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ON THE SATURATION OF THE SUM RULES FOLLOWING FROM THE ALGEBRA OF CURRENTS AND THE ROLE OF SCHWINGER TERMS

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abstract

The relations derived by saturation of the sum rules following from the algebra of currents by the lowest intermediate states (nucleon and isobar) are considered. It is shown that in this approximation the nucleon vertex remains bare and the transition nucleon-isobar vanishes. The results of the papers [1,2,7] are shown to be wrong since the intermediate states of the type $2W\tilde{W}$ are not taken into account in them. General expressions for the Schwinger terms in the quark model are presented. In some cases their contribution is shown to vanish identically.

1. INTRODUCTION

The application of the algebra of currents allowed lately to obtain various sum rules for the characteristics of elementary particles. Such rules were used to obtain a number of conclusions that were sometimes in good agreement with experiment. At the same time the relations proposed by different authors turned out to contradict each other. E.g. considering the expectation value in the one-nucleon state with the momentum $\vec{p}=0$ of the magnetic moments commutator

$$\left[\mathcal{M}_{12}^{+},\mathcal{M}_{12}^{-}\right]=\frac{1}{4}\int d\vec{x}\,(x_1^2+x_2^2)V_0^3(\vec{x},0) \tag{1}$$

$$\mathcal{M}_{12}^{\pm}=\frac{1}{2}\int d\vec{x}\left(x_1V_2^{\pm}(\vec{x},0)-x_2V_1^{\pm}(\vec{x},0)\right)$$

and restricting to the intermediate states with one nucleon and one isobar, Dashen and Gell-Mann/1/ and Lee/2/ obtained the equality

$$\left(\frac{\mu_V+1}{2m}\right)^2-2\left(\frac{\mu^*}{2m}\right)^2=\frac{1}{6}r_V^2+\frac{1}{8m^2} \tag{2}$$

Here μ_V is the isovector anomalous magnetic moment of the nucleon, μ^* is the magnetic moment of the nucleon-isobar transition, r_V is the Sachs isovector electrical radius of the nucleon.

On the other hand from the sum rule derived by Cabibbo and Radicati/3/ and Adler/4/ saturated with the same intermediate states the following relation may be obtained/5/

$$\left(\frac{\mu_V+1}{2m}\right)^2-2\left(\frac{\mu^*}{2m}\right)^2=\frac{1}{3}r_V^2+\frac{1}{4m^2} \tag{3}$$

Whereas the left hand sides of (2) and (3) coincide, the right hand sides differ by the factor 2. Buccella, Veneziano, Gatto and Okubo/6/ conjectured that this contradiction might be removed if Schwinger terms omitted in /1,2/ were taken into account. In derivation of the sum rules from the commutators of the time components of current densities/3,4/ the Schwinger terms symmetrical in the isotopic indices in this case are inessential.

Apparently the most far reaching conclusions were obtained by saturation of the sum rule with the lowest one-particle intermediate states in /7/. Working in the proton rest frame, the authors of this paper obtained the relation

$$F_1^v(t) + \frac{t}{2m} F_2^v(t) = \left(\frac{1 - \frac{t}{2m^2}}{1 - \frac{t}{4m^2}} \right)^{\frac{1}{2}}, \tag{4}$$

where $F_1^v(t)$ and $F_2^v(t)$ are the isovector electrical and magnetic nucleon form-factors. The strange peculiarity of the equalities (2) and (4) consists in their discrepancy with the perturbation theory. Furthermore the form-factor (4) has wrong analitical properties in t. The relation (4) contradicts also the results by Cabibbo - Radicati /3/ and Adler /4/, obtained with the use of the same commutators.

The aim of our work consists in the clearing up the reasons of the mentioned contradictions and the pithiness of the relations obtained by the saturation of sum rules by the lowest intermediate states.

In the section 2 the relation (1) is studied in the perturbation theory. In the lowest order the contribution of Schwinger terms is shown to vanish, so they cannot ensure the correspondence of the Dashen - Gell-Mann - Lee relation with the perturbation theory. It is shown that in this order the three-particle intermediate state with two nucleons and one antinucleon contributes as well as one-nucleon state. Just taking into account the three-particle state removes the mentioned contradiction.

