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TO THE QUESTION OF THE LIMITING TRANSITION TO ZERO MASS AND RENORMALIZABILITY IN THE MASSIVE YANG-MILLS FIELD THEORY

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A b s t r a c t

It is shown that in perturbation theory for the massive Yang-Mills field there is no continuous limiting transition to zero mass and the theory is unrenormalizable. These results are the consequence of the singularity of zero helicity (three-dimensionally longitudinal) quanta interaction, the singularity which is absent for the neutral vector field. The generalization of the radiation gauge to the non-zero mass case is built. In this description the interaction of three-dimensionally longitudinal quanta is extracted explicitly. The consideration in this formalism makes extremely likely the assertion of the existence of continuous limiting transition to zero mass and of renormalizability beyond the perturbation theory.

I. Introduction

Experimental discovery of non-zero mass vector mesons as well as vector structure of weak interaction attracted the attention to the massive Yang-Mills field theory. In particular the question of renormalizability of the theory is being discussed for a long time /I-9/. The point is that in usual description in Proca formalism the vector field propagator $\mathcal{D}_{\mu\nu}(K) = (g_{\mu\nu} - K_\mu K_\nu/\mu^2)(K^2-\mu^2)^{-1}$ tends to constant at high momenta, that corresponds to unrenormalizability of the theory according to standard classification. However as it is well known, current conservation leads to cancellation at least of part of divergent terms. In the case of neutral vector boson these cancellations ensure renormalizability of the theory. As to the massive Yang-Mills field the question turned out more complicated and the results of various works do not coincide.

The proof of unrenormalizability of the massive Yang-Mills field given in the works /2-6/ is based essentially on the transition to vector fields with Feynman Green function $g_{\mu\nu}(K^2-\mu^2)^{-1}$. However this transition was performed incorrectly. The difficulty here is the same as in the case of the massless Yang-Mills field where, as it was shown by Feynman /IO/, the using of Feynman propagator $g_{\mu\nu}/K^2$ demands the introduction of fictitious scalar particles (see /II-I4/). In spite of this substantial shortcoming of the works /2-6/ their main conclusion is correct from our point of view, since the necessary modification does not affect the most divergent terms.

Recently the works appeared where the radiative corrections in low orders of perturbation theory are evaluated /7, 8, 15, 16/. It is proved that till the diagrams describing the process contain no more than one closed loop, the power divergences cancel out.

In the present paper the question of the limiting transition to zero mass and of renormalizability of the massive Yang-Mills field theory is discussed. In the first part of the paper the investigation is carried out in the frame of the usual perturbation theory. The problems of zero mass limit and renormalizability appear to be connected with the interaction of quanta, three-dimensional polarization vector of which is directed along momentum. In contrast to the neutral vector field the cross-sections of the processes with n such quanta do not turn to zero at $\mu \to 0$ but behave in the first nonvanishing approximation in general as μ^{-n+2}. From this fact it follows that in the perturbation theory there is neither limiting transition to $\mu = 0$, nor renormalizability.

In the second part of the paper we build the formalism that is the generalization of the radiation gauge for the massless field /17-20/ to the non-zero mass case. Such generalization for the neutral vector field is given in /21/. This formalism is based on the division of physical degrees of freedom into three-dimensionally transversal (the helicity ± 1) and three-dimensionally longitudinal (the helicity 0) ones. As it is known three-dimensionally longitudinal component is absent for the massless field. In contrast to Proca formalism our description permits the continuous limiting transition to zero mass. Only the interaction vertices of longitudinal quanta appear to be singular in mass. They do lead in the perturbation theory to the absence of continuity in mass at $\mu = 0$ and to the non-renormalizability. The Lagrangian of the system is not a polinome in longitudinal fields and, if it is not expanded in the coupling constant, permits the continuous transition to zero mass. At $\mu = 0$ the longitudinal quanta interaction is switched off and we come to the massless Yang-Mills field theory which is renormalizable. It seems to us very likely therefore that beyond the frame of perturbation theory the Yang-Mills field theory is renormalizable and all the amplitudes in the limit of zero mass turn into corresponding amplitudes of the massless Yang-Mills field. The renormalizability is understood as the possibility of elimination of all the divergences by means of fixing of the finite number of physical parameters.

When the work had been finished it became known to the authors that the analogous results were obtained by Boulware/22/, Slavnov and Faddeev /23/, by Tyutin and Fradkin /24/ in functional integral approach.

