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GREEN FUNCTIONS IN THE THEORIES WITH NON-ABELIAN GAUGE GROUP

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abstract

The Yang-Mills field is considered in the radiation gauge. Its Hamiltonian as a function of the independent canonical variables is written down as infinite series in the coupling constant. Therefore when passing to the diagram technique the graphs without any analogue in quantum electrodynamics arise here. The use of N-products when writing an interaction Hamiltonian is shown to be superfluous in the spinor electrodynamics and inadmissible in the scalar electrodynamics and in the Yang-Mills theory. The general structure of the Green function of the Yang-Mills field is considered. Then the Green function is computed in the second order of perturbation theory. It is shown that neither Yang-Mills, nor gravitational field can acquire mass in perturbation theory. Beyond perturbation theory arising of mass of these fields requires at any rate the satisfaction of the conditions even more stringent than those of the photon mass arising. It is shown how the number of degrees of freedom of the vector field in the radiation gauge increases from two to three when the field acquires mass.

1.INTRODUCTION

The interest in the quantum theory of the fields with non-abelian gauge group stems first of all from attempts to quantize the gravitational field for which the role of such a group play coordinate transformations. Extremely complicated structure of the gravitational field equations compelled theorists to turn firstly to the study of a more simple model - the Yang-Mills theory which possesses the non-abelian gauge group generalizing the isotopic spin group/1/.

On the other hand the interest in models of the Yang-Mills type was stimulated by the discovery of vector mesons forming isotopic and unitary multiplets. It should be noted here that since the vector meson masses differ of course from zero, the question is widely discussed whether a field of the Yang-Mills type as well as neutral one can acquire physical mass when it has no bare one /2,3/. The substantial part of the present paper is dedicated to the consideration of this question.

The investigations by Feynman/4/, and then by DeWitt/5/, Faddeev and Popov/6/ and Mandelstam/7/ showed that the use of covariant gauges in the theories with non-abelian gauge group is connected with serious difficulties. Essentially in this case the unitarity condition is violated when computing the closed loops formed by gravitons or Yang-Mills mesons/4/. To make the ends meet one has to introduce new diagrams with loops formed by some fictitious particle, vector one in the case of the gravitational field and scalar one in the Yang-Mills case/4-7/.

Due to these difficulties resorting to the radiation gauge in theories with non-abelian gauge group becomes especially natural /8/. Well-known loss in the automatism of computations due to noncovariance of the method seems not too substantial on the present stage of investigations since now we cannot go all the same beyond the lowest orders of perturbation theory. But the gain is that in this formulation all the unphysical degrees of freedom, unphysical field variables are excluded.

At the beginning of the second section of the present article the quantization of the electromagnetic field in the radiation gauge is carried out by the scheme considered in/9/; this part of the work is of the methodical character mainly. Then the question of the use of N-products in quantum electrodynamics and Yang-Mills theory is discussed. The diagram technique and the structure of Green functions in electrodynamics in radiation gauge is considered. It is shown how the number of degrees of freedom of the vector field without bare mass that acquires the physical mass (such a possibility was pointed by Schwinger/3/) increases in radiation gauge from two to three.

The third section of the paper is dedicated to the quantization of the Yang-Mills field in the radiation gauge. In distinction from electrodynamics, here the Hamiltonian may be written as an explicit function of independent canonical variables as infinite series in coupling constant only. Due to this circumstance in the Yang-Mills theory new types of diagrams without any analogue in quantum electrodynamics arise.

In the fourth section general properties of the Yang-Mills Green function in the radiation gauge are investigated, its distinction from the photon one is discussed.

The computation of the Yang-Mills Green function in the 2nd order of perturbation theory is carried out in the fifth section. It is shown here that the Yang-Mills field cannot acquire mass in perturbation theory. The same assertion may be made for the gravitational field too. The possibility of arising of the mass of Yang-Mills and gravitational field beyond perturbation theory is discussed.

2. RADIATION GAUGE IN QUANTUM ELECTRODYNAMICS

The Lagrangian density in spinor electrodynamics may be written as

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + eA_\mu j_\mu + \mathcal{L}_s \tag{1}$$

Here $\mathcal{L}_s$ is the Lagrangian density of the spinor particles whose current density j_μ serves as the source of the electromagnetic field.

Introduce the canonical momenta of the field

$$\pi_0 = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_0)} = 0 \tag{2}$$

$$\pi_m = -\frac{\partial \mathcal{L}}{\partial(\partial_0 A_m)} = -(\partial_0 A_m - \partial_m A_0), \quad m = 1,2,3 \tag{3}$$

The difficulty connected with vanishing of the momentum conjugated with the time component of the field is well known. It is the result of the fact that the vector-potential has more components than it is necessary to describe the field, i.e., it is the consequence of the gauge invariance. The equation (2) is called an equation of the primary constraint/9/. The Hamiltonian density is written up to three-dimensional divergence as

$$\begin{aligned} H = \frac{1}{2}\pi_m\pi_m + \frac{1}{2}\partial_m A_n \partial_m A_n - \frac{1}{2}\partial_m A_m \partial_n A_n + \\ + eA_m j_m + A_0(\partial_m \pi_m - e j_0) + H_s \end{aligned} \tag{4}$$

From the equation (2) it follows that

$$\partial_0 \pi_0 = -\frac{\partial H}{\partial A_0} = -(\partial_m \pi_m - e j_0) = 0 \tag{5}$$

This condition which is evidently another form of the Maxwell equation $div\vec{E} = e j_0$ is called usually an equation of the secondary constraint/9/. Taking it into account, the Hamiltonian density is written in such a way

$$H = \frac{1}{2}\pi_m\pi_m + \frac{1}{2}\partial_m A_n \partial A_n - \frac{1}{2}\partial_m A_m \partial_n A_n + eA_m j_m + H_s \tag{6}$$

Take into account now the radiation gauge condition

$$\partial_m A_m = 0 \tag{7}$$

i.e., the space part of the vector-potential will be considered as three-dimensionally transverse. The canonical momentum will be split into parts transverse and longitudinal in the three-dimensional sense

$$\Pi_m = p_m + \partial_m \varphi, \quad \partial_m p_m = 0 \tag{8}$$

The equation of secondary constraint is reduced now to the form

$$\Delta \varphi = e j_0 \tag{9}$$

It is easily seen that φ coincides in fact with A_0. Eliminating φ transform the Hamiltonian density up to three-dimensional divergence to the next form

$$H = \frac{1}{2} p_m p_m + \frac{1}{2} \partial_m A_n \partial_m A_n + e A_m j_m - \frac{1}{2} j_0 \Delta^{-1} j_0 + H_s \tag{10}$$

Thus the interaction Hamiltonian density is equal to

$$\begin{aligned} H_{int} &= e A_m j_m - \frac{1}{2} e^2 j_0 \Delta^{-1} j_0 = \\ &= e A_m(\bar{x},t) j_m(\bar{x},t) + \frac{e^2}{8\pi} \int \frac{dy\, j_0(\bar{x},t)\, j_0(\bar{y},t)}{|\bar{x}-\bar{y}|} \end{aligned} \tag{11}$$

The canonical commutation relations are here

$$[p_m(\bar{x},t), A_n(\bar{y},t)] = i d_{mn} \delta(\bar{x}-\bar{y}), \quad d_{mn} = \delta_{mn} - \partial_m \partial_n \Delta^{-1} \tag{12}$$

It is necessary to stress here the next, extremely important circumstance. We succeeded in writing the Hamiltonian density in quantum electrodynamics as a function of the independent dynamical variables A_m and p_m in a closed form only due to the fact that j_0 does not depend on φ, or A_0, so that the equation (9) is solvable explicitly.