In the section 3 the sum rules following from the algebra of currents are saturated with nucleon and isobar intermediate states taking into account both one-particle and the corresponding three-particle states. In this case instead of (2) we may get the equality that coincides with (3). However if we take into account all the relations, obtained from the current commutators, we arrive, to all appearance, to the conclusion that the nucleon remains bare and the transition nucleon-isobar vanishes

$$F_1^V(t) = 1 \;;\; F_2^V(t) = F^*(t) = 0$$

($F^*(t)$ is the form-factor of the transition $\gamma N \to N^*$).

Lastly in the section 4 the commutation relations between the components of the vector and axial current densities in the quark model are presented taking into account the Schwinger terms. The contribution of Schwinger terms is shown to vanish at any rate in the proton rest frame. For this reason we do not take them into account in the section 3.

2. DASHEN - GELL-MANN - LEE RELATION IN THE PERTURBATION THEORY

To clear up the true reason of the discrepancy between the relations (2) and (3) consider first of all the simplest example - the free nucleon case. Following Schwinger /8/ and Okubo /9/, introduce the nucleon current operator by means of the relation

$$V_\mu^\alpha(x) = \lim_{\varepsilon\to 0} \overline{\Psi}(x-\varepsilon)\gamma_\mu \tfrac{1}{2}\tau^\alpha \Psi(x+\varepsilon) \;;\; \varepsilon^2 < 0 \tag{5}$$

where $\Psi(x)$ is the nucleon field operator. Then taking into account the Schwinger terms, we get

$$\left[\mathcal{M}_{12}^+, \mathcal{M}_{12}^-\right] = \frac{1}{4}\int d\vec{x}\,(x_1^2 + x_2^2) V_0^3(\vec{x},0) + \frac{i}{4}\lim_{\varepsilon\to 0}\int d\vec{x}\,(x_1\varepsilon_2 - x_2\varepsilon_1)\overline{\Psi}(\vec{x}-\vec{\varepsilon},0)\gamma_5\gamma_3\tau^3\Psi(\vec{x}+\vec{\varepsilon},0) \tag{6}$$

The contribution of Schwinger terms may be computed explicitly in the free nucleon case. We are interested in the matrix element over the one-nucleon states

$$\frac{i}{4}\lim_{\varepsilon\to 0}\int d\vec{x}\,(x_1\varepsilon_2 - x_2\varepsilon_1)\langle\vec{p}\,|\overline{\Psi}(\vec{x}-\vec{\varepsilon},0)\gamma_5\gamma_3\tau^3\Psi(\vec{x}+\vec{\varepsilon},0)|\vec{p}\,\rangle =$$

$$=\frac{i}{4}\,\overline{u}(\vec{p}\,')\gamma_5\gamma_3 u(\vec{p})\lim_{\varepsilon\to 0}\int d\vec{x}\,e^{i(\vec{p}-\vec{p}\,')\vec{x}}(x_1\varepsilon_2 - x_2\varepsilon_1)e^{i(\vec{p}+\vec{p}\,')\vec{\varepsilon}} = 0 \text{ I)}$$

Thus, contrary to the conjecture made in /6/ , taking into account the Schwinger terms cannot remove the discrepancy between (2) and (3).*)

Using the relations

$$\left.\frac{\partial\overline{u}(\vec{p}\,')}{\partial p_i'}\right|_{\vec{p}\,'=0} = -\frac{1}{2m}\overline{u}(0)\gamma_i$$

$$\left.\frac{\partial^2\overline{u}(\vec{p}\,')}{\partial p_i'\partial p_e'}\right|_{\vec{p}\,'=0} = -\frac{\delta_{ie}}{4m^2}\overline{u}(0) \tag{7}$$

we find that the matrix element of the right hand side (rhs) of (6) in the proton rest frame is equal to $\frac{1}{8m^2}V$ (V is the normalization volume).