2. The perturbation theory and current conservation

This section deals with the question of limiting transition to $\mu = 0$ and the renormalizability for the massive Yang-Mills field within the usual perturbation theory. Note, that the isotopic triplet of vector fields $\bar{b}_\mu(x)$ interacting with each other and with isodoublet of fermions $N(x)$ is determined by Lagrangian

$$\mathcal{L} = -\frac{1}{4}\bar{f}_{\mu\nu}\bar{f}_{\mu\nu} + \frac{1}{2}\mu^2 \bar{b}_\mu \bar{b}_\mu + \bar{N}\left[\gamma_\mu\left(i\partial_\mu + g\bar{\tau}\bar{b}_\mu\right) - m\right]N$$

$$\bar{f}_{\mu\nu} = \partial_\mu \bar{b}_\nu - \partial_\nu \bar{b}_\mu + 2g\,\bar{b}_\mu \times \bar{b}_\mu \qquad \text{(I)}$$

The equation of four-dimensional transversality $\partial_\mu b_\mu = 0$ follows from the equations of motion.

The description of neutral vector field is obtained merely by means of replacement of isotriplet $\bar{b}_\mu$ by singlet b_μ; the field strength $\bar{f}_{\mu\nu} \to f_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$.

As it is known the polarization of free vector particle with momentum k_μ is determined by vector ε_μ which satisfies the condition $k_\mu \varepsilon_\mu = 0$. It follows from this condition that there are three states of polarization. In the frame where $k_\mu = (\omega, 0, 0, |\bar{k}|)$ these states may be described by vectors $\varepsilon_\mu^{(i)}$ $(i = 1,2,3)$ normalized by the condition $\varepsilon_\mu^{(i)} \varepsilon_\mu^{(k)} = -\delta^{ik}$

$$\varepsilon_\mu^{(1)} = (0,1,0,0), \quad \varepsilon_\mu^{(2)} = (0,0,1,0),$$
$$\varepsilon_\mu^{(3)} = \left(\frac{|\bar{k}|}{\mu}, 0, 0, \frac{\omega}{\mu}\right) = \frac{k_\mu}{\mu}\frac{\omega}{|\bar{k}|} - n_\mu \frac{\mu}{|\bar{k}|} \tag{2}$$

where $n_\mu = (1,0,0,0)$. Emphasize that the singularity of $\varepsilon_\mu^{(3)}$ in mass is a consequence of the conditions $k_\mu \varepsilon_\mu = 0$, $\varepsilon_\mu^{(i)} \varepsilon_\mu^{(k)} = -\delta^{ik}$.

The quanta in the first and the second polarization states we shall call transversal (the helicity ± 1) and that in the third state we shall call longitudinal (the helicity 0) according to three-dimensional properties of polarization vectors $\bar{k}\bar{\varepsilon}^{(1,2)} = 0$, $\bar{\varepsilon}^{(3)} = \frac{\bar{k}}{|\bar{k}|}\frac{\omega}{\mu}$. The singularity in μ of the vector $\varepsilon_\mu^{(3)}$ leads to the singularity of Green function of the vector field

$$\mathcal{D}_{\mu\nu}(k) = \frac{1}{\mu^2 - k^2}\sum_{i=1}^{3} \varepsilon_\mu^{(i)} \varepsilon_\nu^{(i)} = \frac{1}{k^2 - \mu^2}\left(g_{\mu\nu} - k_\mu k_\nu / \mu^2\right) \tag{3}$$

Matrix element of the process with n quanta is written as follows

$$\mathcal{M} = \varepsilon_{\mu_1}(k_1)\varepsilon_{\mu_2}(k_2)\ldots\varepsilon_{\mu_n}(k_n)\,\mathcal{M}_{\mu_1\mu_2\ldots\mu_n} \qquad (4)$$

Here the indices indicating the polarization and charge states of the vector particles are omitted.

We shall consider the amplitudes of the processes in the first non-vanishing approximation in the coupling constant g. Matrix elements in this approximation correspond to the diagrams without closed loops so called "trees". In consideration we use some ideas contained in the Feynman work /IO/ where massless fields mainly are considered.

It will be shown that in the limit $\mu \to 0$ the amplitude $\mathcal{M}$ of process with n Yang-Mills three-dimensionally longitudinal quanta behaves as μ^{-n+2}. As it is known, in neutral case the amplitudes of all the processes with the longitudinal quanta turn to zero at $\mu = 0$.