One can easily ascertain that the independence j_0 from A_0 which is evident in the case of charged spinor field takes place also in the electrodynamics of scalar particles if j_0 is expressed through canonical coordinates and momenta of the charged field

as it is customary for the Hamiltonian description of a system.

One more remark should be made, concerning the use of normal products in field theory, in particular, in quantum electrodynamics. In the spinor case writing of the current density as N-product leads only to the elimination of the diagram 1 which describes the transition of γ-quantum to vacuum. It is clear, however,

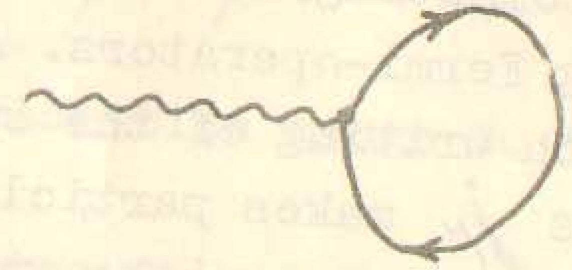

Fig.1.

that this transition is forbidden by the charge parity conservation, and it can be easily seen that its matrix element computed by the usual Feynman rules turns to zero automatically. Thus, in spinor electrodynamics the representation of the current density as N-product is a superfluous, although harmless, operation. As regards, however, scalar electrodynamics, here writing of the part of the interaction Hamiltonian responsible for contact diagrams as N-product leads to the elimination of the diagrams 2 and 2a. But

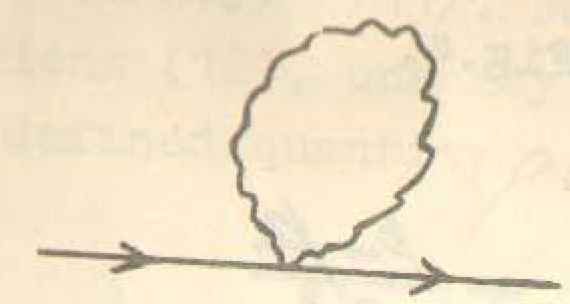

Fig.2.

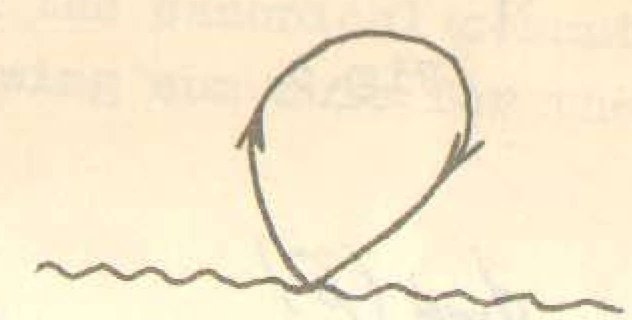

Fig.2a.

it is easily seen that without the diagram 2 the electromagnetic correction to the mass of the charged meson depends on the longitudinal part of the photon Green function/10/. If to place under the sign of N-product the meson operators only as it is suggested in /10/, i.e., exclude only the diagram 2a, then the longitudinal part of the photon polarization operator will arise. And writing of the one-photon vertex in scalar electrodynamics as N-product is unnecessary by the same reason as in spinor case.

In the Yang-Mills theory the application of N-product to the triple vertex, i.e., the elimination of the diagram of the type 1, is superfluous also, since the transition of the Yang-Mills quantum to vacuum is forbidden by the law of isotopic spin conservation, and the corresponding matrix element turns to zero automatically. And the application of the N-product to the four-particle vertices will lead to the same contradictions as in scalar electrodynamics.

Hence writing of the interaction Hamiltonian as N-product is unnecessary in spinor electrodynamics andwrong in scalar electrodynamics and the Yang-Mills theory. It is sufficient to symmetrize used expressions over non-commuting Bose-operators and antisymmetrize over non-anticommuting Fermi-operators. As it is known, in quantum electrodynamics such writing of the current density of spinor and scalar particles j_μ makes particles and antiparticles enter j_μ symmetrically and makes also the vacuum expectation value of j_μ turn to zero. Just this recipe will be used below.

Return to H_{int} given by the formula (11). Such interaction Hamiltonian gives rise to two types of primary diagrams given at the figures 3 and 4. Here the wavy line corresponds to a space photon whose free Green function in momentum representation is

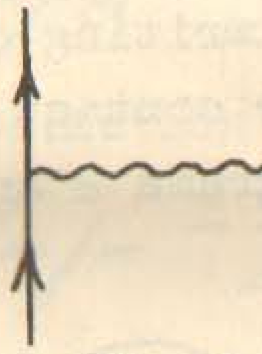

Fig.3.

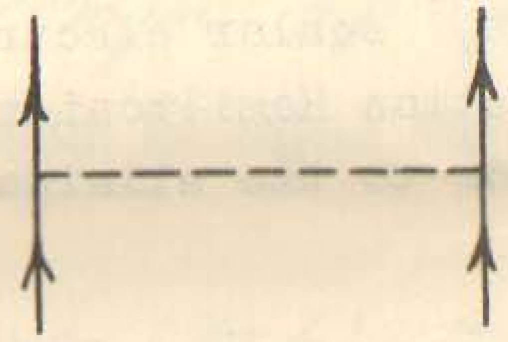

Fig.4.

$$\mathcal{D}^{(0)}_{mn}=\frac{d_{mn}(\bar p)}{\bar p^2-\omega^2},\qquad d_{mn}(\bar p)=\delta_{mn}-\frac{p_m p_n}{\bar p^2} \tag{13}$$

The dotted line denotes the Fourier transform of the inverse Laplace operator

$$\mathcal{D}^{(0)}_{00}=-\frac{1}{\bar p^2} \tag{14}$$

Collecting (13) and (14) into a single non-covariant Green function

$$\mathcal{D}^{(0)}_{\mu\nu}(p,n)=\frac{\delta_{\mu\nu}-\dfrac{(np)(n_\mu p_\nu+p_\mu n_\nu)-p_\mu p_\nu}{(np)^2-p^2}}{p^2} \tag{15}$$

$$p_\mu=(\omega,\bar p),\quad n_\mu=(1,0,0,0),\quad p^2=\omega^2-\bar p^2$$

we come to a diagram technique which differs from a covariant Feynman one in the form of the γ-quantum propagator only; but arising here diagrams are identical topologically to usual Feynman ones. As to the distinction of $\mathcal{D}^{(0)}_{\mu\nu}(p,n)$ from the covariant Green functions, it may be removed with a gauge transformation. E.g., by means of the transformation

$$A'_\mu=A_\mu-\partial_\mu\partial_\nu\Box^{-1}A_\nu \tag{16}$$

we may go from the radiation gauge to the usual diagram technique in the Landau gauge.

