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abstract

Yang-Mills field is considered in the radiation gauge. Its Hamiltonian is written down as an explicit function of independent canonical variables in a form of series in the coupling constant. From the consideration of the Feynman graphs arising from this Hamiltonian a simple explanation is given to the necessity of introduction of additional diagrams with fictitious particles under covariant description of the Yang-Mills field. The correct definition of the Yang-Mills particles self-interaction current is obtained taking into account spatial spreading of arguments proposed by Schwinger. By means of this definition polarization operator of the Yang-Mills field is computed in the second order in the coupling constant. It is shown that in perturbation theory Yang-Mills particles do not acquire mass.

1. It is known that in quantum electrodynamics the usual definition of the current density as bilinear combination of charged field operators taken at the same point leads sometimes to contradictions/1/. As it was shown first by Schwinger/1/, the contradictions may be avoided if one takes the charged field operators at the points, separated by a space-like interval, and let this spreading tend to zero at the last stage of computations only. For this definition not to violate the gauge invariance of the current density, one has to introduce additional explicit dependence of the current on the electromagnetic field operators/1,2/.

The pointed definition allows to get for the photon polarization operator $\Pi_{\mu\nu}(k)$ an expression which in natural way satisfies the requirements of gauge and relativistic invariance/2-4/. In particular, the condition $\Pi(0)=0$ is fulfilled in perturbation theory automatically in this case.

The Schwinger definition of the fermion current density was generalized to the case of a theory with non-abelian gauge group in the works/5-7/. This generalization allows to conclude that the contribution of fermion loops to the polarization operator of the Yang-Mills quantum $\Pi^{f}_{\mu\nu}(k)$ turns to zero at $k=0$, just as it takes place in electrodynamics. Thus, the Yang-Mills quantum cannot acquire mass in perturbation theory by means of the interaction only with fermion field. This conclusion is sufficiently evident since by the usual computation of $\Pi^{f}_{\mu\nu}$ with Feynman diagrams an expression is obtained which differs in the isotopic factor $\delta^{\alpha\beta}$ only from the result of analogous computations for the photon polarization operator.

However, the question of the contribution of the Yang-Mills field self-interaction current to the polarization operator of this field seems by no means so evident/8/. Moreover, one comes across the assertion/9/ that due to its self-interaction the Yang-Mills field acquires mass in natural way even in perturbation theory.

But in the work/10/ it was shown that in perturbation theory the Yang-Mills quantum cannot acquire mass even if its self-interaction is taken into account. Moreover, beyond perturbation theory the conditions of mass arising for the Yang-Mills field are at any rate even more stringent than the conditions of the photon mass arising discussed earlier by Schwinger/11/. The mentioned results were obtained in the article/10/ by consideration of the spectral properties of the Yang-Mills field Green functions and the canonical commutation rules in radiation gauge, so to say in an indirect way; the question of the concrete definition of the Yang-Mills self-interaction current was side-stepped in that paper.

In the present work the Schwinger definition is generalized to the case of the Yang-Mills particles current. Then taking into account the explicit form of this definition, the Yang-Mills quantum polarization operator is directly computed in the lowest order of perturbation theory.

2. Due to well-known difficulties with covariant description of the Yang-Mills field/12-16/, we shall consider it in the radiation gauge/17,15,10/. In the present section, which contains for the sake of completeness some results of the work/10/, we present the description of the Yang-Mills field in radiation gauge. For simplicity we restrict the consideration to the case of the self-interacting Yang-Mills field. Its Lagrangian density may be written as

$$\mathcal{L} = -\frac{1}{4} f^{\alpha}_{\mu\nu} f^{\alpha}_{\mu\nu}$$
$$f^{\alpha}_{\mu\nu} = \partial_\mu b^{\alpha}_{\nu} - \partial_\nu b^{\alpha}_{\mu} + 2g\varepsilon^{\alpha\beta\gamma} b^{\beta}_{\mu} b^{\gamma}_{\nu} \tag{1}$$

The canonical momenta of the field are

$$\pi^{\alpha}_{0} = \frac{\partial \mathcal{L}}{\partial(\partial_0 b^{\alpha}_{0})} = 0 \tag{2}$$

$$\pi^{\alpha}_{m} = -\frac{\partial \mathcal{L}}{\partial(\partial_0 b^{\alpha}_{m})} = -f^{\alpha}_{0m} \tag{3}$$

