USSR ACADEMY OF SCIENCES
SIBERIAN DIVISION
INSTITUTE OF NUCLEAR PHYSICS

preprint

π-MESON PRODUCTION AT THRESHOLD IN NEUTRINO REACTIONS

A.I.Vainshtein, V.V.Sokolov, I.B.Khriplovich

NOVOSIBIRSK
1966

abstract

Using the current algebra and the assumption of smooth dependence on the momenta of the weak hadronic current matrix element, the amplitude of the π-meson production at threshold in the neutrino reactions at the momentum transfer $|k^2| \lesssim \mu^2$ is found.

In connection with the realization of the neutrino experiments the evaluation of the cross-sections of the reactions under investigation becomes important. In the present work the amplitude of the π-meson production at threshold in the reactions

$$\nu + N \to \ell + N' + \pi$$

at momentum transfer $|k^2| \lesssim \mu^2$ is given using the current algebra.

For definiteness consider the π^+-meson production on the proton. The amplitude of the process may be written as

$$M = \frac{G}{\sqrt{2}} \ell_\mu T_\mu = \frac{G}{\sqrt{2}} \ell_\mu \cdot i \int dx\, e^{iqx} (\Box - \mu^2)\, \theta(x_0) \langle p' | [J_\mu^+(0), \varphi^-(x)] | p \rangle \tag{1}$$

Here ℓ_μ is the leptonic weak current, $J_\mu^+ = v_\mu^+ + a_\mu^+$ is the hadronic weak current operator, φ^- is the mesonic field operator, p, p' and q are the momenta of the initial and final protons and meson respectively. Make the substitution $\varphi^- \to c\, \partial_\mu a_\mu^-$ $\left(c = -\frac{g_r K(0)}{\sqrt{2}\, m \mu^2 g_A}\right)$. At $q^2 = \mu^2$ it is possible without any approximations since the only contribution is given by the π-meson pole /1/. Then integrating by parts we get

$$\begin{aligned} T_\mu = &-ic \int dx\, e^{iqx} (\Box - \mu^2)\, \delta(x_0) \langle p' | [J_\mu^+(0), a_0^-(x)] | p \rangle + \\ &+ c q_\nu \int dx\, e^{iqx} (\Box - \mu^2)\, \theta(x_0) \langle p' | [J_\mu^+(0), a_\nu^-(x)] | p \rangle \end{aligned} \tag{2}$$

whence it follows that in the limit $q \to 0$

$$\begin{aligned} \widetilde{T}_\mu = &\lim_{q \to 0} \left\{ T_\mu - c q_\nu \int dx\, e^{iqx} (\Box - \mu^2)\, \theta(x_0) \langle p' | [J_\mu^+(0), a_\nu^-(x)] | p \rangle \right\} = \\ &= ic\mu^2 \langle p' | [J_\mu^+(0), \int d\vec{x}\, a_0^-(\vec{x}, 0)] | p \rangle \end{aligned} \tag{3}$$

Separating in $\widetilde{T}_\mu$ the contributions from the vector and axial parts of J_μ^+, we find

$$\tilde{T}_\mu^V = ic\mu^2 \langle p' | [v_\mu^+(0), \int d\vec{x}\, a_o^-(\vec{x},0)] | p \rangle = ic\mu^2 \langle p' | a_\mu^3(0) | p \rangle \tag{4}$$

$$\tilde{T}_\mu^A = ic\mu^2 \langle p' | [a_\mu^+(0), \int d\vec{x}\, a_o^-(\vec{x},0)] | p \rangle = ic\mu^2 \langle p' | v_\mu^3(0) | p \rangle \tag{5}$$

We use here known commutation relations /2/.

Consideration of the vector part is completely analogous to that of the electroproduction /3/. Contribution to $\tilde{T}_\mu^V$ from the nucleon and meson pole diagrams independently of the way of taking the limit is equal to

$$-\frac{ig_2}{\sqrt{2}m} \bar{u}(p') \Big[\gamma_\mu - \frac{\mathfrak{M}}{2m} \sigma_{\mu\nu} k_\nu - \frac{2m k_\mu}{k^2-\mu^2} \Big] \gamma_5 u(p) \tag{6}$$

Here all the form-factors are assumed to depend smoothly on k^2 and q^2 so that in the region $|k^2| \lesssim \mu^2$ we may use their values at $k^2 = 0$, $q^2 = \mu^2$. Under the same assumptions

$$ic\mu^2 \langle p' | a_\mu^3(0) | p \rangle = -\frac{ig_2}{\sqrt{2}m} \bar{u}(p') \Big(\gamma_\mu - \frac{2m k_\mu}{k^2-\mu^2} \Big) \gamma_5 u(p) \tag{7}$$

The expressions (6) and (7) coincide with the accuracy of $\frac{\mu}{m}$ because in our region all the components of the momentum $k \sim \mu$. Since at $q \to 0$ only the poles contribute to the second term in $\tilde{T}_\mu$, the non-pole part of the matrix element T_μ^V with the accepted accuracy is equal to zero. It may be expected that this part of T_μ^V remains relatively small at the threshold too. In this case the contribution of the vector hadronic current is described in the threshold region by the pole diagrams without the weak magnetic term. It corresponds to the diagrams of the first order in the renormalized coupling constant of the pseudoscalar meson theory. Note that the one-nucleon contribution with the accuracy of $\frac{\mu}{2m}$ does not change under the continuation from $q = 0$ to $q_o = \mu$,

$\vec{q} = 0$, this fact strengthening our hope for the weak variation of the non-pole terms.

