i^z of the projections of the spins on the Z-axis. Each of the projection m_i can take one of three values: 1, 0, -1. We solved the clustering problem by means of sufficiently slow (adiabatic) system evolution with an effective time-dependent Hamiltonian $0 \le t \le T$ [2]:

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$$H(t)=\left(1-\frac{t}{T}\right)H_0+\frac{t}{T}H_f, \tag{1}$$

where $H_0=-h\sum_{i=1}^{n}S_i^x$ is the initial Hamiltonian of the interaction with a transverse magnetic field, the ground state of which easy to prepare, and H_f is the problem Hamiltonian, the ground state of which encodes the solution of our problem.

The problem of clustering is to group data by proximity in the space of some properties. As an example of data, we consider points on a two-dimension plate with Cartesian coordinates (x,y) that represent two properties of that data. The proximity of points with numbers i and j we have characterized by the Euclidean distance R_{ij} between them:

$$R_{ij}=\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2}, \tag{2}$$

where x_i, x_j, y_i and y_j are the coordinates of points i and j on a Cartesian plate. The solution to the clustering problem is to find a partition of the set of n points into k clusters, that minimize the sum of the sums of the distances between the points in each of the clusters.

In previous paper[2], for the simplest case of partitioning into three clusters, we proposed to take the problem Hamiltonian in the following form:

$$H_f=\frac{1}{2}\sum_{i,j}H_{f\,\mathrm{ij}} \tag{3}$$

$$H_{f\,\mathrm{ij}}=R_{ij}\left(2\left[|1,1\rangle\langle 1,1|_{i,j}+|0,0\rangle\langle 0,0|_{i,j}+|-1,-1\rangle\langle -1,-1|_{i,j}\right]-1\right), \tag{4}$$

where $|m_i,m_j\rangle\langle m_i,m_j|_{ij}=|m_i\rangle\langle m_i|_i\otimes|m_j\rangle\langle m_j|_j=|m_i\rangle\langle m_i|_i|m_j\rangle\langle m_j|_j$ is the projector onto eigenstate of two spins at points i and j with projections m_i and m_j respectively. In Hamiltonian (4), each data point i is associated with an operator S_i^z.

As a result of adiabatic evolution with Hamiltonian (1) system pass to the ground state of Hamiltonian (3), which corresponds to the energy minimum (cost function minimum[1, 2]). The energy minimum determines the values of spin projection m_i at the points. The points with the same spin projections belong to the same clusters. Since in (4) three projections 1, 0, -1 are equivalent, the ground state is sixfold degenerate. To remove the threefold degeneracy, one could fix a value of the projection 1 at the first spin, for example, by applying a strong magnetic field to it. Taking this circumster into account, we can simplify calculation and decrease the Hilbert space dimension by factor of three by applying Hamiltonian (3) in following form:

$$H_f=\frac{1}{2}\sum_{i,j\neq 1}H_{f\,\mathrm{ij}}+\sum_{j\neq 1}R_{1j}\left(2|1\rangle\langle 1|_j-1\right). \tag{5}$$

For a simulation closer to the experiment, we express the projection operators in terms of the spin operators of individual spin *i* (*j*):

$$|-1\rangle\langle -1|_i=-S_i^z\frac{1-S_i^z}{2},\quad |0\rangle\langle 0|_i=1-(S_i^z)^2,\quad |1\rangle\langle 1|_i=S_i^z\frac{1+S_i^z}{2}. \tag{6}$$

After substitution equations (6) into the Hamiltonian (4) of the interaction between two points takes the form:

$$H_{f\,\mathrm{ij}}=R_{ij}\left(S_i^zS_j^z+3S_i^zS_i^zS_j^zS_j^z-2S_i^zS_i^z-2S_j^zS_j^z+1\right), \tag{7}$$

The solution to our problem $\langle\Psi|$ has been found in the following form:

$$\langle\Psi|=\langle\psi|\hat{Q}\exp\left(-i\int_0^T H(t)dt\right)\cong\langle\psi|\prod_{l=0}^{N}U_l, \tag{8}$$

where $\hat{Q}$ is the ordering operator in time, and $\langle\psi|$ is the ground state of the initial Hamiltonian H_0. Following[2.22-24], the operator of adiabatic evolution over time $T = \Delta t N$ with the Hamiltonian varying according to the linear law (1), we presented in the form of the product of evolution operators by a sequence of N small time intervals Δt. On each time interval, we neglect the Hamiltonian changing (1)

$$U_l \cong \exp\left\{-i\Delta t\left(\frac{l}{N}H_f + \left(1-\frac{l}{N}\right)H_0\right)\right\}, \quad \textbf{(9)}$$

where l is discrete time $(0 \le l \le N)$.