The Hamiltonian density is written up to three-dimensional divergence as

$$H=\frac{1}{2}\pi_m^\alpha \pi_m^\alpha+\frac{1}{4}f_{mn}^\alpha f_{mn}^\alpha+b_0^\alpha\left(\partial_m \pi_m^\alpha-2g\varepsilon^{\alpha\beta\gamma}\pi_m^\beta b_m^\gamma\right) \tag{4}$$

From the equation (2) it follows that

$$\partial_0 \pi_0^\alpha=-\frac{\partial H}{\partial b_0^\alpha}=-\left(\partial_m \pi_m^\alpha+2g\varepsilon^{\alpha\beta\gamma}b_m^\beta \pi_m^\gamma\right)=0 \tag{5}$$

Therefore, the last term in (4) may be omitted.

Impose now the radiation gauge condition

$$\partial_m b_m^\alpha=0 \tag{6}$$

and split the canonical momentum into the three-dimensionally transverse and longitudinal parts

$$\pi_m^\alpha=p_m^\alpha+\partial_m\varphi^\alpha, \qquad \partial_m p_m^\alpha=0 \tag{7}$$

Then the Hamiltonian density may be written neglecting three-dimensional divergence in the next form

$$\begin{aligned}H=\frac{1}{2}p_m^\alpha p_m^\alpha+\frac{1}{2}\partial_n b_m^\alpha \partial_n b_m^\alpha+2g\varepsilon^{\alpha\beta\gamma}\partial_m b_n^\alpha b_m^\beta b_n^\gamma+\\+g^2\left(b_m^\alpha b_m^\alpha b_n^\beta b_n^\beta-b_m^\alpha b_m^\beta b_n^\alpha b_n^\beta\right)-\frac{1}{2}\varphi^\alpha\Delta\varphi^\alpha\end{aligned} \tag{8}$$

And φ^α due to the relation (5) satisfies the equation

$$\Delta\varphi^\alpha=-2g\varepsilon^{\alpha\beta\gamma}b_m^\beta\left(p_m^\gamma+\partial_m\varphi^\gamma\right) \tag{9}$$

Solving the equation (9) by the iterations in the coupling constant, one can easily get the following expression for the nth term of the φ^α expansion

$$\varphi_{(n)}^\alpha=(-2g)^n D^{\alpha\alpha_1}\varepsilon^{\alpha_1\beta_1\gamma_1}b_{m_1}^{\beta_1}\partial_{m_1}D^{\gamma_1\alpha_2}\varepsilon^{\alpha_2\beta_2\gamma_2}b_{m_2}^{\beta_2}\partial_{m_2}D^{\gamma_2\alpha_3}_{\dots}\varepsilon^{\alpha_n\beta_n\gamma_n}b_{m_n}^{\beta_n}p_{m_n}^{\gamma_n} \tag{10}$$

where for convenience the integral operator $D^{\alpha\alpha_1} = \delta^{\alpha\alpha_1}\Delta^{-1}$ is introduced. Inserting the obtained series in coupling constant for φ^α into (8), we express the Hamiltonian density through the independent canonical variables b_m^α and p_m^α as the following expansion

$$
\begin{aligned}
H = {} & \frac{1}{2} p_m^\alpha p_m^\alpha + \frac{1}{2}\partial_n b_m^\alpha \partial_n b_m^\alpha + 2g\varepsilon^{\alpha\beta\sigma}\partial_m b_n^\alpha b_m^\beta b_n^\sigma + \\
& + g^2 (b_m^\alpha b_m^\alpha b_n^\beta b_n^\beta - b_m^\alpha b_m^\beta b_n^\alpha b_n^\beta) - \\
& - \frac{1}{2}\sum_{n=1}^{\infty} (-2g)^{n+1} n (\varepsilon^{\alpha\beta\sigma} b_m^\beta p_m^\sigma) D^{\alpha\alpha_1} \varepsilon^{\alpha_1\beta_1\sigma_1} b_{m_1}^{\beta_1} \partial_{m_1} D^{\sigma_1\alpha_2} \ldots (\varepsilon^{\alpha_n\beta_n\sigma_n} b_{m_n}^{\beta_n} p_{m_n}^{\sigma_n})
\end{aligned}
\tag{11}
$$