It follows from the mentioned result that, firstly, the limiting transition to zero mass is absent in the perturbation theory for the Yang-Mills field. Secondly, the cross-sections of the processes with four or more longitudinal quanta increase with energy as it is clear already from dimensional considerations. The rate of growth increases with the quantity of longitudinal quanta. This fact shows the unrenormalizability of the theory. Really, while considering the radiative corrections, we may pass from the closed loops to the integrals of the quantities expressed through the amplitudes of the processes without radiative corrections. The increase with energy of such amplitudes leads to the divergence of dispersion integrals.

It follows very simply from above in particular that power divergences are absent in the amplitudes of the processes with two-particle intermediate states, that is the processes determined by diagrams with one closed loop. Besides, the amplitudes of such processes without external longitudinal quanta remain finite at $\mu = 0$.

Below we shall prove the assertion of the singularity of matrix elements in mass. The result is corroborated by the concrete processes calculation. On the concrete example we illustrate also the covariant technique /IO-I4/ for the massless Yang-Mills field.

Consider the matrix element (see eq.(4)) of reaction with n quanta. It is clear that the amplitude is the most singular in the case of longitudinal quanta the polarization of which is determined by vector $\varepsilon_\mu^{(3)} \sim \frac{1}{\mu}$. Let the first quantum have such a polarization. Using the transversality condition

$$K_{1\mu} \varepsilon_{\mu_2}(K_2) \varepsilon_{\mu_3}(K_3) \ldots \varepsilon_{\mu_n}(K_n) \mathcal{M}_{\mu_1 \mu_2 \mu_3 \ldots \mu_n} = 0 \tag{5}$$

which results from the current conservation, we obtain for the amplitude $\mathcal{M}$ the following expression:

$$\mathcal{M} = -\frac{\mu}{|\vec{K}_1|} n_{\mu_1} \varepsilon_{\mu_2}(K_2) \ldots \varepsilon_{\mu_n}(K_n) \mathcal{M}_{\mu_1 \mu_2 \ldots \mu_n} \tag{6}$$

If all the other quanta are three-dimensionally transversal then at $\mu = 0$ the amplitude $\mathcal{M}$ turns to zero, that is in this limit the matrix elements of the processes with one longitudinal quantum vanish. Note, that the singularity in μ of tensor $\mathcal{M}_{\mu_1 \mu_2 \ldots \mu_n}$ due to the propagators of virtual bosons (see eq.(3)) is unessential as it will be shown later.

In the theory of the neutral vector field interacting with the conserved current the analogous conclusion is true for the processes with any number of longitudinal quanta. The point is that in neutral case the transversality condition (5) may be strengthened

$$k_{i\mu_i} \mathcal{M}_{\mu_1 \dots \mu_i \dots \mu_n} = 0 \quad (i = 1, \dots, n) \tag{7}$$

The transition from (5) to (7) is equivalent to the inclusion of unphysical polarizations (not satisfying eq. $k_\mu \varepsilon_\mu = 0$) in(5). This inclusion does not break the transversality condition since the neutral vector particles are not the sources for each other. The longitudinal part of the tensor $\mathcal{M}_{\mu_1 \dots \mu_n}$ is different from zero only in the case when the charged particles are in unphysical states. Then it is determined by generalized Ward identity. In the Yang-Mills theory vector quanta are charged and they are the sources for each other, so that it becomes impossible to pass to unphysical polarization in the transversality condition (5).

Let us return to the expression (6) for $\mathcal{M}$. Here instead of physical vector of polarization $\varepsilon_{\mu_1}(k_1)$ there is vector $-n_\mu \frac{\mu}{|\vec{k}_1|}$ nonorthogonal to momentum (in four-dimensional sense). So if one more longitudinal quantum, e.g. the second one, takes part in the process, then in the expression for the matrix element

$$\mathcal{M} = -\frac{\mu}{|\vec{k}_1|} n_{\mu_1} \left(\frac{k_{2\mu_2}}{\mu} \frac{\omega_2}{|\vec{k}_2|} - n_{\mu_2} \frac{\mu}{|\vec{k}_2|} \right) \varepsilon_{\mu_3} \dots \varepsilon_{\mu_n} \mathcal{M}_{\mu_1 \mu_2 \mu_3 \dots \mu_n} \tag{8}$$

the term proportional to $k_{2\mu_2}$ does not vanish in contrast to the neutral case. This statement takes place under the condition that the first and the second particles may interact directly with each other.

Thus the amplitude of the process with the emission of two longitudinal quanta of the same charge turns to zero at $\mu \to 0$ and in the case of different charges it remains finite in the limit $\mu \to 0$.

At further increasing the number of zero helicity particles taking part in the process the current conservation does not lead, generally speaking, to any new cancellation of singular in μ terms.