Vacuum expectation value of the product of two photon operators in the radiation gauge may be represented in the form /11,3/

$$\langle A_\mu(x)A_\nu(y)\rangle=-i\int d\varkappa^2\rho(\varkappa^2)\{\delta_{\mu\nu}-\Delta^{-1}[(n\partial)(n_\mu\partial_\nu+n_\nu\partial_\mu)-\partial_\mu\partial_\nu]\}\Delta^+(x-y,\varkappa^2) \tag{17}$$

A term proportional to $n_\mu n_\nu$ is absent , otherwise no gauge transformation of A_μ will reduce the expectation value to a covariant form. By the same reason the spectral function $\rho(\varkappa^2)$ cannot depend on $(n\partial)^2$ /11/. Note that using the canonical commutation relations (12), one may get the following sum rule for the positively defined quantity $\rho(\varkappa^2)$ /11,3/:

$$\int d\varkappa^2\rho(\varkappa^2)=1 \tag{18}$$

The complete Green function in the momentum representation of the space components of the vector-potential may be represented by means of (17) in the form

$$D_{mn}(p)=d_{mn}(\bar p)D(p^2),\qquad D(p^2)=\int\frac{d\varkappa^2\rho(\varkappa^2)}{\varkappa^2-p^2} \tag{19}$$

As for the time part of the Green function, the next expression follows for it from (17)

$$\frac{1}{\bar p^2}\int\frac{d\varkappa^2\varkappa^2\rho(\varkappa^2)}{\varkappa^2-p^2} \tag{20}$$

This quantity turns to zero when the interaction is switched off. But it is convenient to define the Green function of the Coulomb quantum in such a way that it reduces to (14) in the absence of the interaction. Proceeding from this consideration, the quantity $-\frac{1}{\bar{p}^2}$, which may be interpreted as the bare Coulomb quantum propagator containing directly in H_{int}, should be added to the expression (20) which contains vacuum loops. Thus, the expression

$$\mathcal{D}_{00}(p)=-\frac{1}{\bar{p}^2}\left[1-\int\frac{d\varkappa^2\varkappa^2\rho(\varkappa^2)}{\varkappa^2-p^2}\right]=\frac{p^2}{\bar{p}^2}\int\frac{d\varkappa^2\rho(\varkappa^2)}{\varkappa^2-p^2} \tag{21}$$

will be taken as the complete Green function of the Coulomb quantum. When going to the last form of $\mathcal{D}_{00}$ we use the sum rule (18).

The components $\mathcal{D}_{on}$ of the Green function in the radiation gauge are equal to zero. Really, because of three-dimensional invariance $\mathcal{D}_{on}=p_n\mathcal{D}(p^2,\bar{p}^2)$, and due to the condition (7) $p_n\mathcal{D}_{on}(p)=\bar{p}^2\mathcal{D}(p^2,\bar{p}^2)=0$ at arbitrary p^2 and $\bar{p}^2$.

Schwinger/3/ has shown that at sufficiently large value of the charge e the photon can in principle acquire mass. If the spectral function is represented in the form

$$\rho(\varkappa^2)=Z\delta(\varkappa^2)+\sigma(\varkappa^2) \tag{22}$$

then at $e=0$ $\sigma(\varkappa^2)=0$ and $Z=1$. As the charge e increases, the contribution of the continuous spectrum may increase to such extent that Z due to the condition (18) turns to zero, so the massless photon disappears. If simultaneously a sufficiently sharp maximum developes in $\sigma(\varkappa^2)$, it may be interpreted as a massive photon. The question of change of the number of degrees of freedom when the mass arises was not discussed by Schwinger.

Show in what manner the number of degrees of freedom of the field described in radiation gauge increases from two to three when the mass arises. Consider firstly for simplicity the model in which $\sigma(\varkappa^2)=\delta(\varkappa^2-\mu^2)$, $Z=0$. In it

$$\mathcal{D}(p^2)=\frac{1}{\mu^2-p^2},\qquad \mathcal{D}_{00}(p)=\frac{p^2}{\bar{p}^2(\mu^2-p^2)} \tag{23}$$

It is easily seen that $\mathcal{D}_{00}$ has now a pole in ω which was absent in $\mathcal{D}_{00}^{(0)}$ (see (14)). Thus the time component of the vector-potential becomes in essence a new dynamical variable. The complete number of degrees of freedom of the field (taking into account two

space ones) equals now to three. On the other hand, in the static limit $\mathcal{D}_{00}$ becomes equal to

$$\mathcal{D}_{00}\Big|_{\omega\to 0}=-\frac{1}{\mu^2+\bar{p}^2} \tag{24}$$

And it is the Fourier transform of the Yukawa potential as it should be expected.

In general case for arbitrary structure of the spectral function $\rho(\varkappa^2)$, the Green function's components may be represented in the form

$$\mathcal{D}_{mn}(p)=\frac{d_{mn}(\bar{p})}{\Pi(p^2)-p^2},\qquad \mathcal{D}_{00}(p)=\frac{p^2}{\bar{p}^2(\Pi(p^2)-p^2)} \tag{25}$$

The absence of the state with zero mass in the spectrum corresponds to the condition $\Pi(0)\neq 0$. It is easily seen that under this condition the potential arising in the static limit is a short-range one even if $\sigma(\varkappa^2)\neq\delta(\varkappa^2-\mu^2)$. And if at some $p^2>0$ $\mathcal{D}(p^2)$ has a sharp maximum, it may be interpreted as an unstable particle with non-zero mass/3/ and with three polarization states.

At last, one more remark on the quantum electrodynamics in the radiation gauge. The vacuum expectation value of the product of Fermi-operators, whose gauge transformation is homogenious, multiplicative one, depends also on the vector n_μ and moreover in not such a trivial manner as the vacuum expectation value (17) of the product of photon operators, for which the gauge transformation is an additive one, reduces tothe mere shift, does. For the product of Fermi-operators $\psi(x)$, $\bar{\psi}(y)$ the vacuum expectation value has the following form/12/, /13/:

$$\langle\psi(x)\bar{\psi}(y)\rangle=i\int d\varkappa^2\{w_1(\varkappa^2,(n\partial)^2)+w_2(\varkappa^2,(n\partial)^2)i\hat{\partial}+ \\ +w_3(\varkappa^2,(n\partial)^2)i\hat{n}(n\hat{\partial})\}\Delta^+(x-y,\varkappa^2) \tag{26}$$

Presence of three spectral functions, instead of the usual two, in this expectation value in the radiation gauge and these functions' dependence on $(n\partial)^2$ is confirmed by the direct computation in the second order of perturbation theory/13/.

3. QUANTIZATION OF THE YANG-MILLS FIELD IN THE RADIATION GAUGE

For simplicity we restrict to the consideration of the self-interacting Yang-Mills field. Taking into account interaction with other fields does not change qualitatively the results got below. Write the Lagrangian density as

$$\mathcal{L} = -\frac{1}{4}\left(\mathcal{D}_\mu b_\nu^\alpha - \mathcal{D}_\nu b_\mu^\alpha\right)\left(\mathcal{D}_\mu b_\nu^\alpha - \mathcal{D}_\nu b_\mu^\alpha\right)$$
$$\mathcal{D}_\mu b_\nu^\alpha = \partial_\mu b_\nu^\alpha + g\varepsilon^{\alpha\beta\gamma} b_\mu^\beta b_\nu^\gamma \qquad (27)$$

The upper indices are the isotopic ones and attain values 1,2,3.