Calculate now the matrix element of the left hand side (lhs) of (6). It is convenient to introduce the notations

$$\mathcal{M}_{ie}^{\pm} = m_{ie}^{\pm} - m_{ei}^{\pm}$$

$$m_{ie}^{\pm} = \frac{1}{2}\int d\vec{x}\,x_i V_e^{\pm}(\vec{x},0) \tag{8}$$

Then

$$[\mathcal{M}_{12}^{+},\mathcal{M}_{12}^{-}] = [m_{12}^{+},m_{12}^{-}] + [m_{21}^{+},m_{21}^{-}] - [m_{12}^{+},m_{21}^{-}] - [m_{21}^{+},m_{12}^{-}] \tag{9}$$

Consider the quantity

$$\langle\vec{p}\,'|[m_{ie}^{+},m_{kn}^{-}]|\vec{p}\,\rangle =$$

$$=\sum_n\left\{\langle\vec{p}\,'|m_{ie}^{+}|n\rangle\langle n|m_{kn}^{-}|\vec{p}\,\rangle - \langle\vec{p}\,'|m_{kn}^{-}|n\rangle\langle n|m_{ie}^{+}|\vec{p}\,\rangle\right\} \equiv S_1 + S_2 \tag{10}$$

Clearly, the only term in the sum S_1 , different from zero, in the perturbation theory, corresponds to one neutron in the intermediate state. Taking this into account, we easily get by means of (7) at $\vec{p}\,' = \vec{p} = 0$

I) The conclusion that the contribution of the Schwinger terms vanishes in the lowest order of perturbation theory was made also by B.L.Ioffe (private communication).

$$S_1=\begin{cases} \frac{1}{16m^2}V & i=k\neq l=n \\ -\frac{1}{16m^2}V & i=n\neq l=k \end{cases} \tag{11}$$

On the other hand, in S_2 the one-particle contribution is absent. If on this ground we omit S_2 altogether (that corresponds exactly to the approximation with the one-nucleon state made in /1,2/), the lhs of (6) becomes eqal to $\frac{1}{4m^2}V$, which is in two times larger than the rhs. This is just the contradiction which the formula (2) displays in perturbation theory. Meanwhile, it is quite evident that $S_2\neq 0$ even in perturbation theory. Indeed, the term with two protons and an antineutron in the intermediate state differs from zero in this sum. Simple computations show that

$$S_2=\begin{cases} 0 & i=k\neq l=n \\ \frac{1}{16m^2}V & i=n\neq l=k \end{cases} \tag{12}$$

and in result the lhs of (6) coincides with the rhs.

Now we can make the following conclusions. Firstly, taking into account the Schwinger terms does not remove the contradictions mentioned in introduction. Their contribution vanishes in the lowest order of perturbation theory. Secondly, when saturating the sum rules with the lowest intermediate states, one should take into account along with one-particle states with nucleon and isobar (n and N^*), the corresponding three-particle states $2p\tilde{n}$ and $2p\tilde{N}^*$ (see the diagrams 1,2).

In connection with it one should note that such consideration of the intermediate states contributions is analogous to the non-covariant perturbation theory. It is well-known (see, e.g. /10/) that in the latter two diagrams, which internal lines describe particle and antiparticle correspondingly, are equivalent to a single Feynman pole diagram. Note also that the necessity of taking into account three-particle intermediate states with relatively large mass means that to speak of saturation with states with the lowest masses is possible in covariant technique only.

3. SATURATION OF THE SUM RULES

WITH THE NUCLEON AND ISOBAR INTERMEDIATE STATES

As it was pointed in /4,7/, for derivation of the sum rules, following from local commutation relations, it is convenient to use not the momenta but the Fourie transforms of the currents, which constitute algebra if the Schwinger terms are neglected. But when computing the contribution of the intermediate states, the difficulty arises due to the fact that in the vertices of the non-covariant diagrams of the type 2 energy is not conserved. Therefore the question of the arguments of form-factors at these vertices becomes unclear. Really, in this case the momentum transfer at the vertex cannot be defined as the difference of the outgoing and ingoing momenta. To avoid this difficulty we begin with the discussion of the amplitude

$$T_{\mu\nu} = i\int dx\, e^{ikx}\,\theta(x_0)\langle\vec{p}|[V^-_\mu(0), V^+_\nu(x)]|\vec{p}\rangle \tag{13}$$