Thus, in general case the amplitude of the process with n longitudinal quanta behaves as μ^{-n+2} at $\mu \to 0$. The reservation "generally speaking" is caused by the fact that at any number of longitudinal quanta there exist the processes in which for example all the quanta are neutral; the amplitudes of such processes are equal to zero at $\mu = 0$.

For the conclusion of the proof we shall show now that the singularity in mass of tensor $M_{\mu_1 \mu_2 \dots \mu_n}$ connected with the propagators $D_{\mu\nu}$ (see (3)) of virtual particles is unessential.

Feynman /IO/ noted that in the amplitudes of the processes determined by the tree diagrams the "tails" of propagators $-k_\mu k_\nu \mu^{-2}(k^2 - \mu^2)^{-1}$ may be omitted if all the external particles are physical. We need the generalization of this statement for the case when one of the external vector quanta is unphysical. This generalization will justify the neglect of singular part of $M_{\mu_1 \mu_2 \dots \mu_n}$ in the above consideration.

For the proof we divide the diagrams determining the process (the tree diagrams are considered) into two groups by the type of the vertices (see eq.(I) for Lagrangian) in which unphysical external quantum enters.

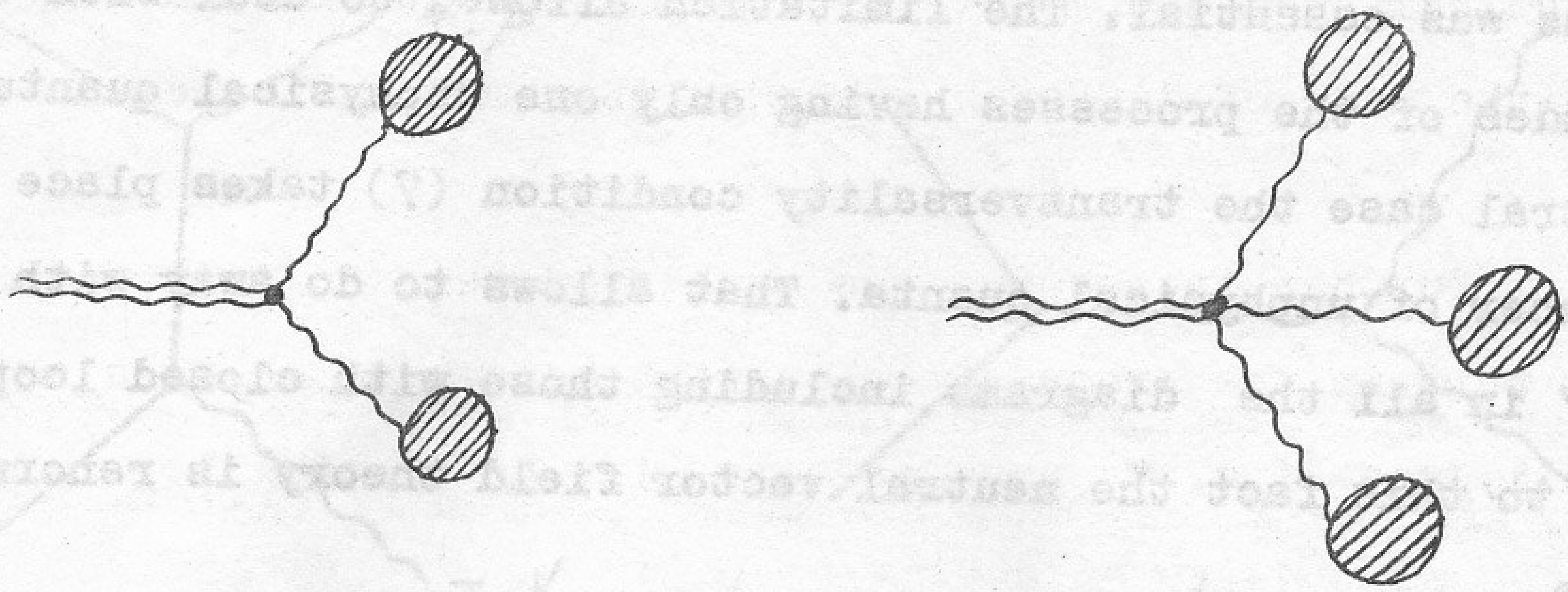

Fig.I

In Fig.I this quantum is represented by double line. (Here we do not consider the interaction with fermion field the consideration of which leads only to unessential modification of the proof). Every shaded block determines the definite process with one virtual and some real particles (the lines designating real particles are omitted in Fig.I). In particular the line ended in block may correspond to the physical quantum.