The canonical momenta of the field are

$$\pi_0^\alpha = \frac{\partial \mathcal{L}}{\partial(\partial_0 b_0^\alpha)} = 0 \qquad (28)$$

$$\pi_m^\alpha = -\frac{\partial \mathcal{L}}{\partial(\partial_0 b_m^\alpha)} = -\left(\mathcal{D}_0 b_m^\alpha - \mathcal{D}_m b_0^\alpha\right) = \qquad (29)$$
$$= -\left(\partial_0 b_m^\alpha - \partial_m b_0^\alpha + 2g\varepsilon^{\alpha\beta\gamma} b_0^\beta b_m^\gamma\right)$$

The equation of the primary constraint $\pi_0^\alpha = 0$ takes place here by the same reason as in quantum electrodynamics. The Hamiltonian density is written (neglecting the three-dimensional divergence) as

$$H = \frac{1}{2}\pi_m^\alpha \pi_m^\alpha + \frac{1}{2}\partial_m b_n^\alpha \partial_m b_n^\alpha - \frac{1}{2}\partial_m b_m^\alpha \partial_n b_n^\alpha +$$
$$+ 2g\varepsilon^{\alpha\beta\gamma}\partial_m b_n^\alpha b_m^\beta b_n^\gamma + g^2\left(b_m^\alpha b_m^\alpha b_n^\beta b_n^\beta - b_m^\alpha b_m^\beta b_n^\alpha b_n^\beta\right) \qquad (30)$$
$$+ b_0^\alpha\left(\partial_m \pi_m^\alpha - 2g\varepsilon^{\alpha\beta\gamma}\pi_m^\beta b_m^\gamma\right)$$

Due to the condition (28) we come to the secondary constraint equation

$$\partial_0 \pi_0^\alpha = -\frac{\partial H}{\partial b_0^\alpha} = -\left(\partial_m \pi_m^\alpha - 2g\varepsilon^{\alpha\beta\gamma}\pi_m^\beta b_m^\gamma\right) = 0 \qquad (31)$$

which allows to omit the last term in the expression for H.

Impose now the condition fixing radiation gauge

$$\partial_m b_m^\alpha = 0 \qquad (32)$$

Split the canonical momentum π_m^α into the three-dimensionally transverse and longitudinal parts

$$\pi_m^\alpha = p_m^\alpha + \partial_m \varphi^\alpha, \quad \partial_m p_m^\alpha = 0 \qquad (33)$$

The secondary constraint equation (31) is reduced now to the form

$$\Delta\varphi^\alpha = 2g\varepsilon^{\alpha\beta\gamma}\left(p_m^\beta + \partial_m \varphi^\beta\right) b_m^\gamma \qquad (34)$$

Comparing the equation (34) with the time component of the Lagrangian equations of motion, it can be easily seen that φ^α coincides with b_0^α. Taking into account the condition (32) and neglecting the three-dimensional divergence, the Hamiltonian density may be expressed through b_n^α, p_n^α and φ^α in the following way

$$H = \frac{1}{2}p_m^\alpha p_m^\alpha + \frac{1}{2}\partial_m b_n^\alpha \partial_m b_n^\alpha + 2g\varepsilon^{\alpha\beta\gamma}\partial_m b_n^\alpha b_m^\beta b_n^\gamma +$$
$$+ g^2\left(b_m^\alpha b_m^\alpha b_n^\beta b_n^\beta - b_m^\alpha b_m^\beta b_n^\alpha b_n^\beta\right) - \frac{1}{2}\varphi^\alpha \Delta \varphi^\alpha \qquad (35)$$

However, in distinction from the equation (9) in quantum electrodynamics, the equation (34) does not allow to express φ^α in closed form through the independent canonical variables p_m^β and b_m^γ. Solving the equation (34) by iterations in g and eliminating then φ^α from (35), we get the Hamiltonian of the Yang-Mills field expressed through p_m^α and b_n^β only as a series in the interaction constant g. In distinction from the Hamiltonian in quantum electrodynamics, the Yang-Mills field Hamiltonian cannot be expressed in closed form through independent dynamical variables in the radiation gauge.

Iterations of the equation (34) in the coupling constant g give

$$\varphi_1^\alpha = 2g\varepsilon^{\alpha\beta\gamma}\Delta^{-1}\left(p_m^\beta b_m^\gamma\right)$$
$$\varphi_2^\alpha = 4g^2\Delta^{-1}\left[b_m^\beta \partial_m \Delta^{-1}\left(p_n^\beta b_n^\alpha - p_n^\alpha b_n^\beta\right)\right] \qquad (36)$$
$$\varphi_3^\alpha = -8g^3\varepsilon^{\alpha\beta\gamma}\Delta^{-1}\left\{b_m^\beta \partial_m \Delta^{-1}\left[b_n^\delta \partial_n \Delta^{-1}\left(p_k^\delta b_k^\gamma - p_k^\gamma b_k^\delta\right)\right]\right\}$$

These expressions should be still symmetrized over non-commuting operators. It can be easily shown, however, that the symmetrization does not influence the above result. Therefore we shall use the expression (36) for φ^α. The Hamiltonian density of the Yang-Mills field is written after some transformations as

$$
\begin{aligned}
H = {} & \tfrac{1}{2} p_m^\alpha p_m^\alpha + \tfrac{1}{2}\partial_m b_n^\alpha \partial_m b_n^\alpha + 2g\varepsilon^{\alpha\beta\gamma}\partial_m b_n^\alpha b_m^\beta b_n^\gamma + \\
& + g^2 (b_m^\alpha b_m^\alpha b_n^\beta b_n^\beta - b_m^\alpha b_m^\beta b_n^\alpha b_n^\beta) - g^2 (p_m^\alpha b_m^\beta - p_m^\beta b_m^\alpha) \times \\
& \times \Delta^{-1}(p_n^\alpha b_n^\beta - p_n^\beta b_n^\alpha) - 4g^3 \varepsilon^{\alpha\beta\gamma}\{ p_m^\beta b_m^\gamma \Delta^{-1}[b_n^\delta \partial_n \Delta^{-1}(p_k^\delta b_k^\alpha - \\
& - p_k^\alpha b_k^\delta)] + \Delta^{-1}[b_n^\delta \partial_n \Delta^{-1}(p_k^\delta b_k^\alpha - p_k^\alpha b_k^\delta)] p_m^\beta b_m^\gamma \} - \\
& - 24 g^4 [b_m^\beta \partial_m \Delta^{-1}(p_n^\beta b_n^\alpha - p_n^\alpha b_n^\beta)] \Delta^{-1}[b_k^\gamma \partial_k \Delta^{-1}(p_\ell^\gamma b_\ell^\alpha - p_\ell^\alpha b_\ell^\gamma)] + \ldots
\end{aligned}
\tag{37}
$$

In this expression the terms of the fifth and higher orders in the coupling constant are omitted.

To go to the interaction representation in this Hamiltonian density it is sufficient in practice to identify p_m^α with the canonical three-dimensionally transverse momentum of the free field, i.e., with the quantity $-\partial_0 b_m^\alpha$.