If we consider in (13) the Feynman graphs which have a pole in the S-channel (nucleon, isobar etc.), the form-factors at the vertices will depend evidently on $t=k^2$ only. On the other hand, inserting into (13) the complete set of states, we represent the amplitude $T_{\mu\nu}$ as the sum of non-covariant diagrams, and as it was noted in the previous section in general some of them correspond to a single Feynman diagram. Hence it is clear that for both non-covariant pole diagrams the argument of the form-factors is k^2.

To get the sum rule from (13) we set $\mu = 0$ and find the longitudinal part of $T_{\mu\nu}$. Using the vector current conservation and the local commutation relation (Schwinger terms are omitted)

$$\langle\vec{p}|[V^+_0(\vec{x},0), V^-_0(0)]|\vec{p}\rangle = \langle\vec{p}|V^3_0|\vec{p}\rangle\,\delta(\vec{x}) \tag{14}$$

we get

$$k_\nu T_{0\nu} = \langle \vec{p} | V_0^3 | \vec{p} \rangle = 1 \tag{15}$$

On the other hand,

$$k_\nu T_{0\nu} = \sum_n \int d\ell_0^n \Big\{ \frac{k_\nu \langle \vec{p} | V_\nu^+ | n, \vec{p}+\vec{k}, \ell_0^n \rangle \langle n, \vec{p}+\vec{k}, \ell_0^n | V_0^- | \vec{p} \rangle}{p_0 + k_0 + \ell_0^n} + \frac{\langle \vec{p} | V_0^- | n, \vec{p}-\vec{k}, \ell_0^n \rangle k_\nu \langle n, \vec{p}-\vec{k}, \ell_0^n | V_\nu^+ | \vec{p} \rangle}{p_0 - k_0 - \ell_0^n} \Big\}, \tag{16}$$

and due to the vector current conservation

$$k_\nu \langle \vec{p} | V_\nu^+ | n, \vec{p}+\vec{k}, \ell_0^n \rangle = (k_0 + p_0 - \ell_0^n) \langle \vec{p} | V_0^+ | n, \vec{p}+\vec{k}, \ell_0^n \rangle$$
$$k_\nu \langle n, \vec{p}-\vec{k}, \ell_0^n | V_\nu^+ | \vec{p} \rangle = (k_0 - p_0 + \ell_0^n) \langle n, \vec{p}-\vec{k}, \ell_0^n | V_0^+ | \vec{p} \rangle \tag{17}$$

Inserting (17) into (16), we find taking into account (15)

$$\sum_n \int d\ell_0^n \Big\{ \langle \vec{p} | V_0^+ | n, \vec{p}+\vec{k}, \ell_0^n \rangle \langle n, \vec{p}+\vec{k}, \ell_0^n | V_0^- | \vec{p} \rangle - \langle \vec{p} | V_0^- | n, \vec{p}-\vec{k}, \ell_0^n \rangle \langle n, \vec{p}-\vec{k}, \ell_0^n | V_0^+ | \vec{p} \rangle \Big\} = 1, \tag{18}$$

which coincides of course with the relation (8) in /7/.

If we now restrict the sum (18) to the one- neutron intermediate state (diagram 1a), then in the proton rest frame the lhs in (18) is equal to

$$\left(\frac{1 - \frac{t}{4m^2}}{1 - \frac{t}{2m^2}} \right) \left[F_1^V(t) + \frac{t}{2m} F_2^V(t) \right]^2 \tag{19}$$