3. Partition of a set of six points into three clusters

In this section, we have performed a numerical simulation of the proposed quantum algorithm using a simple example. Using a random number generator from the range [-10, 10], we obtained the coordinates of six points

(4,-2), (-7,7), (6,-9), (-6,8), (-2,-6), (-9,5), **(10)**

which we assign numbers from 1 to 6 in the order of the following. Before the simulation, we calculated the distance R_{ij} and substitute them in Hamiltonian (5) and (7) (we assign a value $S^z = 1$ to the first point (4,-2)). The simulation was performed with the following parameters: $N = 4000$, $\Delta t = 0{,}05$, $h = 2$. The result of solving this problem is shown in Fig. 1, it can be seen that our algorithm coped with the solution of this problem.

The calculation result (8) is obtained as a superposition of $3^5 = 243$ states of the computational basis (we fixed the first spin in the state with projection value $S^z = 1$)

$$\langle\Psi(t)| = \sum_{m_2,m_3,...,m_6} C_{1,m_2,m_3,...,m_6}(t)\langle 1, m_2, m_3, ..., m_6|. \quad \textbf{(11)}$$

We calculated the time variation of the three lowest instantaneous energy levels of Hamiltonian (5) on the interval $(0,T)$ and the probability of finding the system in these three states of the computational basis $\left|C_{1,m_2,m_3,...,m_6}(t)\right|^2$. At the end of evolution at $t = T$, the system located in state $\langle 1,-1,0,-1,1,-1|$ with the fidelity of 0.95 (for the chosen values of the parameters). The corresponding clustering result is shown in Fig. 1. Note that the same clustering result corresponds to the state $\langle 1,0,-1,0,1,0|$ resulting from the rearranging the spin projections 0 and -1. At the end of evolution at $t = T$, the system will be in this state with a probability of 0.05. Such a difference in probabilities was a consequence of the fact that the curve for the instantaneous energy level corresponding to this state passes higher over the entire evolution interval. The coincidence of the energies of the two states occurs only at $t = T$. Therefore, the probability of finding the system in this state is small. The violation of symmetry occurred due to the fixation of the projection value of the first spin. If we assign the projection value 0 to the first spin, then the probabilities of finding the system in each of the two states will equalize.

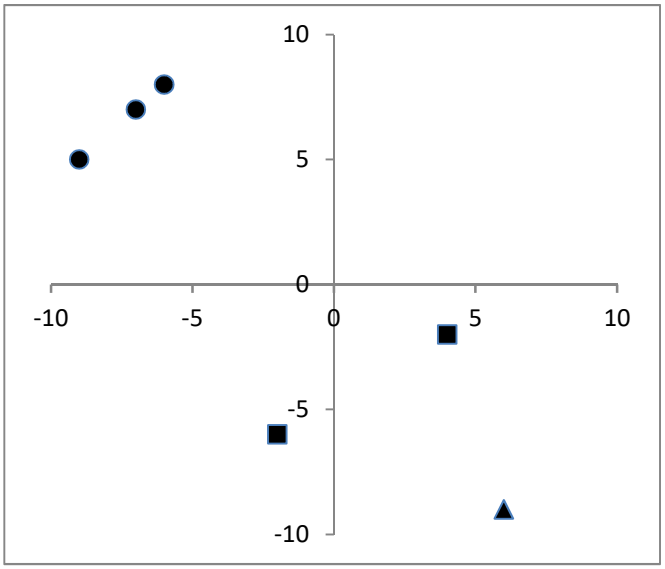

Fig 1. Result of partition of a set of six data points into three clusters. The first cluster: (–9, 5), (–7, 7), and (–6, 8) is shown by circles; the second cluster: (–2, –6) and (4, –2) by squares; and the third cluster (6, –9) by a triangle.