The first three terms of the series were written down in the work /10/ in the less convenient form. Taking into account the three-dimensional transversality of the field, the canonical commutation relations are

$$
\begin{aligned}
& [p_m^\alpha(\bar{x},t), b_n^\beta(\bar{y},t)] = \\
& = i\delta^{\alpha\beta}\left(\delta_{mn}\delta(\bar{x}-\bar{y}) + \frac{1}{4\pi}\partial_m\partial_n \frac{1}{|\bar{x}-\bar{y}|}\right) \equiv i\delta^{\alpha\beta} d_{mn}\delta(\bar{x}-\bar{y})
\end{aligned}
\tag{12}
$$

The expression (11) for H still should be symmetrized over non-commuting operators. It can be shown however that the terms in which the symmetrized expression differs from (11) reduce to the three-dimensional divergence and hence may be omitted.

For going over to the interaction representation in the Hamiltonian density (11) it is sufficient to identify p_m^α with the canonical three-dimensionally transverse momentum of the free field, i.e. with the quantity $-\partial_0 b_m^\alpha$. Note that as it was shown in /10/, the interaction Hamiltonian should not be written as N-product.

For construction of diagram technique in the formalism under consideration, besides the Green function of the physical three-dimensionally transverse quantum which is equal in the momentum representation to

$$
\mathcal{D}_{mn}^{\alpha\beta(0)} = \delta^{\alpha\beta}\frac{d_{mn}(\bar{p})}{\bar{p}^2-\omega^2}, \qquad d_{mn}(\bar{p}) = \delta_{mn} - \frac{p_m p_n}{\bar{p}^2}
\tag{13}
$$

and is represented on diagrams by the solid line, it is convenient to introduce the Fourier transform of the integral operator

$$\mathcal{D}_{00}^{\alpha\beta(0)} = -\delta^{\alpha\beta}\frac{1}{p^2} \tag{14}$$

which is represented on graphs by the dotted line.

Then along with the usual diagrams 1 and 2, the Hamiltonian

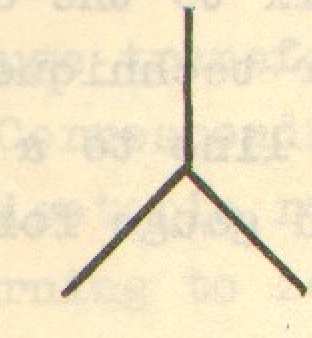

Fig.1.

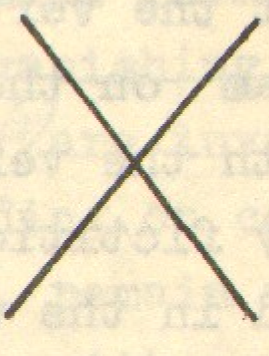

Fig.2.

(11) raises the infinite series of primary diagrams, the first three of them are represented at the figures 3,4,5. The explicit form of the primary diagrams is evident from the consideration of the Hamiltonian (11).

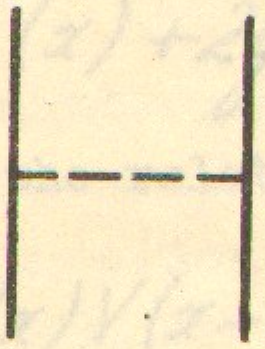

Fig.3.

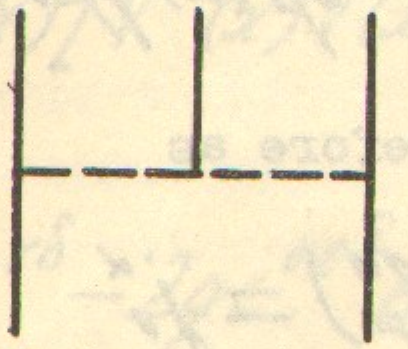

Fig.4.

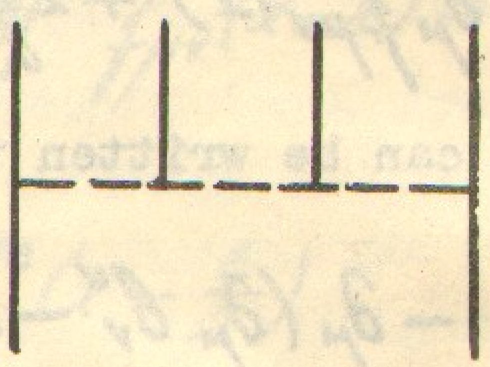

Fig.5.