Inserting it into (18), the authors of /7/ obtained the relation (4). But taking into account the diagram 2a gives in the same frame

$$- \frac{\frac{t}{4m^2}}{1 - \frac{t}{2m^2}} \left[F_1^V(t) + 2m \left(1 - \frac{t}{4m^2}\right) F_2^V(t) \right]^2 \tag{20}$$

and together with (19) it leads to

$$F_1^{V2}(t) - t\left(1 - \frac{t}{4m^2}\right) F_2^{V2}(t) = 1 \tag{21}$$

In contrast to (4) the last relation contradicts neither perturbation theory, nor the analytical properties of form-factors; but it does not agree in general with the equality

$$F_1^{V2}(t) - t F_2^{V2}(t) = 1 \tag{22}$$

which follows from the Adler sum rule. However in the frame with $|\vec{p}| \to \infty$ /11/ the contribution of the diagram 1a becomes equal to $F_1^{V2}(t) - t F_2^{V2}(t)$, and that of the diagram 2a turns to zero, and so we come to the relation (22) too. The conditions (21) and (22) may agree only if $F_1^V(t) = 1$, $F_2^V(t) = 0$. Thus saturation of the sum rule

(18) with the nucleon intermediate state is consistent for free nucleons only.

Show now that this conclusion is not altered if we take into account the isobar intermediate state. In the nucleon-isobar transition current we retain the lowest multipole which corresponds to the magnetic dipole transition

$$\langle N^*, \vec{p}+\vec{k}, p_0+k_0 | V_\mu^- | \vec{p} \rangle = a(k^2)\, \overline{w}_\nu(\vec{p}+\vec{k})\, \gamma_5 (k_\nu \gamma_\mu - g_{\mu\nu} \hat{k})\, u(\vec{p}) \tag{23}$$

It may be easily checked that the coefficient $a(k^2)$ coincides with the form-factor of the electromagnetic transition current. (When comparing the Clebsch-Gordan coefficients for transitions with different change of the third component of the isotopic spin, one should have in mind that they connect $T^\pm$ with $\sqrt{2}\, T^3$ and not directly with T^3)

It is convenient to put $a(k^2) = \sqrt{\frac{3}{2}}\, F^*(k^2)$. Then from the consideration of the $N \to N^*$ transition in the static magnetic field (we neglect the N-N^* mass difference) it is evident that $F^*(0) = \frac{\mu^*}{2m}$. The summation over the spin of the intermediate isobar is performed by means of the projection operator[12]

$$\frac{\hat{q}+m^*}{2q_0}\left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3m^*}(\gamma_\mu q_\nu - \gamma_\nu q_\mu) + \frac{2}{3m^{*2}} q_\mu q_\nu\right] \tag{24}$$

where m^* is the isobar mass and q is its momentum. Insertion of (23) and (24) into (18) leads, if we take into account the nucleon intermediate state too, to the relation true in an arbitrary frame

$$F_1^{v2}(t) + \vec{k}^2 F_2^{v2}(t) - 2\vec{k}^2 F^{*2}(t)\left(1 + \frac{\vec{p}^2 \sin^2\theta}{m^*}\right) = 1 \tag{25}$$

Here θ is the angle between momenta $\vec{p}$ and $\vec{k}$. In the frame where $|\vec{p}| \to \infty$ and $\theta \neq 0$ this relation can take place only under the condition

$$F^*(t) = 0 \tag{26}$$

so that inclusion of isobar in itself cannot indeed make a nucleon clothed. This result seems to us quite natural from the physical point of view since the existence of the nucleon-nucleon and nucleon-isobar transitions only cannot make a nucleon clothed. To make it clothed we need take into account intermediate states containing e.g. π-mesons besides a baryon. If we introduce from the very beginning the lagrangian of nucleon and isobar interaction with vector field, taking into account in it the corresponding form-factors, then the algebra of currents will be broken. Due to these reasons the saturation of the sum rules with arbitrary number of baryon resonances apparently cannot make the nucleon vertex clothed.

Note that the formula (3) is a corollary of (25). To obtain it it is sufficient to differentiate (25) on t at $t=0$ in the proton rest frame (or in the frame with $|\vec{P}| \to \infty$ and $\theta = 0$). Therefore the discussion of the correspondence between the relation (3) and the experimental data has no meaning at all.