At first we shall get rid of singularity in μ which arises from the "tail" of propagator corresponding to one of virtual lines in Fig.I. The contribution of the singularity is equal to zero since it is proportional to four-dimensionally longitudinal part of the amplitude of corresponding block. The longitudinal part vanishes because of (5) since all the other particles entering the block are physical.

Now only the singularities connected with "tails" of propagators of virtual lines inside blocks remain. Again every block determines the process with one unphysical quantum. Thus the above consideration may be repeated for this block and so on up to the complete driving out of "tails".

Note, that for the above proof the limitation to the tree diagrams was essential. The limitation allowed to deal with the amplitudes of the processes having only one unphysical quantum. In neutral case the transversality condition (7) takes place at any number of unphysical quanta. That allows to do away with "tails" in all the diagrams including those with closed loops. Thanks to this fact the neutral vector field theory is renormalizable.

The question arises if there are no more cancellations of the terms singular in mass besides those accounted for. We calculated matrix elements of annihilation of nucleon-anti-nucleon pair into two, three and four longitudinal quanta in the limit of $\mu \to 0$ and found that the amplitudes are really proportional to μ^{-n+2}. The calculations are carried out rather simply by means of generalized Ward identities and are given in Appendix. Note, that in spite of noncovariance of polarization description the final expressions for amplitudes in the limit of $\mu \to 0$ are covariant.

In conclusion of this section we would illustrate the complications arising at covariant description of the massless Yang-Mills field /10-14/ though it has no direct relation to our work.

Let us consider the annihilation of the proton-anti-proton pair into pair of charged massless quanta $p(p_1) + \bar{p}(p_2) \to b^-(k_1) + b^+(k_2)$. The reaction is described by two diagrams (Fig.2). Since both vertices of the diagram b are transverse in the 4-momentum of virtual quanta, its propagator may be gauged arbitrarily, for example, by Feynman gauge $g_{\mu\nu}/k^2$.