The characteristic feature of the graphical technique under discussion is, as one can ascertain easily by consideration of the primary diagrams, the absence of the graphs with closed loops formed by the dotted lines only. In other words, in a closed loop at least one line should correspond to a physical quantum. From our point of view, this circumstance explains the necessity to introduce additionary diagrams with closed loops formed by fictitious scalar particles and having so to say the wrong common sign, the necessity that was discovered in the works /12-16/. Really, under covariant description of the Yang-Mills field (as well as in the radiation gauge formalism developed in /16/) the Yang-Mills line on diagrams corresponds to the sum of the Green functions of physical and non-physical (time-like, longitudinal) quanta. Hence, in such a technique closed loop formed by these lines is in essence a set of loops each of which contains, along with Green functions that describe the propagation of physical quanta, Green functions of unphysical particles. And this sum contains a closed loop formed by unphysical quanta only, a loop which is absent in reality. Additionary diagrams introduced in /12-16/ are necessary just for cancellation of these superfluous loops. Note also that the vertex which joins a solid appendix to the dotted cross-beam on the graphs of the type 4,5 in our technique coincides with the vertex which joins a Yang- Mills line to a loop, formed by fictitious particles, in the radiation gauge formalism developed in the work /15/.

3.Pass now to the question of the Yang-Mills particles current density definition. By the current density $j_\nu^\alpha(x)$ we shall mean the sum of all non-linear terms in the equation of motion

$$\partial_\mu f_{\mu\nu}^\alpha(x) + 2g\varepsilon^{\alpha\beta\gamma} b_\mu^\beta(x) f_{\mu\nu}^\gamma(x) = 0 \tag{15}$$

which can be written therefore as

$$-\partial_\mu(\partial_\mu b_\nu^\alpha - \partial_\nu b_\mu^\alpha) = j_\nu^\alpha \tag{16}$$

Note that in distinction from the fermion current in theories with non-abelian gauge group/5-7/, the self-interaction current of the Yang-Mills field even without the Schwinger spatial spreading of arguments is not a vector with respect to isotopic transformations with parameters depending on coordinates. Therefore, we shall proceed from the requirement that the equation (15) as a whole must preserve correct transformation properties even after the spreading of arguments in the operator products.

Introduce the operator $V^{\alpha\delta}(x)$ which satisfies the equation /18,16/

$$\partial_\mu V^{\alpha\delta}(x) = 2g\varepsilon^{\beta\gamma\delta} V^{\alpha\beta}(x) b_\mu^\gamma(x) \tag{17}$$

The solution of this equation which satisfies the boundary condition $V^{\alpha\delta}\big|_{x\to-\infty}=\delta^{\alpha\delta}$ can be written as a series in the coupling constant in the following way

$$V^{\alpha\beta}=\delta^{\alpha\beta}+2g\varepsilon^{\alpha\delta\beta}\int_{-\infty}^{x}d\xi_\mu\, b_\mu^\delta(\xi)+\dots \tag{18}$$

The series (18) is evidently an analogue of the quantity $\exp\left(ie\int_{-\infty}^{x}d\xi_\mu A_\mu(\xi)\right)$ multiplication by which in quantum electrodynamics makes the operators of the charged fields invariant under gauge transformations with variable phase vanishing at $-\infty$ /19/. Correspondingly, the quantities $V^{\alpha\beta}(x)f_{\mu\nu}^\beta(x)$ are invariant under isotopic rotations with parameters depending on coordinates and turning to zero at $-\infty$. However, $V^{\alpha\beta}f_{\mu\nu}^\beta$ remain vectors with respect to rotations in the isotopic space with parameters which are independent of coordinates.

It is clear now that the quantity $V^{-1}(x)V(x-\varepsilon)f_{\mu\nu}^{\alpha\delta}$ is transformed in the same way as $f_{\mu\nu}(x)$. Therefore the equation of motion with the correctly defined product of b_μ^β and $f_{\mu\nu}^\delta$ is written as

$$\partial_\mu f_{\mu\nu}^\alpha(x)+2g\varepsilon^{\alpha\beta\sigma}b_\mu^\beta(x)\left[V^{-1}(x)V(x-\varepsilon)\right]^{\sigma\delta}f_{\mu\nu}^\delta(x-\varepsilon)=0 \tag{19}$$