We can use not only (18) but analogous relations following from commutators of identical space components. The investigation of them does not lead however to new results and we shall not discuss it in detail. Note only that the contribution of diagrams of the type 2 does not always turn to zero even in the frame with $|\vec{p}| \to \infty$. E.g. the contribution of the diagram 2a to the matrix element of the commutator of space components orthogonal to $\vec{p}$ turns out to be finite in this frame, and that of the diagram 1a vanishes. The contribution of the diagram 2b is distinct from zero (and infinite!) in the case of the commutator of time components too. Thus in distinction from the theories of the type $\lambda\varphi^3$ or $\lambda\varphi^4$ /10/, in the case of particles with non-zero spin we have to take into account the diagrams of this kind even in the frame with infinite momentum.

4. THE ROLE OF THE SCHWINGER TERMS

In the quark model the vector and axial current densities are to be defined by means of the limiting process

$$V_\mu^\alpha(x) = \lim_{\varepsilon\to 0} \bar{\Psi}(x-\varepsilon)\gamma_\mu \tfrac{1}{2}\lambda^\alpha \Psi(x+\varepsilon) \qquad \varepsilon^2 < 0$$
$$A_\mu^\alpha(x) = \lim_{\varepsilon\to 0} \bar{\Psi}(x-\varepsilon)\gamma_5\gamma_\mu \tfrac{1}{2}\lambda^\alpha \Psi(x+\varepsilon) \tag{27}$$

Making use of the canonical commutators for Ψ and $\bar{\Psi}$, and taking into account the Schwinger terms, we get the following commutation relations

$$\begin{aligned}
&\left[V_\mu^\alpha(x), V_\nu^\beta(y)\right]_{x_0=y_0} = \left[A_\mu^\alpha(x), A_\nu^\beta(y)\right]_{x_0=y_0} = \Big\{ i f^{\alpha\beta\sigma}\left[g_{\mu 0}V_\nu^\sigma(x) + g_{\nu 0}V_\mu^\sigma(x) - g_{\mu\nu}V_0^\sigma(x)\right] + \\
&+ i\varepsilon_{0\mu\nu\lambda}\left[d^{\alpha\beta\sigma}A_\lambda^\sigma(x) + \tfrac{1}{3}\delta^{\alpha\beta}A_\lambda(x)\right]\Big\}\delta(\vec{x}-\vec{y}) + \lim_{\varepsilon\to 0}\Big\{ i\varepsilon_{0\mu\nu\lambda}\bar{\Psi}(x-\varepsilon)\gamma_5\gamma_\lambda \tfrac{1}{4}[\lambda^\alpha,\lambda^\beta]\Psi(x+\varepsilon) + \\
&+ \bar{\Psi}(x-\varepsilon)\left(g_{\mu 0}\gamma_\nu + g_{\nu 0}\gamma_\mu - g_{\mu\nu}\gamma_0\right)\tfrac{1}{4}\{\lambda^\alpha,\lambda^\beta\}\Psi(x+\varepsilon)\Big\}\tfrac{1}{2}\left[\delta(\vec{x}-\vec{y}+\vec{\varepsilon}) - \delta(\vec{x}-\vec{y}-\vec{\varepsilon})\right]
\end{aligned} \tag{28}$$

$$\begin{aligned}
&\left[V_\mu^\alpha(x), A_\nu^\beta(y)\right]_{x_0=y_0} = \Big\{ i f^{\alpha\beta\sigma}\left[g_{\mu 0}A_\nu^\sigma(x) + g_{\nu 0}A_\mu^\sigma(x) - g_{\mu\nu}A_0^\sigma(x)\right] + i\varepsilon_{0\mu\nu\lambda}\left[d^{\alpha\beta\sigma}V_\lambda^\sigma(x) + \right. \\
&\left. + \tfrac{1}{3}\delta^{\alpha\beta}V_\lambda(x)\right]\Big\}\delta(\vec{x}-\vec{y}) + \lim_{\varepsilon\to 0}\Big\{ i\varepsilon_{0\mu\nu\lambda}\bar{\Psi}(x-\varepsilon)\gamma_\lambda \tfrac{1}{4}[\lambda^\alpha,\lambda^\beta]\Psi(x+\varepsilon) + \\
&+ \bar{\Psi}(x-\varepsilon)\gamma_5\left(g_{\mu 0}\gamma_\nu + g_{\nu 0}\gamma_\mu - g_{\mu\nu}\gamma_0\right)\tfrac{1}{4}\{\lambda^\alpha,\lambda^\beta\}\Psi(x+\varepsilon)\Big\}\tfrac{1}{2}\left[\delta(\vec{x}-\vec{y}+\vec{\varepsilon}) - \delta(\vec{x}-\vec{y}-\vec{\varepsilon})\right]
\end{aligned} \tag{29}$$