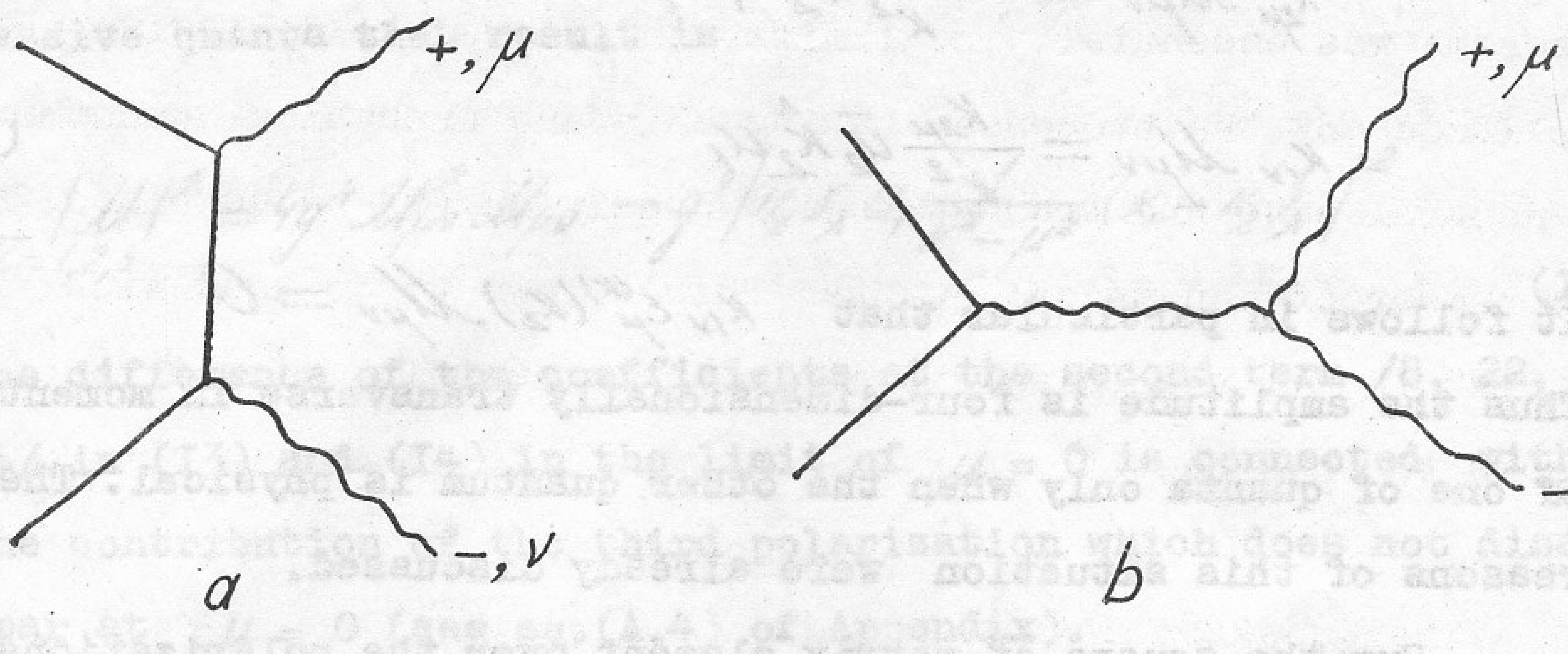

Fig.2

(It corresponds to the general statement /IO/ of gauge invariance of tree diagrams). The expression for the amplitude may be written as

$$\mathcal{M} = 2g^2 \varepsilon_\mu^{(i)}(k_2)\, \varepsilon_\nu^{(\kappa)}(k_1)\, \mathcal{M}_{\mu\nu}$$

$$\mathcal{M}_{\mu\nu} = -\bar{u}_2 \gamma_\nu \frac{1}{\hat{p}_1 - \hat{k} - m} \gamma_\mu u_1 + \bar{u}_2 \gamma_\lambda u_1 \frac{1}{k^2} \Gamma_{\lambda\mu\nu} \tag{9}$$

where $\varepsilon_\mu^{(i)}(k_2)$, $\varepsilon_\nu^{(\kappa)}(k_1)$ are three-dimensionally transverse polarizations of massless particles (see eq.(2)for $i, \kappa = 1,2$), u_1 , $\bar{u}_2$ are spinors determining proton and antiproton, $k = -k_1 - k_2$ is momentum of virtual quantum. The vector particles interaction vertex $\Gamma_{\lambda\mu\nu}$ is

$$\Gamma_{\lambda\mu\nu} = -g_{\mu\nu}(k_2 - k_1)_\lambda - g_{\nu\lambda}(k_1 - k)_\mu - g_{\lambda\mu}(k - k_2)_\nu \tag{IO}$$

It is clear that

$$k_{2\mu} M_{\mu\nu} = -\frac{k_{1\nu}}{k^2} \bar{u}_2 \hat{k}_1 u_1$$

$$k_{1\nu} M_{\mu\nu} = \frac{k_{2\mu}}{k^2} \bar{u}_2 \hat{k}_2 u_1 \qquad (\mathrm{II})$$

It follows in particular that $k_{1\nu} \varepsilon_\mu^{(k)}(k_2) M_{\mu\nu} = 0$.
Thus the amplitude is four-dimensionally transverse in momentum of one of quanta only when the other quantum is physical. The reasons of this situation were already discussed.

Sum the square of matrix element over the polarizations of final particles by means of relation

$$d_{\mu\mu'} = \sum_{i=1,2} \varepsilon_\mu^{(i)} \varepsilon_{\mu'}^{(i)} = -g_{\mu\mu'} + \frac{(kn)(k_\mu n_{\mu'} + k_{\mu'} n_\mu) - k_\mu k_{\mu'}}{(kn)^2 - k^2} \qquad (\mathrm{I2})$$

The summation over polarizations of the first quantum because of relation $k_{1\nu} \varepsilon_\mu^{(k)}(k_2) M_{\mu\nu} = 0$ comes to substitution $d_{\nu\nu'} \longrightarrow -g_{\nu\nu'}$. But after that it is impossible to sum by means of tensor $-g_{\mu\mu'}$ over polarizations of the second quantum as it is seen from (II). It is impossible, in essence, because we introduced by the substitution $d_{\nu\nu'} \longrightarrow -g_{\nu\nu'}$ unphysical polarizations of the first particle. With the help of (II) the next expression for summed over polarizations square of matrix element may be obtained

$$\sum_{i,k=1,2} |M|^2 = 4g^4 M_{\mu\nu}^* M_{\mu\nu} - 2g^4 \left| \bar{u}_2 \gamma_\lambda u_1 \frac{1}{k^2} (k_1 - k_2)_\lambda \right|^2 \qquad (\mathrm{I3})$$

It is clear that the analogous result is obtained for imaginary part of the process with two massless quanta in intermediate state. In covariant technique /I0-I4/ the second term is connected with interaction of fictitious scalar particles.