The canonical commutation relations for the Yang-Mills field are

$$[p_m^\alpha(\bar{x},t),\, b_n^\beta(\bar{y},t)] = i\delta^{\alpha\beta} d_{mn}\, \delta(\bar{x}-\bar{y}) \tag{38}$$

The characteristic feature of the obtained interaction Hamiltonian of the Yang-Mills field is that along with the usual diagrams 5 and 6, and the simple Coulomb diagram 7, it raises the in-

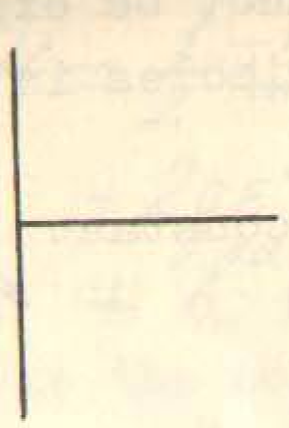

Fig.5.

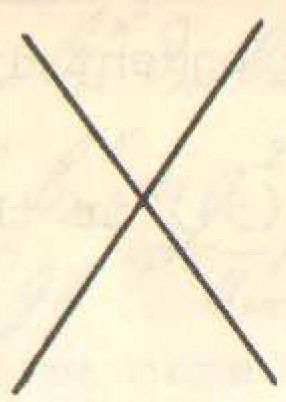

Fig.6.

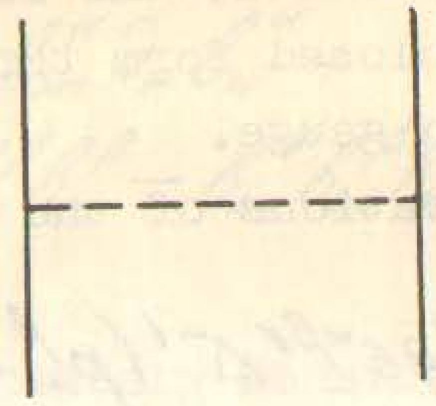

Fig.7.

finite series of the primary diagrams. The first representatives ofthis series are the diagrams 8 and 9 representing the terms of

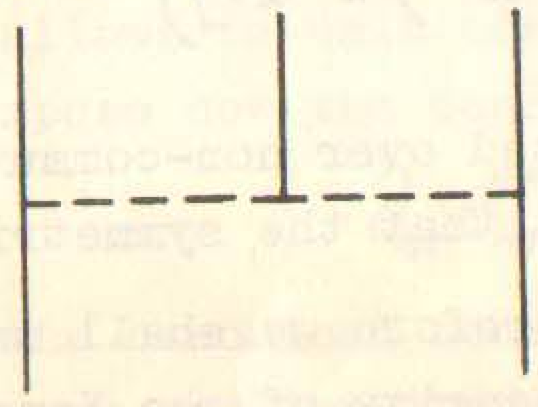

Fig.8.

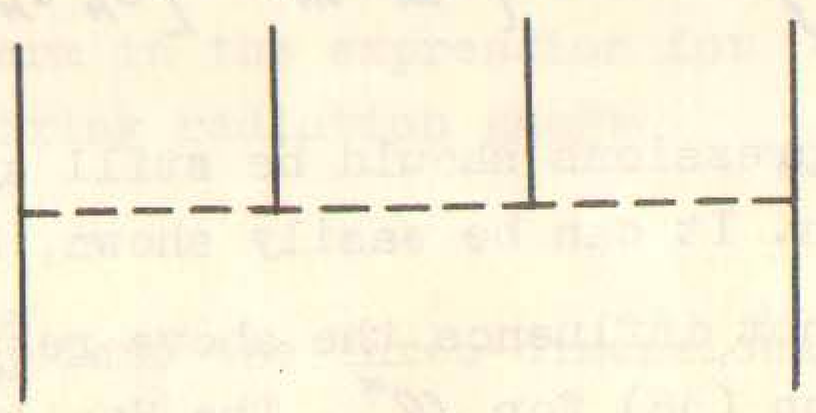

Fig.9.

the third and fourth orders in the coupling constant correspondingly in the Hamiltonian. Every next diagram in this series is obtained from the previous one by the addition of one more solid appendix to the dotted cross-beam. On the presented diagrams a solid line corresponds to a three-dimensionally transverse space quantum whose Green function in the momentum representation is

$$\mathcal{D}_{mn}^{\alpha\beta(0)} = \delta^{\alpha\beta}\frac{d_{mn}(\bar{p})}{\bar{p}^2-\omega^2} \tag{39}$$

A dotted line denotes the analogue of the Green function of a bare Coulomb quantum

$$\mathcal{D}_{00}^{\alpha\beta(0)} = -\delta^{\alpha\beta}\frac{1}{\bar{p}^2} \tag{40}$$

The presence of the infinite series of primary diagrams in radiation gauge distinguishes essentially the Yang-Mills theory from the quantum electrodynamics. Evidently, the presence of these graphs is closely connected with the necessity to introduce diagrams with fictitious particles when using covariant gauges/4-7/. The characteristic feature of the diagram technique in the radiation gauge is, as one can ascertain easily by consideration of the primary diagrams, the absence of the graphs with closed loops formed by the dotted lines only.

In the quantum theory of gravitation the number of primary diagrams is also infinite. The first, obvious, reason of it is that already the Lagrangian of the gravitational field (in distinction, e.g., from the Yang-Mills Lagrangian) is represented as infinite series in the coupling constant, i.e., in the Einstein gravitational constant. And the second reason is quite analoguous to that which leads to the infinite number of primary diagrams for the Yang-Mills theory in the radiation gauge. The self-interaction of the gravitational field makes the elimination of superfluous degrees of freedom of the field possible by means of iterations only. Therefore, even if the expansion of the Lagrangian is restricted to a finite number of terms, the Hamiltonian of the gravitational field is represented as infinite series in the Einstein gravitational constant. Just this series of primary graphs is, evidently, closely connected with the presence of diagrams with fictitious particles which should be introduced when using covariant gauges in the quantum theory of gravitation/4-7/.

4. GREEN FUNCTION OF THE YANG-MILLS FIELD

The vacuum expectation value of the product of two Yang-Mills operators in the radiation gauge is written in the following way

$$\langle b_\mu^\alpha(x) b_\nu^\beta(y)\rangle = -i\delta^{\alpha\beta}\int d\varkappa^2 \Big\{\rho(\varkappa^2,\Delta)\Big[\delta_{\mu\nu} - \frac{(n\partial)(n_\mu\partial_\nu + n_\nu\partial_\mu) - \partial_\mu\partial_\nu}{\Delta}\Big] + \rho_1(\varkappa^2,\Delta) n_\mu n_\nu\Big\}\Delta^+(x-y,\varkappa^2) \tag{41}$$

Such a form of the vacuum expectation value guarantees evidently validity of the condition (32) that fixes the radiation gauge. New features, in comparison with the corresponding quantity (17) in quantum electrodynamics, are here arising of the new tensorial structure containing $n_\mu n_\nu$ and the dependence of the spectral functions not only on $\varkappa^2$, but on $(n\partial)^2$ also, or what is the same on the Laplace operator Δ. This distinction is connected with the fact that the gauge transformation for the Yang-Mills field does not reduce to a shift only as it is the case for electromagnetic vector-potential, but contains also a multiplicative part similarly to the transformation of a charged field in electrodynamics. Therefore, the spectral representation (41) contains characteristic features not only of the relation (17), but of the spectral representation (26) for charged fields also, of course taking into account the distinction in transformation properties of b_μ^α and ψ.