These results are contained partly in the work by Adler and Callan[13]

When obtaining (28) and (29) the limiting process (27) was taken into account in one of the currents only. Sometimes (e.g. in (6)) it appears to be convenient to act in more symmetrical way moving apart the arguments in both currents.

If we take into account the Schwinger terms, the relations used in the section 3 become

$$\begin{aligned}
&\sum_n \int d\ell_0^n \Big\{ \langle \vec{p}|V_\mu^+|n,\vec{p}+\vec{k},\ell_0^n\rangle\langle n,\vec{p}+\vec{k},\ell_0^n|V_\mu^-|\vec{p}\rangle - \\
&- \langle \vec{p}|V_\mu^-|n,\vec{p}-\vec{k},\ell_0^n\rangle\langle n,\vec{p}-\vec{k},\ell_0^n|V_\mu^+|\vec{p}\rangle\Big\} = 1 - \\
&- i\lim_{\varepsilon\to 0}\sin \vec{k}\vec{\varepsilon}\,\langle\vec{p}|\bar{\Psi}(-\vec{\varepsilon})\gamma_0\Psi(\vec{\varepsilon})|\vec{p}\rangle
\end{aligned} \tag{30}$$

In this case the Schwinger term is isotopically even.

In the proton rest frame the matrix element $\langle \vec{p}=0 | \overline{\Psi}(-\vec{\varepsilon})\gamma_0 \Psi(\vec{\varepsilon}) | \vec{p}=0 \rangle$ is a scalar in the three-dimensional space and depends on a single vector $\vec{\varepsilon}$, therefore it must be a function of $\vec{\varepsilon}^2$. Thus after averaging over the angles the Schwinger term in (30) vanishes. Unfortunately we cannot prove this result in an arbitrary frame from general considerations. To ground the neglect of the Schwinger term in the frame with $|\vec{p}| \to \infty$, we have therefore to appeal to the lowest order of perturbation theory.

Note for completeness that the Schwinger term in the commutator of the time and space components of the vector (or axial) current is equal to

$$-i \lim_{\varepsilon \to 0} \sin \vec{k}\vec{\varepsilon} \langle \vec{p} | \overline{\Psi}(-\vec{\varepsilon}) \gamma_n \Psi(\vec{\varepsilon}) | \vec{p} \rangle$$

and can be distinct from zero even in the rest frame. At the same time the Schwinger term in the commutator of different space components is

$$-\varepsilon_{mnk} \lim_{\varepsilon \to 0} \sin \vec{k}\vec{\varepsilon} \langle \vec{p}=0 | \overline{\Psi}(-\vec{\varepsilon}) \gamma_5 \gamma_k \Psi(\vec{\varepsilon}) | \vec{p}=0 \rangle = 0$$