Using the relation

$$\left[V^{-1}(x)V(x-\varepsilon)\right]^{\sigma\delta}\approx\delta^{\sigma\delta}-2g\varepsilon^{\sigma\varepsilon\delta}\varepsilon_\lambda b_\lambda^\varepsilon\left(x-\frac{\varepsilon}{2}\right) \tag{20}$$

which is obtained by the expansion of both factors in the left hand side near the point $x-\frac{\varepsilon}{2}$, we come to the following equation

$$\partial_\mu f_{\mu\nu}^\alpha(x)+2g\varepsilon^{\alpha\beta\sigma}b_\mu^\beta(x)f_{\mu\nu}^\sigma(x-\varepsilon)-$$
$$-4g^2\varepsilon^{\alpha\beta\sigma}\varepsilon^{\sigma\varepsilon\delta}\varepsilon_\lambda b_\lambda^\varepsilon\left(x-\frac{\varepsilon}{2}\right)b_\mu^\beta(x)f_{\mu\nu}^\delta(x-\varepsilon)=0 \tag{21}$$

To get the correct expression for the current density we have still to define field strength $f_{\mu\nu}^\alpha(x)$ which themself depend on the operators non-linearly (see (1)). This problem is complicated by the fact that in distinction from the equation of motion (15) which can be written as $\mathcal{D}_\mu^{\alpha\beta}f_{\mu\nu}^\beta=0$, where the covariant derivative $\mathcal{D}_\mu^{\alpha\beta}=\partial_\mu\delta^{\alpha\beta}+2g\varepsilon^{\alpha\delta\beta}b_\mu^\delta$ is introduced, the expression

(1) for $f^{\alpha}_{\mu\nu}$ cannot be represented as covariant curl of some quantity.

It is convenient for this definition to use the matrix operator

$$\tau^{\alpha} f^{\alpha}_{\mu\nu}(x) = \frac{1}{2}\left[\hat{\mathcal{D}}_{\mu}(x)\tau^{\alpha} b^{\alpha}_{\nu}(x) - \hat{\mathcal{D}}_{\nu}(x)\tau^{\alpha} b^{\alpha}_{\mu}(x)\right] + h.c.$$

$$\hat{\mathcal{D}}_{\mu}(x) = \partial_{\mu} - ig\tau^{\alpha} b^{\alpha}_{\mu}(x) \tag{22}$$

Here τ^{α} are Pauli matrices. Analogously to (17), introducing the operator $\hat{Y}(x)$ that satisfies the equation

$$\partial_{\mu}\hat{Y}(x) = ig\hat{Y}(x)\tau^{\alpha} b^{\alpha}_{\mu}(x) \tag{23}$$

one can obtain correctly defined quantity $\tau^{\alpha} f^{\alpha}_{\mu\nu}(x)$

$$\tau^{\alpha} f^{\alpha}_{\mu\nu}(x) = \frac{1}{2}\hat{Y}^{-1}(x, x-\varepsilon)\left\{\hat{\mathcal{D}}_{\mu}(x-\varepsilon)\left[\hat{Y}(x, x-\varepsilon)\tau^{\alpha} b^{\alpha}_{\nu}(x)\right] - \hat{\mathcal{D}}_{\nu}(x-\varepsilon)\left[\hat{Y}(x, x-\varepsilon)\tau^{\alpha} b^{\alpha}_{\mu}(x)\right]\right\} + h.c.; \tag{24}$$

$$\hat{Y}(x, x-\varepsilon) = \hat{Y}^{-1}(x-\varepsilon)\hat{Y}(x) \approx 1 - ig\tau^{\alpha}\varepsilon_{\lambda} b^{\alpha}_{\lambda}(x-\tfrac{\varepsilon}{2}); \quad \hat{Y}^{-1} = \hat{Y}^{+}$$

We do not dwell at length on the details of the computations that lead to (24), since the Schwinger addition to $f^{\alpha}_{\mu\nu}$ appears inessential at any rate for the computation of the polarization operator. Combining (24) with (21) and comparing the result with (16), one can obtain the final expression for the current density that takes into account the Schwinger spreading. But we do not write it down explicitly since it is rather lengthy.

Note in conclusion that the obtained expression for $j^{\alpha}_{\nu}(x)$ should be symmetrized over non-commuting operators.