Covariant equation of motion got by the variation of the Lagrangian (27) may be written as

$$\partial_\mu(\partial_\mu b_\nu^\alpha - \partial_\nu b_\mu^\alpha) = j_\nu^\alpha \tag{42}$$

Using the current conservation $\partial_\nu j_\nu^\alpha = 0$ following immediately from (42) and the condition $\partial_m b_m^\alpha = 0$, these equations may be transformed to

$$\Delta b_0^\alpha = j_0^\alpha \tag{43}$$

$$-\Box b_m^\alpha = d_{mn} j_n^\alpha \tag{44}$$

Vacuum expectation value of the product of current density components taking into account the conservation law $\partial_\nu j_\nu^\alpha = 0$ may be written in the following way

$$\langle j_\mu^\alpha(x) j_\nu^\beta(y)\rangle = -i\delta^{\alpha\beta}\int d\varkappa^2\Big\{\tau(\varkappa^2,\Delta)(\varkappa^2\delta_{\mu\nu} + \partial_\mu\partial_\nu) + \tau_1(\varkappa^2,\Delta)\Big[(n\partial)^2\partial_\mu\partial_\nu + \varkappa^2(n\partial)(n_\mu\partial_\nu + n_\nu\partial_\mu) + \varkappa^4 n_\mu n_\nu\Big]\Big\}\Delta^+(x-y,\varkappa^2) \tag{45}$$

Using the equations of motion (43),(44), the functions τ and τ_1 can be easily connected with the spectral functions of the vacuum expectation value $\langle b_\mu^\alpha(x) b_\nu^\beta(y)\rangle$ (see (41))

$$\tau(\varkappa^2,\bar{p}^2) = \varkappa^2\rho(\varkappa^2,\bar{p}^2),\quad \tau_1(\varkappa^2,\bar{p}^2) = \rho_1(\varkappa^2,\bar{p}^2) \tag{46}$$

In electrodynamics the equations for the vector-potential in the radiation gauge may be written in the form coinciding with (43),(44), but of course without isotopic indices. Therefore, using (17) one can easily get in electrodynamics for the quantity $\langle j_\mu(x) j_\nu(y)\rangle$ an expression fully covariant even in radiation gauge and gauge-invariant at all

$$\langle j_\mu(x) j_\nu(y)\rangle = -i\int d\varkappa^2\,\varkappa^2\rho(\varkappa^2)(\varkappa^2\delta_{\mu\nu} + \partial_\mu\partial_\nu)\Delta^+(x-y,\varkappa^2) \tag{47}$$

And it must be so of course since in quantum electrodynamics the Fourier transform of the vacuum expectation value is connected directly with cross-sections of physical processes/14/, so this quantity cannot depend neither on the gauge, nor on the arbitrary vector n_μ. E.g., this quantity, computed taking into account the two-meson intermediate state only, defines unambiguously the total cross-section of the meson pair production in the annihilation of the pair e^+e^- in the lowest order in α. This process is described by the diagrams 10 and 10a. (Note that in the centre-of-mass system the diagram 10a with the virtual Coulomb quantum turns to zero.)

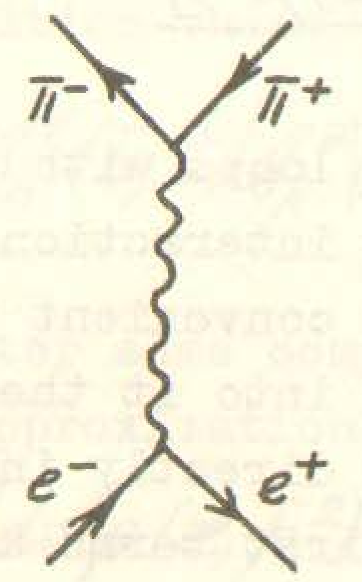

Fig.10.

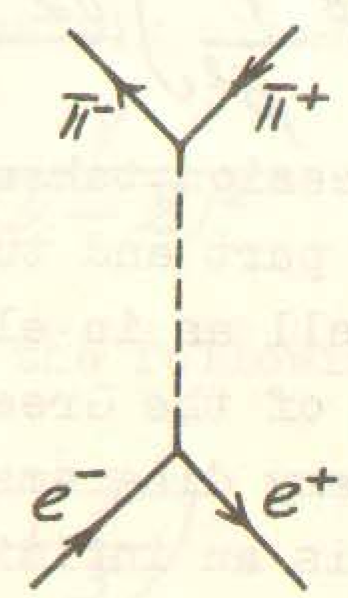

Fig.10a.

The situation in the Yang-Mills theory is rather distinct. E.g., the process of fermion annihilation into a pair of Yang-Mills quanta is described here in the second order in g by the diagrams 11, 11a and 12 where double lines correspond to fermions.

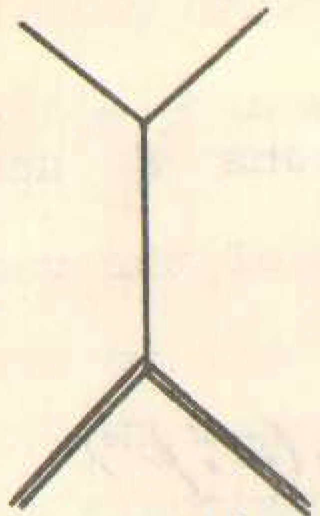

Fig.11.

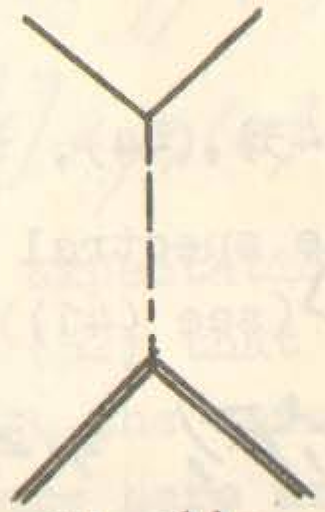

Fig.11a.

Fig.12.

The requirement of the gauge and Lorentz invariance must be satisfied generally speaking only by thesummary contribution to the cross-section of allthe three graphs. Meanwhile, the vacuum expectation value (45) in the two-particle approximation is determined by the diagrams 11 and 11a only, so there are no physical grounds to demand the gauge and Lorentz invariance of this quantity.

Using (41), it is easy to find the Green function of the Yang-Mills field's space components. It can be represented conveniently in the following form

$$\mathcal{D}^{\alpha\beta}_{mn}(p) = \delta^{\alpha\beta} d_{mn}(\bar p)\mathcal{D}(p^2, \bar p^2)$$

$$\mathcal{D}(p^2, \bar p^2) = \int \frac{d\varkappa^2 \rho(\varkappa^2, \bar p^2)}{\varkappa^2 - p^2} = -\frac{1}{p^2}\left\{\int d\varkappa^2 \rho(\varkappa^2, \bar p^2) - \int \frac{d\varkappa^2 \varkappa^2 \rho(\varkappa^2, \bar p^2)}{\varkappa^2 - p^2}\right\} \tag{48}$$

The last form of $\mathcal{D}$ given in (48) appears convenient for the computations in perturbation theory.