4. Engineering of effective interaction

In the simulation performed in the previous section, we use the product of evolution operator (9) with Hamiltonian (7). However, in a real system[15-19] , the Hamiltonian has another form. In particular, the interaction between qutrit has the form of an Ising interaction:

$$H_{ss} = J_{12}S_1^zS_2^z + J_{13}S_1^zS_3^z + J_{14}S_1^zS_4^z + J_{15}S_1^zS_5^z + J_{16}S_1^zS_6^z + J_{23}S_2^zS_3^z + J_{24}S_2^zS_4^z + \\ + J_{25}S_2^zS_5^z + J_{26}S_2^zS_6^z + J_{34}S_3^zS_4^z + J_{35}S_3^zS_5^z + J_{36}S_3^zS_6^z + J_{45}S_4^zS_5^z + J_{46}S_4^zS_6^z + J_{56}S_5^zS_6^z, \quad \textbf{(12)}$$

and does not contain interactions between the squares of the spin operators included in the Hamiltonian (7). Now we find rules for transformation from one evolution operator into another.

First of all, let us approximately represent the evolution operator (9) in the form of a product of three non-commuting operators:

$$U_l = \exp\left[-i\left(1-\frac{l}{N}\right)\Delta t H_0/2\right]\exp\left[-i\Delta t H_f\frac{l}{N}\right]\exp\left[-i\left(1-\frac{l}{N}\right)\Delta t H_0/2\right], \quad \textbf{(13)}$$

where l is discrete time $\left(0 \le l \le N\right)$.

As a transformation tool we take the selective rotation operators[20, 22, 25] $\left\{\Omega\right\}_{\alpha,j}^{k\leftrightarrow n}$, which in matrix representation have the form:

$$\{\Omega\}_{z,j}^{1\leftrightarrow 2}=\begin{pmatrix}\exp\left[-i\frac{\Omega}{2}\right] & 0 & 0\\ 0 & \exp\left[i\frac{\Omega}{2}\right] & 0\\ 0 & 0 & 1\end{pmatrix},\{\Omega\}_{z,j}^{2\leftrightarrow 3}=\begin{pmatrix}1 & 0 & 0\\ 0 & \exp\left[-i\frac{\Omega}{2}\right] & 0\\ 0 & 0 & \exp\left[i\frac{\Omega}{2}\right]\end{pmatrix},$$

$$\{\Omega\}_{y,j}^{1\leftrightarrow 2}=\begin{pmatrix}\cos\frac{\Omega}{2} & -\sin\frac{\Omega}{2} & 0\\ \sin\frac{\Omega}{2} & \cos\frac{\Omega}{2} & 0\\ 0 & 0 & 1\end{pmatrix},\{\Omega\}_{y,j}^{2\leftrightarrow 3}=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\frac{\Omega}{2} & -\sin\frac{\Omega}{2}\\ 0 & \sin\frac{\Omega}{2} & \cos\frac{\Omega}{2}\end{pmatrix}, \quad \textbf{(14)}$$

where Ω is the angle of rotation around axis $\alpha\,(\alpha=x,y,z)$, the k and n are level numbers, j is spin number. The X-rotation matrices differ from Y-rotation by the coefficients $(-i)$ in front of the sine. To implement selective rotation between the levels k and n by an angle $\Omega=\sqrt{2}h_f t_p$ an MW field with the amplitude h_f (in frequency units) and frequency equal to the energy difference of the level $\omega=\omega_{nk}=\varepsilon_n-\varepsilon_k$ [13, 20, 25] is switched on for a finite time $t_p\,(t_p >> 1/\omega)$. The direction of the axis of rotation is determined by the phase of the MW field.

At last, acting simultaneously on two transitions in the course of time t_p by two MW fields with frequencies ω_{12} and ω_{23}, and amplitudes h_f respectively, one can produce a non-selective spin rotation produce a non-selective spin rotation[20, 22] specified by the operator $\exp(-i\theta S^x)$ (or $\exp(-i\theta S^y)$), where: $\theta=t_p h_f$. Using this rotation we obtained the factors $\exp\left[-i\left(1-\frac{l}{N}\right)\Delta t H_0/2\right]$ in the evolution operators U_l (13).

4.1 Transformation of one-spin evolution operators

Consider the second term $\exp\left[-i\Delta t\frac{l}{N}H_f\right]$ in the evolution operators U_l (13). Let us substitute Hamiltonian (5) and (7) into this operator. We split it into the product of evolution operators for each interaction, which is possible, since all terms in (3) commute with each other.