4. We begin the computation of the Yang-Mills field polarization operator from its space components. Using the usual reduction formula and the space components of the equations of motion which can be written in the radiation gauge taking into account the current conservation as

$$\Box b^{\alpha}_{m} = d_{mn} j^{\alpha}_{n} \tag{25}$$

we get for the space part of the Green function $\mathcal{D}^{\alpha\beta}_{mn}(p)$ the following relation

$$p^4\mathcal{D}^{\alpha\beta}_{mn}(p)=i\int d^4x\, e^{-ipx}\{d_{mk}(\bar p)d_{nl}(\bar p)\langle T j^{\alpha}_{k}(x) j^{\beta}_{l}(0)\rangle-$$

$$-\delta(x_0)d_{nl}(\bar p)\langle[\partial_0 b^{\alpha}_{m}(x), j^{\beta}_{l}(0)]\rangle- \tag{26}$$

$$-d_{nl}(\bar p)\partial_0\left(\delta(x_0)\langle[b^{\alpha}_{m}(x), j^{\beta}_{l}(0)]\rangle\right)+p^2\delta(x_0)\langle[b^{\alpha}_{m}(x),\partial_0 b^{\beta}_{n}(0)]\rangle\}$$

We shall make the computations in the second order of perturbation theory restricting to the divergent terms.

In the second order in the coupling constant the first term in (26) corresponds to the usual diagram 6 that describes vacuum

Fig.6.

polarization. Free Green functions (13) correspond to every line on this graph, and the form of the vertex can be found easily from the expression for the current in the first order in g

$$j^{\alpha}_{k}(x)\approx 2g\varepsilon^{\alpha\beta\gamma}b^{\beta}_{m}(x)[\partial_k b^{\gamma}_{m}(x-\varepsilon)-2\partial_m b^{\gamma}_{k}(x-\varepsilon)] \tag{27}$$

When computing this diagram one should integrate firstly over time component of the internal momentum, and then over space ones /3,4/. The Schwinger spreading (it is sufficient to make it in one current only) leads to arising of the factor $e^{iq\varepsilon}$ under integral so that $1/\varepsilon^2$ acts as a cut-off parameter. After rather lengthy calculations we find

$$i d_{mk}(\bar p)d_{nl}(\bar p)\int d^4x\, e^{-ipx}\langle T j^{\alpha}_{k}(x) j^{\beta}_{l}(0)\rangle=$$

$$=\delta^{\alpha\beta}d_{mn}(\bar p)\frac{g^2}{3\pi^2}\left\{\frac{4}{\varepsilon^2}+\left(\frac{27}{10}\bar p^2+\frac{1}{2}\omega^2\right)\ln\frac{1}{\bar p^2\varepsilon^2}\right\};\quad p_\mu=(\omega,\bar p) \tag{28}$$

To compute the commutators that enter the other terms in (26) one has to express $\partial_0 b^{\alpha}_{m}$ and j^{β}_{l} through canonical variables. In particular

$$\partial_0 b_n^\beta(0) \approx -p_n^\beta(0) - \frac{g^2}{\pi}\varepsilon^{\alpha\delta\varepsilon}\varepsilon^{\beta\gamma\delta} d_{nk}\left\{ b_k^\gamma(0)\int\frac{dz}{z}\, p_l^\varepsilon(\bar z-\bar\varepsilon)\, b_l^\alpha(\bar z)\right\} \tag{29}$$

Here the Schwinger spreading of arguments is carried out also in the expression for the zero component of the current density in the first order in g

$$j_0^\alpha(x) \approx 2g\varepsilon^{\alpha\beta\gamma} p_l^\beta(x-\varepsilon)\, b_l^\gamma(x) \tag{30}$$

Using (29) we find the following expression for the last term in (26)

$$\delta^{\alpha\beta} d_{mn}(\bar p)(-p^2)\left[1+\frac{2g^2}{3\pi^2}\ln\frac{1}{\bar p^2\bar\varepsilon^2}\right] \tag{31}$$

When computing with our accuracy the second term in (26), one must take $\partial_0 b_m^\alpha$ equal to $-p_m^\alpha$. Without presenting rather cumbersome calculations we shall write down the contributions of different parts of $j_l^\beta(0)$ into this term. Note once more that as the computations show, the Schwinger additions to $f_{\mu\nu}$ do not influence the result at all. The only term in $p^4 D_{mn}^{\alpha\beta}$ which appears from the Schwinger definition of the current, is due to the last term in (21) and equals to

$$\delta^{\alpha\beta} d_{mn}(\bar p)\,\frac{8g^2}{3\pi^2}\,\frac{1}{\bar\varepsilon^2} \tag{32}$$