Passnow to the time part of the Green function. The contribution to it from (41) is equal to

$$\delta^{\alpha\beta}\frac{1}{\bar p^2}\int \frac{d\varkappa^2[\varkappa^2 \rho(\varkappa^2, \bar p^2) - \bar p^2 \rho_1(\varkappa^2, \bar p^2)]}{\varkappa^2 - p^2} \tag{49}$$

This expression takes into account vacuum loops with non-vanishing imaginary part and turns to zero when the interaction is switched off. As well as in electrodynamics, it is convenient to define the time part of the Green function including into it the contribution from primary diagrams which are contained directly in the Hamiltonian. It is an infinite series, and its first terms are $-\delta^{\alpha\beta}\frac{1}{\bar p^2}$ from the diagram 7 and the quantity obtained from the graph 9 by closing the two internal appendices (other different from zero line closings on this graph correspond to radiation corrections either to the vertex of transition of two space quanta into a time one, or to the process of the scattering of two space quanta). It can be easily seen that the diagrams entering this sum do not possess an imaginary part. When taking into account the summary contribution of primary diagrams, which we shall denote through $-\delta^{\alpha\beta}\frac{K(\bar p^2)}{\bar p^2}$, the time component of the Green function is equal to

$$\mathcal{D}^{\alpha\beta}_{00} = \delta^{\alpha\beta}\mathcal{D}_{00}(p)$$

$$\mathcal{D}_{00}(p) = -\frac{1}{\bar p^2}\left\{K(\bar p^2) - \int \frac{d\varkappa^2[\varkappa^2 \rho(\varkappa^2, \bar p^2) - \bar p^2 \rho_1(\varkappa^2, \bar p^2)]}{\varkappa^2 - p^2}\right\} \tag{50}$$

5. THE GREEN FUNCTION IN PERTURBATION THEORY; PHYSICAL MASS OF YANG-MILLS QUANTUM

Compute the Green function of the Yang-Mills field in the second order of perturbation theory. The quantity $\int d\varkappa^2 \rho(\varkappa^2, \bar p^2)$ we shall find considering the equality

$$\langle [b^\alpha_m(x), \partial_0 b^\beta_n(y)]\big|_{x_0 = y_0}\rangle = i\delta^{\alpha\beta} d_{mn}(\bar p)\int d\varkappa^2 \rho(\varkappa^2, \Delta)\,\delta(\bar x - \bar y) \tag{51}$$

following from (41). To find the left-hand side of (51) express firstly $\partial_0 b^\beta_n$ through canonical variables. Taking into account the relations (29),(33),(36) and that $\varphi^\alpha = b^\alpha_0$, we get in g^2 approximation

$$\partial_0 b^\beta_n(\bar y) = -p^\beta_n(\bar y) - \frac{g^2}{\pi}(\delta^{\varepsilon\beta}\delta^{\sigma\varkappa} - \delta^{\varepsilon\sigma}\delta^{\varkappa\beta})\, b^\sigma_n(\bar y)\int \frac{dz\, p^\varepsilon_e(\bar z)\, b^\varkappa_e(\bar z)}{|\bar y - \bar z|} \tag{52}$$

Using then the canonical commutation relations (38), express the vacuum expectation value in (51) through the matrix element

$$\langle b^\sigma_n(\bar y, t)\, b^\varkappa_k(\bar z, t)\rangle = \delta^{\sigma\varkappa} d_{nk}\frac{1}{4\pi^2}\frac{1}{|\bar y - \bar z|^2} \tag{53}$$

Ultimately after some computations we come to the following sum rule in g^2 approximation

$$\int d\varkappa^2 \rho(\varkappa, \bar p^2) = 1 + \frac{2g^2}{3\pi^2}\left(\ln\frac{\Lambda_1^2}{\bar p^2} + \frac{8}{3}\right) \tag{54}$$

Here Λ_1 is the maximal value of the modulus of the three-dimensional integration momentum.

It should be stressed that in distinction from the quantum electrodynamics, in the Yang-Mills theory the integral of the spectral function is not equal to unity.

As it can be seen from (45) and (46), the quantity $\varkappa^2\rho(\varkappa^2,\bar{p}^2)$ can be found by means of the unitarity condition. It is convenient to compute it as a coefficient at $\varkappa^2\delta_{mn}$ in the imaginary part of the vacuum polarization diagram of a space Yang-Mills quantum. The computations lead to the following result

$$\varkappa^2\rho(\varkappa^2,\bar{p}^2)=\frac{g^2}{4\pi^2}\Big\{-\frac{43}{3}-3x-(x+1)^{-1}-\frac{3x^3+20x^2+32x+16}{(x+1)^{3/2}}\ln\frac{(x+1)^{1/2}-1}{x^{1/2}}\Big\},\quad x=\frac{\varkappa^2}{\bar{p}^2} \tag{55}$$

Note that since by means of the unitarity condition we compute directly $\varkappa^4\rho(\varkappa^2,\bar{p}^2)$, the quantity $\varkappa^2\rho(\varkappa^2,\bar{p}^2)$ may in principle contain still a term with $\delta(\varkappa^2)$. It will lead, however, to a pole of the second order in the Green function (48) and that is inadmissible.

Computing by means of the unitarity condition the imaginary part of the vacuum polarization diagram of a time Yang-Mills quantum, we get

$$\varkappa^2\rho(\varkappa^2,\bar{p}^2)-\bar{p}^2\rho_1(\varkappa^2,\bar{p}^2)=\frac{g^2}{6\pi^2}\Big\{1+9x+\frac{3x(3x+2)}{(x+1)^{1/2}}\ln\frac{(x+1)^{1/2}-1}{x^{1/2}}\Big\} \tag{56}$$

Asymptotical expressions for the spectral functions in the region $\varkappa^2\gg\bar{p}^2$ are

$$\varkappa^2\rho(\varkappa^2,\bar{p}^2)=\frac{g^2}{6\pi^2}\Big(1+\frac{32}{5}x^{-1}-\frac{16}{35}x^{-2}+\dots\Big) \tag{57}$$

$$\bar{p}^2\rho_1(\varkappa^2,\bar{p}^2)=\frac{6g^2}{5\pi^2}x^{-1}\Big(1-\frac{4}{21}x^{-1}+\dots\Big) \tag{58}$$

Contribution to the quantity $R(\bar{p}^2)$ (see (50)) from the term of the fourth order in the coupling constant in the Hamiltonian (37) can be obtained, as it was noted in the end of the fourth section, from the diagram 9 by closing its internal appendices. Together with the contribution of the diagram 7 which is equal to unity, it gives

$$R(\bar{p}^2)=1+\frac{2g^2}{\pi^2}\Big(\ln\frac{\Lambda_1^2}{\bar{p}^2}+\frac{2}{3}\Big) \tag{59}$$