We start with the single-spin terms contained in Hamiltonian (5):

$$\sum_{j\neq 1}R_{1j}\left(2|1\rangle\langle 1|_j-1\right)=\sum_{j\neq 1}R_{1j}\left(S_j^z+S_j^zS_j^z-1\right). \quad \textbf{(15)}$$

Take the evolution operator: $\exp\left[-i\frac{l}{N}\Delta tR_{1,j}S_j^z\right]$. Such operators can be obtained by the formula[20, 25]:

$$\exp\left[-i\frac{l}{N}\Delta tR_{1j}S_j^z\right]=\left\{\frac{2l}{N}\Delta tR_{1j}\right\}_{z,j}^{1\leftrightarrow 2}\left\{\frac{2l}{N}\Delta tR_{1j}\right\}_{z,j}^{2\leftrightarrow 3}. \quad \textbf{(16)}$$

The next single-spin interactions in (15) are quadratic in the spin operators: $\exp\left[-i\frac{l}{N}\Delta tR_{1,j}S_j^zS_j^z\right]$. The corresponding evolution operators can be obtained by the formula[22, 25]:

$$\exp\left[-i\frac{l}{N}\Delta tR_{1,j}S_j^zS_j^z\right]=\left\{\frac{2l}{3N}\Delta tR_{1,j}\right\}_{z,j}^{1\leftrightarrow 2}\left\{-\frac{2l}{3N}\Delta tR_{1,j}\right\}_{z,j}^{2\leftrightarrow 3}\exp\left[-i\frac{2l}{3N}\Delta tR_{1,j}I\right], \quad \textbf{(17)}$$

where I is unit matrix.

4.2 Transformation of two-spin evolution operators

We will now consider obtaining multipliers of the form $\exp\left[-i\Delta t\frac{l}{N}3R_{ij}S_i^zS_i^zS_j^zS_j^z\right]$ from multipliers of the form $\exp\left[-i\Delta t\frac{l}{N}J_{ij}S_i^zS_j^z\right]$.

At the first stage, we will carry out the following transformation:

$$\begin{aligned}&\exp\left[-3i\Delta t\frac{l}{N}R_{ij}S_i^zS_i^zS_j^zS_j^z\right]=\exp\left[-2i\Delta t\frac{l}{N}R_{ij}S_i^zS_i^z\right]\left\{-\pi\right\}_{y,j}^{2\leftrightarrow3}\times\\&\times\exp\left[-i\Delta t\frac{l}{N}R_{ij}S_i^zS_i^zS_j^z\right]\left\{-\pi\right\}_{y,j}^{1\leftrightarrow2}\exp\left[-i\Delta t\frac{l}{N}R_{ij}S_i^zS_i^zS_j^z\right]\left\{\pi\right\}_{y,j}^{1\leftrightarrow2}\left\{\pi\right\}_{y,j}^{2\leftrightarrow3}\end{aligned}. \tag{18}$$

Then we repeat it for the resulting multipliers: $\exp\left[-i\Delta t\frac{l}{N}R_{ij}S_i^zS_i^zS_j^z\right]$:

$$\begin{aligned}&\exp\left[-3i\Delta t\frac{l}{N}R_{ij}S_i^zS_i^zS_j^zS_j^z\right]=\exp\left[-2i\Delta t\frac{l}{N}R_{ij}S_i^zS_i^z\right]\left\{-\pi\right\}_{y,j}^{2\leftrightarrow3}\times\\&\times\exp\left[-i\frac{2}{3}\Delta t\frac{l}{N}R_{ij}S_j^z\right]\left\{-\pi\right\}_{y,i}^{2\leftrightarrow3}\exp\left[-i\frac{1}{3}\Delta t\frac{l}{N}R_{ij}S_i^zS_j^z\right]\left\{-\pi\right\}_{y,i}^{1\leftrightarrow2}\times\\&\times\exp\left[-i\frac{1}{3}\Delta t\frac{l}{N}R_{ij}S_i^zS_j^z\right]\left\{\pi\right\}_{y,i}^{1\leftrightarrow2}\left\{\pi\right\}_{y,i}^{2\leftrightarrow3}\left\{-\pi\right\}_{y,j}^{1\leftrightarrow2}\exp\left[-i\frac{2}{3}\Delta t\frac{l}{N}R_{ij}S_j^z\right]\times\\&\times\left\{-\pi\right\}_{y,i}^{2\leftrightarrow3}\exp\left[-i\frac{1}{3}\Delta t\frac{l}{N}R_{ij}S_i^zS_j^z\right]\left\{-\pi\right\}_{y,i}^{1\leftrightarrow2}\exp\left[-i\frac{1}{3}\Delta t\frac{l}{N}R_{ij}S_i^zS_j^z\right]\times\\&\times\left\{\pi\right\}_{y,i}^{1\leftrightarrow2}\left\{\pi\right\}_{y,i}^{2\leftrightarrow3}\left\{\pi\right\}_{y,j}^{1\leftrightarrow2}\left\{\pi\right\}_{y,j}^{2\leftrightarrow3}\end{aligned}. \tag{19}$$