Note, that in the case of the same process but for the massive quanta the result is

$$\sum_{i,k=1,2,3} |M|^2 = 4g^4 M^*_{\mu\nu} M_{\mu\nu} - g^4 \left| \bar{u}_2 \gamma_\lambda u_1 \frac{1}{k^2-\mu^2} (k_1 - k_2)_\lambda \right|^2 \tag{I4}$$

The difference of the coefficients at the second term /8, 22, 23. I6/ in (I3) and (I4) in the limit of $\mu = 0$ is connected with the contribution of the third polarization which does not disappear at $\mu = 0$ (see eq.(A.4) of Appendix).

3. The massive Yang-Mills field in the radiation gauge.

It is seen from the results of the above section that the singularity of amplitude in mass is connected with the interaction of longitudinal quanta that does not disappear at $\mu = 0$ in the perturbation theory. So it is natural to pass directly in the Lagrangian description from $\bar{b}_\mu(x)$ operators to $\bar{a}_\mu(x)$ operators describing three-dimensionally transverse quanta $\partial_m \bar{a}_m = 0 \quad (m = 1,2,3)$ and to $\sigma(x)$ operators determining three-dimensionally longitudinal quanta. The arising formalism is generalization of radiation gauge to the nonzero mass case. The analogous description of neutral vector field continuously turns at $\mu = 0$ into formalism of radiation gauge for quantum electrodynamics /2I/. The above transition to the fields $\bar{a}_\mu$ and $\bar{\sigma}$ is realized conveniently with the help of substitution which is represented as gauge transformation

$$b_\mu = S^{-1}\left(a_\mu + \frac{i}{g}\partial_\mu S \cdot S^{-1}\right)S, \quad \partial_m a_m = 0$$
$$N = S^{-1}\psi \tag{I5}$$

For shortening in writing we use matrix designations in (I5) and below : every isotopic vector $\bar{c}$ corresponds to matrix $c=\bar{\tau}\bar{c}=$ $=\tau^\alpha c^\alpha$ $(Sp\,\tau^\alpha\tau^\beta=2\delta^{\alpha\beta};\ \alpha,\beta=1,2,3)$. Matrix S depends on σ -field and is unitary and unimodular. The various parametrizations are known to be possible for such matrix. We shall use two of them

$$S=exp\left[-i\frac{g}{\mu}\sigma(x)\right] \tag{I6a}$$

$$S=\frac{1-i\frac{g}{2\mu}\sigma(x)}{1+i\frac{g}{2\mu}\sigma(x)} \tag{I6b}$$

The Lagrangian of the fields a_μ and σ is obtained by substituting (I5) into the Lagrangian of the b_μ field (see(I)) (the substitution changes the form of the massive term only) :

$$\mathcal{L}=\frac{1}{2}Sp\left\{-\frac{1}{4}f_{\mu\nu}(a)f_{\mu\nu}(a)+\frac{1}{2}(\mu a_\mu+\Sigma_\mu)(\mu a_\mu+\Sigma_\mu)+\right.$$
$$\left.+a_\mu j_\mu\right\}+\bar{\psi}(i\hat{\partial}-m)\psi \tag{I7}$$

The field strength $f_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu-ig[a_\mu,a_\nu]$ depends on a_μ just as $\bar{\tau}\bar{f}_{\mu\nu}$ (see (I)) depends on b_μ ; the operator $j_\mu=\bar{\tau}\bar{j}_\mu,\ \bar{j}_\mu=g\bar{\psi}\gamma_\mu\bar{\tau}\psi$ is fermion current. The expression for the operator $\Sigma_\mu=i\frac{\mu}{g}\partial_\mu S\cdot S^{-1}$ at the parametrizations (I6a,b)is

$$\Sigma_\mu=\partial_\mu\sigma-\frac{i}{2}\frac{g}{\mu}[\sigma,\partial_\mu\sigma]\frac{1+2i\frac{g}{\mu}\sigma-exp\left(2i\frac{g}{\mu}\sigma\right)}{2\frac{g^2}{\mu^2}\sigma^2} \tag{I8a}$$

$$\Sigma_\mu = \frac{1}{1+i\frac{g}{2\mu}\sigma}\,\partial_\mu\sigma\,\frac{1}{1-i\frac{g}{2\mu}\sigma} \tag{18b}$$

Note, that the Lagrangian of self-interacting scalar σ-field $\mathcal{L}(\sigma)=\frac{1}{2}\Sigma_\mu\Sigma_\mu$ coincides with the phenomenological Lagrangian of π-mesons. In particular at the algebraic parametrization (18b) we obtain Lagrangian in the Weinberg form /25/

$$\mathcal{L}(\sigma)=\frac{1}{2}\,\frac{\partial_\mu\bar\sigma\,\partial_\mu\bar\sigma}{\left(1+\frac{g^2}{4\mu^2}\bar\sigma^2\right)^2}$$

Now we shall formulate the results obtained in the present section and then return to the construction of canonical formalism and the perturbation theory.