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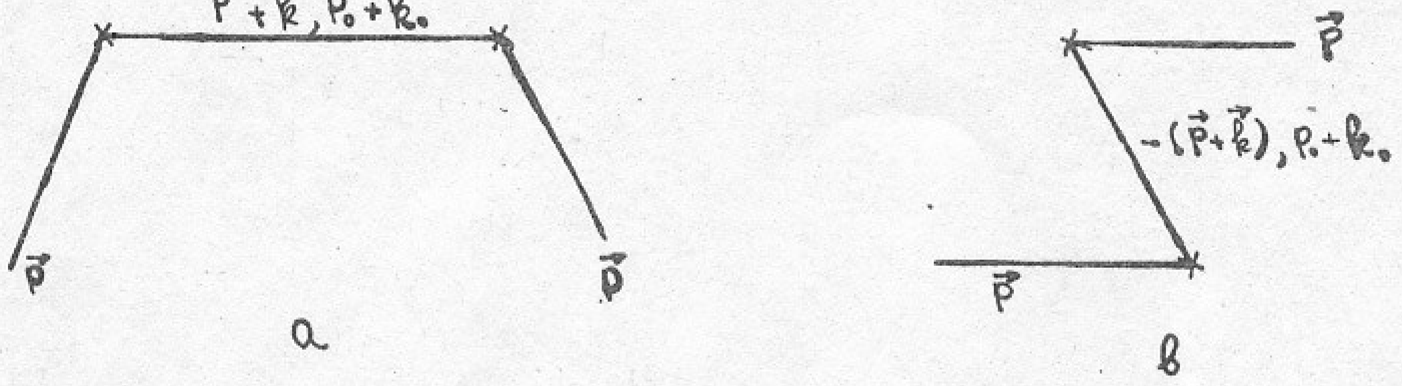
$\vec{p}+\vec{k}, p_0+k_0$
$\vec{p}$
$\vec{p}$
a
$\vec{p}$
$-(\vec{p}+\vec{k}), p_0+k_0$
$\vec{p}$
b

Fig 1

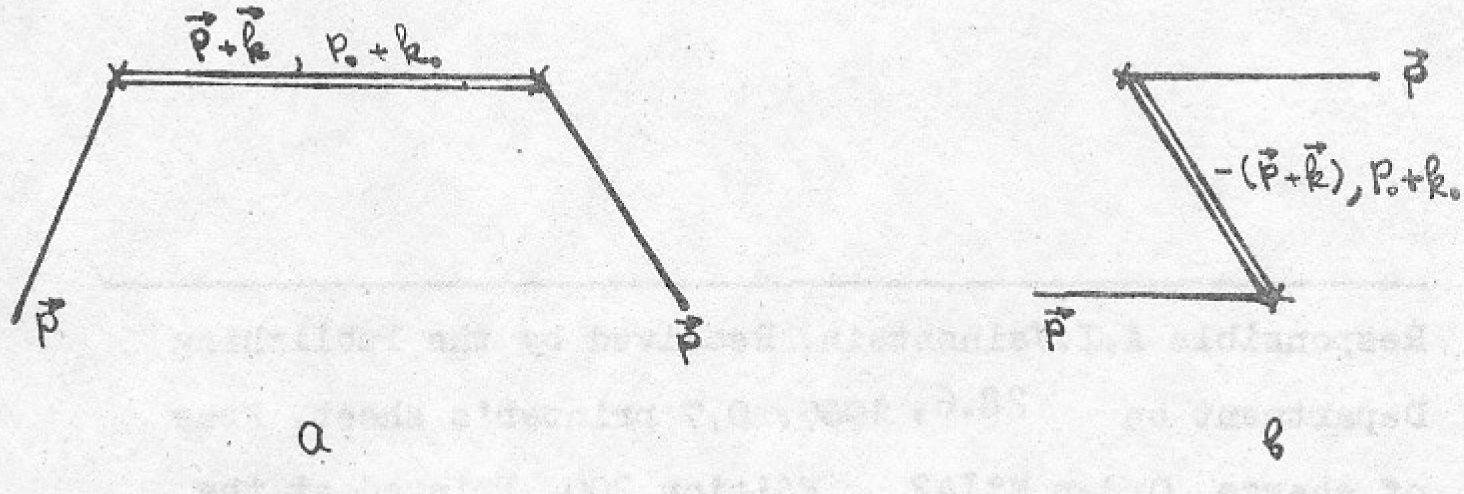
$\vec{p}+\vec{k}$, p_0+k_0
$\vec{p}$
$\vec{p}$
a
$\vec{p}$
$-(\vec{p}+\vec{k}), p_0+k_0$
$\vec{p}$
b

Fig 2

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