At the second stage, we will use formula (16) to transform the first factor of the right-hand side with the help of selective operators of rotation around the *Z*-axis:

$$\begin{aligned}&\exp\left[-3i\Delta t\frac{l}{N}R_{ij}S_i^zS_i^zS_j^zS_j^z\right]=\left\{\frac{4}{3}\Delta t\frac{l}{N}R_{ij}\right\}_{z,i}^{1\leftrightarrow2}\left\{-\frac{4}{3}\Delta t\frac{l}{N}R_{ij}\right\}_{z,i}^{2\leftrightarrow3}\times\\&\times\exp\left[-i\frac{4}{3}\Delta t\frac{l}{N}R_{ij}\right]\left\{-\pi\right\}_{y,j}^{2\leftrightarrow3}\times\exp\left[-i\frac{2}{3}\Delta t\frac{l}{N}R_{ij}S_j^z\right]\left\{-\pi\right\}_{y,i}^{2\leftrightarrow3}\times\\&\times\exp\left[-i\frac{1}{3}\Delta t\frac{l}{N}R_{ij}S_i^zS_j^z\right]\left\{-\pi\right\}_{y,i}^{1\leftrightarrow2}\exp\left[-i\frac{1}{3}\Delta t\frac{l}{N}R_{ij}S_i^zS_j^z\right]\left\{\pi\right\}_{y,i}^{1\leftrightarrow2}\times\\&\times\left\{\pi\right\}_{y,i}^{2\leftrightarrow3}\left\{-\pi\right\}_{y,j}^{1\leftrightarrow2}\exp\left[-i\frac{2}{3}\Delta t\frac{l}{N}R_{ij}S_j^z\right]\left\{-\pi\right\}_{y,i}^{2\leftrightarrow3}\exp\left[-i\frac{1}{3}\Delta t\frac{l}{N}R_{ij}S_i^zS_j^z\right]\times\\&\times\left\{-\pi\right\}_{y,i}^{1\leftrightarrow2}\exp\left[-i\frac{1}{3}\Delta t\frac{l}{N}R_{ij}S_i^zS_j^z\right]\left\{\pi\right\}_{y,i}^{1\leftrightarrow2}\left\{\pi\right\}_{y,i}^{2\leftrightarrow3}\left\{\pi\right\}_{y,j}^{1\leftrightarrow2}\left\{\pi\right\}_{y,j}^{2\leftrightarrow3}\end{aligned} \tag{20}$$

At the next stage, we will transform another resulting multiplier of the evolution operator $\exp\left[-i\frac{2}{3}\Delta t\frac{l}{N}R_{ij}S_j^z\right]$ also with the help of selective operators of rotations around the Z-axis, which is transformed using formula (16):

$$\exp\left[-i\frac{2}{3}\Delta t\frac{l}{N}R_{ij}S_j^z\right]=\left\{\frac{4}{3}\Delta t\frac{l}{N}R_{ij}\right\}_{z,i}^{1\leftrightarrow2}\left\{\frac{4}{3}\Delta t\frac{l}{N}R_{ij}\right\}_{z,i}^{2\leftrightarrow3}, \tag{21}$$

At the last stage, it's necessary to obtain a factor of the form $\exp\left[-i\Delta t\frac{l}{N}J_{ij}S_i^zS_j^z\right]$ included in the formulas, containing only the interaction of an individual pair of spins, from the general evolution of the system with Hamiltonian (12),

containing the sum of all interactions. For simplicity, in Hamiltonian (12), we set the coupling constants between spins taking into account the distances between them:

$$H_{ss} = -\frac{\sqrt{425}}{24} S_2^z S_3^z - \frac{\sqrt{2}}{24} S_2^z S_4^z - \frac{\sqrt{194}}{24} S_2^z S_5^z - \frac{\sqrt{8}}{24} S_2^z S_6^z - \frac{\sqrt{433}}{24} S_3^z S_4^z - \\ - \frac{\sqrt{73}}{24} S_3^z S_5^z - \frac{\sqrt{421}}{24} S_3^z S_6^z - \frac{\sqrt{212}}{24} S_4^z S_5^z - \frac{\sqrt{18}}{24} S_4^z S_6^z - \frac{\sqrt{170}}{24} S_5^z S_6^z \quad . \tag{22}$$

Since the first spin was assigned the value of projection 1, we transferred the interactions with it to the field terms. It was possible to take other values of the constants (12), and the value of the exponent could be adjusted by choosing a time interval Δt.