We wish to note that for the validity of the stated result the assumption of slow variation of the non-pole contribution as the function of k_μ is sufficient. Really, all the six transverse covariants from which T_μ^V is constructed /4/ vanish at $k = 0$, the only non-zero contribution coming from the pole diagrams that contain the characteristic infrared factors of the type $\frac{1}{2pk}$. This observation refers to the electroproduction process too /3/.

Pass now to the investigation of the axial part of the amplitude. Sharp dependence on k^2 in T_μ^A is connected only with the diagrams that contain the π-meson pole. Their sum is equal to

$$\frac{1}{c\mu^2} \frac{k_\mu}{k^2-\mu^2} T_{\pi N} \tag{8}$$

where $T_{\pi N}$ is the πN scattering amplitude. Using the substitution $\varphi^\pm \to c\,\partial_\mu a_\mu^\pm$ and the expression (1) for T_μ, it is easily shown that

$$T_{\pi N} = c\, k_\nu T_\nu^A (k^2-\mu^2) \tag{9}$$

In this equality we omit the commutator $\langle p' | [\int d\vec{x}\, a_o(\vec{x},0), \int d\vec{y}\, \dot{a}_o(\vec{y},0)] | p \rangle$ that determines at $q = k = 0$ the isotopic even part of the πN scattering. In correspondence with the anomalous smallness of the isotopic even πN scattering length, known from experiment /5/, we consider this part as negligibly small.

Passing to $q \to 0$, we get with the use of (8), (9), (5)

$$\frac{1}{c\mu^2} \frac{k_\mu}{k^2-\mu^2} \tilde{T}_{\pi N} = \frac{k_\mu k_\nu}{\mu^2} \tilde{T}_\nu^A = -ic\mu^2 k_\mu k_\nu \frac{1}{\mu^2} \langle p' | v_\nu^3(0) | p \rangle = 0 \tag{10}$$

Thus the contribution of the π-meson pole diagrams to $\tilde{T}_\mu^A$ vanishes. Again the nucleon pole graph (without the effective pseudoscalar taken into account in the previously considered diagrams) varies slowly under the continuation to the threshold. The previous assumptions on the character of the variation of the non-pole terms lead to the conclusion that at the threshold

$$T_\mu^A = \tilde{T}_\mu^A + \frac{i}{c\mu^2}\frac{k_\mu}{k^2-\mu^2}T_{\pi N} \;; \quad (k = p'-p+q) \,, \tag{11}$$

where with the accepted accuracy

$$\tilde{T}_\mu^A = -\frac{ig_2}{\sqrt{2}\, m g_A}\bar{u}(p')\gamma_\mu u(p) \tag{12}$$

From (9), (11) and (12) we find

$$T_\mu^A = -\frac{ig_2}{\sqrt{2}\, m}\frac{1}{g_A}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2-\mu^2}\right)\bar{u}(p')\gamma_\nu u(p) \tag{13}$$

$$T_{\pi N} = -\frac{ig_2^2}{2m^2}\frac{1}{g_A^2}k_\nu \bar{u}(p')\gamma_\nu u(p) \tag{14}$$

Note that the relative contribution of the second term in (13) to the cross-section of the process decreases with the neutrino energy as $\frac{\mu}{\varepsilon}$.

Our consideration shows that both the vector and axial parts of the matrix element T_μ are isotopic odd at the threshold. Hence the production amplitudes of charged π-mesons may differ in sign only, and for π^0-meson the amplitude contains the additional factor $\sqrt{2}$.

The formula (14) determines the isotopic odd πN scattering length[6]. On the other hand using for the πN forward scattering amplitude the dispersion relation without subtractions we may easily get from it the Adler-Weisberger sum rule[7].

Making use of (13) we may in principle determine g_A from the neutrino experiment. However the accuracy of the method allows only to verify the closeness of g_A to the unity. This remark refers evidently to the Adler-Weisberger sum rule too.

It is interesting to note that at $\mu^2 = 0$ T_μ^A becomes transverse. It corresponds to the fact that at $\mu^2 = 0$ the constant c vanishes and the axial current is conserved.

We wish to stress that for obtaining the relations (13), (14) it is basically important to use the commutator (5) non-linear in a_μ whereas we may dispence with the linear commutation relation (4).

REFERENCES

1. S.Okubo. Nuovo Cim., 41,586,1966.
2. M.Gell-Mann. Phys.Rev., 125,1067,1962.
3. A.I.Vainshtein, V.V.Sokolov, I.B.Khriplovich. JETP Lett., 4, No.4, 1966.
4. P.Dennery. Phys.Rev., 124,2000,1961.
5. J.Hamilton, W.S.Woolcock. Rev.Mod.Phys., 35,737,1963.
6. Y.Tomozawa. Preprint, Princeton, 1966.
7. S.L.Adler. Phys.Rev.Lett., 14,1051,1965.
   W.I.Weisberger. Phys.Rev.Lett., 14,1047,1965.

Printed at the Institute of Nuclear Physics, Siberian Division, USSR Academy of Sciences. Received by the Publishing Department on July , 1966. Responsible V.V.Sokolov. Edition 200.