The second term in (21) gives into $p^4 D_{mn}^{\alpha\beta}$ contributions of two kinds. The first of them is due to the term $-4g^2\varepsilon^{\alpha\beta\gamma}\varepsilon^{\gamma\delta\varepsilon} b_m^\beta(0)\, b_m^\delta(-\varepsilon)\, b_l^\varepsilon(-\varepsilon)$ and can be described by "bubble" - the Feynman diagram 7. It is

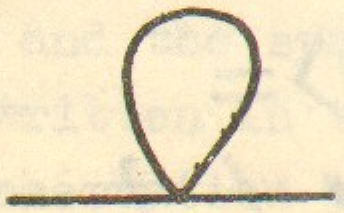

Fig.7.

Fig.8.

equal to

$$-\delta^{\alpha\beta} d_{mn}(\bar p)\,\frac{8g^2}{3\pi^2}\,\frac{1}{\bar\varepsilon^2} \tag{33}$$

Another contribution corresponds to the term

$$2g\varepsilon^{\alpha\beta\gamma}b_o^\beta(0)f_{o\ell}^\gamma(-\varepsilon)\approx 4g^2\varepsilon^{\alpha\beta\gamma}\varepsilon^{\gamma\delta\varepsilon}\Delta^{-1}(b_\kappa^\delta p_\kappa^\varepsilon)p_\ell^\gamma$$

in the current, is described by the diagram 8 and equals to

$$-\delta^{\alpha\beta}d_{mn}(\bar p)\frac{g^2}{\pi^2}\left(\frac{4}{3\bar\varepsilon^2}+\frac{2}{5}\bar p^2\ln\frac{1}{\bar p^2\bar\varepsilon^2}\right) \qquad (34)$$

And the last contribution into $p^4\mathcal{D}_{mn}^{\alpha\beta}$ from the second term in (26) is due to, so to say, non-canonical part of the current that arises when we take into account the non-linearity of $f_{\mu\nu}^\alpha$ in the first item in (21). The non-canonical part of $j_\ell^\alpha(0)$, which is essential for us, equals to $2g\varepsilon^{\alpha\beta\gamma}\partial_o(b_o^\beta(0)b_\ell^\gamma(-\varepsilon))$ and can be transformed taking into account current conservation to the form

$$2g\varepsilon^{\alpha\beta\gamma}\{\Delta^{-1}(\partial_m j_m^\beta)\cdot b_\ell^\gamma-\Delta^{-1}j_o^\beta\cdot p_\ell^\gamma\}.$$

Its contribution to (26) is described by the diagram 8 also and is equal to

$$-\delta^{\alpha\beta}d_{mn}(\bar p)\frac{2g^2}{3\pi^2}\bar p^2\ln\frac{1}{\bar p^2\bar\varepsilon^2} \qquad (35)$$

Still there is the third term in (26). But if taking into account the symmetrization of the current over non-commuting operators, it can be expressed through the vacuum expectation value of the simultaneous anticommutators of b_m^α with p_n^β which are evidently equal to zero. Therefore, the whole this term in (26) turns to zero.

Introducing the polarization operator $\Pi_s(p)$ of physical quanta by means of the relation

$$\mathcal{D}_{mn}^{\alpha\beta}(p)=\delta^{\alpha\beta}d_{mn}(\bar p)\frac{1}{\Pi_s(p)-p^2} \qquad (36)$$

we find for it the following expression

$$\Pi_s(p)=\frac{g^2}{2\pi^2}p^2\ln\frac{1}{\bar p^2\bar\varepsilon^2} \qquad (37)$$

Taking into account the Schwinger term (32) let us obtain the expression for $\Pi_s(p)$ that indeed turns to zero at $p^2=0$ so that the physical quantum does not acquire mass in perturbation theory. The non- covariance of the result is due to the use of the radiation gauge and is discussed in detail in the paper /10/.

Pass now to the time part of the Yang-Mills Green function $\mathcal{D}_{00}^{\alpha\beta}$. By this quantity we shall mean the integral operator that corresponds to the "Coulomb" interaction between real physical charges. In other words, it is dressed dotted line that turns on the right and on the left into two solid ones.