As an example, we get $\exp\left[\frac{i}{3}\Delta t \frac{l}{N}\sqrt{170} S_5^z S_6^z\right]$. To do this, let's create spin inversion operators:

$$P_k^{-1} = \{-\pi\}_{y,k}^{1\leftrightarrow 2} \{-\pi\}_{y,k}^{2\leftrightarrow 3} \{-\pi\}_{y,k}^{1\leftrightarrow 2}, \\ P_k = \{\pi\}_{y,k}^{1\leftrightarrow 2} \{\pi\}_{y,k}^{2\leftrightarrow 3} \{\pi\}_{y,k}^{1\leftrightarrow 2}, \\ P_k^{-1} S_k^z P_k = -S_k^z. \tag{23}$$

First, let's do the inversion of the second spin:

$$P_2^{-1} \exp\left[-i\Delta t \frac{i}{N} H_{ss}\right] P_2 \exp\left[-i\Delta t \frac{i}{N} H_{ss}\right]. \tag{24}$$

As a result, the next terms will be removed from the effective Hamiltonian: $-\frac{\sqrt{425}}{12} S_2^z S_3^z - \frac{\sqrt{2}}{12} S_2^z S_4^z - \frac{\sqrt{194}}{12} S_2^z S_5^z - \frac{\sqrt{8}}{12} S_2^z S_6^z$.

Then we multiply the sequence (24) into the sequence:

$$P_3^{-1} P_2^{-1} \exp\left[-i\Delta t \frac{i}{N} H_{ss}\right] P_2 \exp\left[-i\Delta t \frac{i}{N} H_{ss}\right] P_3, \tag{25}$$

As result, we remove the terms: $-\frac{\sqrt{433}}{6} S_3^z S_4^z - \frac{\sqrt{73}}{6} S_3^z S_5^z - \frac{\sqrt{421}}{6} S_3^z S_6^z$.

Finally, we multiply the product of sequences (24) and (25) by the following sequence

$$P_4^{-1} P_2^{-1} \exp\left[-i\Delta t \frac{i}{N} H_{ss}\right] P_2 \exp\left[-i\Delta t \frac{i}{N} H_{ss}\right] P_3^{-1} \times \\ \times P_2^{-1} \exp\left[-i\Delta t \frac{i}{N} H_{ss}\right] P_2 \exp\left[-i\Delta t \frac{i}{N} H_{ss}\right] P_3 P_4 \quad , \tag{26}$$

which removes the terms: $-\frac{\sqrt{212}}{3} S_4^z S_5^z - \frac{\sqrt{18}}{3} S_4^z S_6^z$.

To obtain the multiplier $\exp\left[i\Delta t \frac{l}{N}\sqrt{170} S_5^z S_6^z\right]$, it is necessary to perform the same transformations, but the time interval must be increased three times.

5. Calculation and discussion

Using the rules described in the previous sections, we found the complete sequence of selective rotation operators and evolution intervals with the Hamiltonian of the spin-spin interaction, which is necessary for clustering (8). With the help of the found sequence, we calculated the clustering of six points into three groups on five qutrits, considered in Section 3. At the end of evolution at $t = T$, the probability that the system will be in the state $\langle 1,-1,0,-1,1,-1|$ decreased from 0.95 to 0.89, for the same parameter values. This was a consequence of the transformation from (9) to (13). For

increasing fidelity, we should divide evolution (8) on a larger number of time intervals, i.e. increase N and decrease Δt .

6. Conclusions

Thus, we have developed the theory of controlling the qutrit system with the help of rotation operators, selective for the transitions between the energy levels of the qutrits. We have found sequences of selective rotation operators to engineer effective interactions that are linear and quadratic in the spin operators. In particular, the interaction between the squares of the operators of two spins was obtained from the Ising interaction. We have demonstrated the possibilities of controlling a system of five qutrits by the example of clustering a set of 6 points. The sequences obtained will allow experimentally clustering on systems of qutrits controlled by selective operators, for example, on ions in a trap[14], on NV-centers in diamond[15 - 17], or on transmons in a superconductor[18, 19].

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