The final expressions for the Green function's components, obtained by the substitution of the quantities (54)-(56),(59) intothe relations (48),(50), are

$$\begin{aligned}\mathcal{D}(p)=&-\frac{1}{p^2}\Big[1+\frac{g^2}{6\pi^2}\Big(4\ln\frac{\Lambda_1^2}{\bar{p}^2}-\ln\frac{\Lambda^2}{-p^2}+\frac{38}{3}\Big)\Big]-\\&-\frac{g^2}{2\pi^2}\frac{1}{\bar{p}^2}\Big\{3\ln\frac{-p^2}{4\bar{p}^2}+6-\frac{1}{2}\Big(3\frac{p^2}{\bar{p}^2}+2\Big)\Big(\frac{\bar{p}^2}{\bar{p}^2+p^2}\Big)^{1/2}\times\\&\times\Big[\ln\frac{-p^2}{4\bar{p}^2}\ln\frac{|\bar{p}|+\sqrt{\bar{p}^2+p^2}}{\big||\bar{p}|-\sqrt{\bar{p}^2+p^2}\big|}+\Phi\Big(\frac{2|\bar{p}|}{|\bar{p}|+\sqrt{\bar{p}^2+p^2}}\Big)-\Phi\Big(\frac{2|\bar{p}|}{|\bar{p}|-\sqrt{\bar{p}^2+p^2}}\Big)\Big]\Big\}\end{aligned} \tag{60}$$

$$\begin{aligned}\mathcal{D}_{00}(p)=&-\frac{1}{\bar{p}^2}\Big\{1+\frac{g^2}{\pi^2}\Big[2\ln\frac{\Lambda_1^2}{\bar{p}^2}-\frac{1}{6}\ln\frac{\Lambda^2}{-p^2}-\frac{19}{3}-\frac{3}{2}\frac{p^2}{\bar{p}^2}-\\&-\Big(4+\frac{p^2}{\bar{p}^2}\Big)\ln\frac{-p^2}{4\bar{p}^2}\Big]+\frac{g^2}{8\pi^2}\Big(\frac{\bar{p}^2}{\bar{p}^2+p^2}\Big)^{3/2}\Big(3\frac{p^6}{\bar{p}^6}+20\frac{p^4}{\bar{p}^4}+32\frac{p^2}{\bar{p}^2}+16\Big)\times\\&\times\Big[\ln\frac{-p^2}{4\bar{p}^2}\ln\frac{|\bar{p}|+\sqrt{\bar{p}^2+p^2}}{\big||\bar{p}|-\sqrt{\bar{p}^2+p^2}\big|}+\Phi\Big(\frac{2|\bar{p}|}{|\bar{p}|+\sqrt{\bar{p}^2+p^2}}\Big)-\Phi\Big(\frac{2|\bar{p}|}{|\bar{p}|-\sqrt{\bar{p}^2+p^2}}\Big)\Big]\Big\}\end{aligned} \tag{61}$$

Here Λ^2 is the upper limit of integration in the dispersion integral. $\Phi(z)=-\int_0^z dy\,\frac{\ln|1-y|}{y}$ is the so-called Spence function.

The logarithmic character of divergencies in the Green function means that in the polarization operator the constant part which diverges as Λ^2 is also absent. In Π_{00} it automatically does not arise if the Feynman integral is computed non-covariantly (firstly the integration over q_0 , and then over $\bar{q}$), but in Π_{mn} this constant part is cancelled with the Schwinger term, i.e., with the quantity $-ig\int d^4x\,e^{-ipx}\delta(x_0)\langle[\partial_0 b_m^\alpha(x), j_n^\beta(0)]\rangle$ which is contained in the polarization operator. The distinction of this vacuum expectation value from zero is due not only to the apparent dependence of j_n^β on b_m^α , which reflects the necessity of taking into account the contact diagrams like 2 and 2a; it is due also to the additional definition of the Yang-Mills self-interaction current. The concrete form of this definition will be considered in a separate paper. The corresponding definition of the fermion isovector current was pointed in the articles/15,16/. The scheme of computations used in the present work takes into account the presence of the Schwinger terms automatically.

For the constant part of the polarization operator to appear, for the massless particle to disappear in the physical spectrum, it is necessary (see(25)) that the expressions in the curly brackets in (48) and (50) turn to zero when $p^2 \to 0$. If the spectral function ρ is represented in the form

$$\rho(\varkappa^2, \bar{p}^2) = Z(p^2)\delta(\varkappa^2) + \sigma(\varkappa^2, \bar{p}^2), \tag{62}$$

these conditions may be written evidently as

$$\int d\varkappa^2 \sigma(\varkappa^2, \bar{p}^2) = \int d\varkappa^2 \rho(\varkappa^2, \bar{p}) \tag{63}$$

$$\int d\varkappa^2 \sigma(\varkappa^2, \bar{p}^2) = k(\bar{p}^2) + \int d\varkappa^2 \frac{\bar{p}^2}{\varkappa^2} \rho_1(\varkappa^2, \bar{p}^2) \tag{64}$$

The first of them, (63), means simply that $Z(\bar{p}^2) = 0$. It is quite clear that in perturbation theory no one of these conditions can be satisfied. Thus, contrary to the doubts which are contained in/1/ and contrary to the certitude which is expressed in /2/, the Yang-Mills field cannot acquire mass in perturbation theory. The analoguous statement can be made also for the gravitational field in perturbation theory.

But can the Yang-Mills field acquire mass if it is not described by perturbation theory? For this purpose the charge has to satisfy simultaneously two equations (63) and (64). The relation (63) is the condition that the mass is acquired by the two states with chiralities ± 1 which are possessed by a massless vector particle; and the relation (64) conditions arising of the new, third state with chirality 0 which is necessary for a vector particle with mass. But these conditions at any rate do not coincide identically. One can easily check the last statement in perturbation theory, comparing the limit of the expression in curly brackets in (64) at $p^2 \to 0$ with the residue of the expression (63) at the point $p^2 = 0$

$$Z(\bar{p}^2) = 1 + \frac{g^2}{6\pi^2}\left(4\ln\frac{\Lambda_1^2}{\bar{p}^2} - \ln\frac{\Lambda^2}{\varkappa_0^2} + \frac{38}{3}\right) \tag{65}$$

Here $\varkappa_0^2$ is the infrared cut-off parameter.

Moreover, despite of the dependence of the equations (63) and (64) on $\bar{p}^2$, the charge which is the solution of these equations cannot depend on this parameter. As for the quantity $\varkappa_0^2$, if the mass arises indeed, all exact expressions should allow limiting transition $\varkappa_0^2 \to 0$.

Therefore, here the conditions of the mass arising are at any rate even more stringent than in quantum electrodynamics where the only equation $Z = 0$ should be satisfied, which in addition does not depend on such quantities as $\bar{p}^2$ and $\varkappa_0^2$.

It is natural to expect that in the quantum theory of gravitation beyond perturbation theory, for the graviton's mass arising the Einstein constant should satisfy already three equations simultaneously. They are the conditions of the mass arising for the states with chiralities ± 2 and of the arising of the new states with mass and with chiralities ± 1 and 0. (Evidently to the chiralities, which differ in the sign only, the same condition corresponds.) As for the dependence on $\bar{p}^2$ and $\varkappa_0^2$, the same remarks as in the Yang-Mills case can be made. Therefore, here the conditions of the mass arising are apparently more stringent than those for the Yang-Mills field.

And now the last remark concerning the possibility of the mass arising for the Yang-Mills and gravitational fields. If reasonable assumption is made that with the growth of the charge the contribution of the continuous spectrum σ to the spectral density ρ increases, then in quantum electrodynamics, where the sum rule (18) takes place, the vanishing of Z at a sufficiently large value of e is more natural than in the Yang-Mills or gravitation theories, where the right-hand side of the sum rule (see (54)) may by itself appear an increasing function of the coupling constant beyond perturbation theory also.

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