The dressing of the dotted line is described by the diagrams

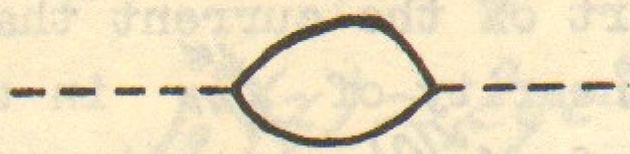

Fig.9.

Fig.10.

9 and 10; the first of them is obtained by the iteration of the diagram 3, and the second one - by closing the internal solid appendices on the diagram 5 /10/. Their contributions to $\mathcal{D}_{00}^{\alpha\beta}$ are respectively

$$\delta^{\alpha\beta}\frac{g^2}{6\pi^2}\frac{1}{\bar{p}^2}\ln\frac{1}{\bar{p}^2\bar{\varepsilon}^2} \tag{38}$$

$$-\delta^{\alpha\beta}\frac{2g^2}{3\pi^2}\frac{1}{\bar{p}^2}\ln\frac{1}{\bar{p}^2\bar{\varepsilon}^2} \tag{39}$$

Hence we get for the polarization operator $\Pi_t(p)$ of the time-like quanta taking into account the contribution to $\mathcal{D}_{00}^{\alpha\beta}(p)$ of the diagram 3, which is equal to $-\delta^{\alpha\beta}\frac{1}{\bar{p}^2}$ /10/, the following expression

$$\Pi_t(p)=\frac{g^2}{2\pi^2}\bar{p}^2\ln\frac{1}{\bar{p}^2\bar{\varepsilon}^2} \tag{40}$$

The polarization operator $\Pi_t(p)$ is defined by means of the relation /10/

$$\mathcal{D}_{00}^{\alpha\beta}(p)=\delta^{\alpha\beta}\frac{p^2}{\bar{p}^2\Pi_t(p)-p^2} \tag{41}$$

Note that the Schwinger terms do not contribute to $\Pi_t(p)$ at all. The point is that the contribution of the diagram 9 which is equal in the adopted approximation to

$$i\int d^4x\, e^{-ipx}\langle T j_0^\alpha(x) j_0^\beta(0)\rangle \tag{42}$$

can be obtained by analogy with (29) by means of the reduction formula and the equation of motion

$$\Delta b_0^\alpha = j_0^\alpha \tag{43}$$

But since the Laplace operator commutes with the symbol of T-ordering, simultaneous commutators do not arise here; so the Schwinger terms are inessential.

Note that $\Pi_t(p)$ also turns to zero at $p^2=0$ so that the Coulomb long-range interaction of charges in the static limit is retained even if the radiation corrections to $\mathcal{D}_{00}^{\alpha\beta}$ are taken into account.

May this conclusion change if we take into account the radiation corrections tothe Green function of the physical quantum and to the vertex of the dotted line transition into two solid lines? The source of the dotted line $-2g\varepsilon^{\alpha\beta\sigma} b_m^\beta p_m^\sigma$ differs from the charge density $j_0^\alpha = -2g\varepsilon^{\alpha\beta\sigma} b_m^\beta (p_m^\sigma + \partial_m \varphi^\sigma + \partial_m b_0^\sigma)$ in the three-dimensional divergence only. Hence the space integral of this source coincides with the integral of j_0^α, i.e., with the isotopic charge operator which commutes of course with the Hamiltonian of the system and therefore is not changed at all if one takes into account the radiation corrections. The question how the radiation corrections to the vertex and to the Green function of the real quantum influence the interaction on large distances, reduces to the question how the Fourier-transform of the quantity $-2g\varepsilon^{\alpha\beta\sigma} b_m^\beta p_m^\sigma(x)$ behaves at $\bar{p}\to 0$. But this last quantity coincides with the isotopic charge so that the radiation corrections do not change it at all.

Therefore, given calculations show that in perturbation theory the physical Yang-Mills quantum cannot acquire mass and the Coulomb long-range interaction of charges in the static limit cannot disappear.

Note that in the work /15/ the expression for the polarization operator of the Yang-Mills quantum in covariant gauge is given. But the question of the constant part of the polarization operator is not in fact discussed in the paper /15/.

The expression for the polarization operator found in the present work corresponds with the adopted accuracy to the result of the paper /10/ obtained by other method. This fact supports the definition of the Yang-Mills particles self-interaction current proposed in the present work and simultaneously confirms the consistency of the spectral conditions and canonical commutation relations used in the paper /10/.

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