The point of the above transformation is that the description of free fields a_μ and σ permits of the limiting transition to zero mass in contrast to Proca formalism. The σ-field interaction vertices arising at the expansion of Lagrangian in the coupling constant g are singular in mass. Then the unrenormalizability in the perturbation theory becomes obvious. For example at the algebraic parametrization (18b) Lagrangian is as follows

$$\mathcal{L}=\mathcal{L}(a,\psi)+\frac{1}{2}\,Sp\Big\{\mu a_\mu\frac{1}{1+\frac{ig}{2\mu}\sigma}\,\partial_\mu\sigma\,\frac{1}{1-\frac{ig}{2\mu}\sigma}+$$
$$+\frac{1}{2}\,\frac{\partial_\mu\sigma\,\partial_\mu\sigma}{\left(1+\frac{g^2}{4\mu^2}\sigma^2\right)^2}\Big\}=\mathcal{L}(a,\psi)+\frac{1}{2}\,Sp\Big\{\mu a_\mu\partial_\mu\sigma-$$
$$-\frac{1}{2}g a_\mu[\sigma,\partial_\mu\sigma]-\frac{g^2}{4\mu}\,a_\mu\left(\sigma^2\partial_\mu\sigma+\sigma[\sigma,\partial_\mu\sigma]\right)+$$
$$+\frac{1}{2}\partial_\mu\sigma\,\partial_\mu\sigma-\frac{g^2}{4\mu^2}\,\sigma^2\partial_\mu\sigma\,\partial_\mu\sigma+\ldots\Big\} \tag{19}$$

Here all the cancellations of terms singular in mass considered above are taken into account automatically. Note, that just as in the case of using radiation gauge for the massless field, in our case a_0 is not an independent dynamic variable, but is expressed through a_m and σ and their first time derivatives. (However at the canonical description in Proca formalism b_0 is also expressed through the momentum conjugated to b_m. But in contrast to our formalism this connection does not allow the transition to zero mass). The isolation of definite helicity used by us depends on the coordinate frame. But it becomes covariant in the zero mass limit.

Though in the perturbation theory the limiting transition to $\mu = 0$ is absent, it is allowed in the complete Lagrangian(17) including (18a, b). In this limit the interaction of σ-field with the other fields vanishes and a_μ operator describes the massless Yang-Mills field in radiation gauge. Thus it seems to be quite natural that in the zero mass limit the amplitudes of the processes with the helicity ± 1 quanta and fermions turn into corresponding amplitudes of the massless theory which is known to be renormalizable. Since in the perturbation theory for the Yang-Mills field with mass the divergences form the series in $g^2\Lambda^2/\mu^2$ (Λ is the cut-off parameter) then the existence of the continuous limiting transition to $\mu = 0$ means that while summing these series the power divergences disappear and the correct theory is renormalizable at $\mu \neq 0$ also. Note, that the obtained Lagrangian satisfies the renormalizability criteria beyond the frame of perturbation theory which were suggested in /26, 27/ (see also /28/ where these criteria are applied to the massive Yang-Mills field theory in Boulware formalism /22/).

Emphasize that the requirement of continuity in mass at $\mu = 0$ is natural from physical point of view if the considered theories have sense. Indeed it seems strange that say the process of fermion annihilation into the massless particles may be distinguished physically from the analogous process with the massive particles with Compton length essentially exceeding the size of the experimental set.

If zero mass limit exists then at high energies (apparently at $E \gg \frac{\mu}{g}$) the amplitudes of the processes without the longitudinal quanta coincide with the corresponding amplitudes of the massless theory and the quanta of zero helicity are not produced.

Now let us construct the canonical formalism for Lagrangian (I7). The calculation is given for the case of algebraic parametrization (I8b) where it is more simple. The canonical momenta for a_μ and σ fields are

$$p_0 = \frac{\partial \mathcal{L}}{\partial(\partial_0 a_0)} = 0 \tag{20}$$

$$p_m = \frac{\partial \mathcal{L}}{\partial(\partial_0 a_m)} = (\delta_{mn} - \Delta^{-1}\partial_m \partial_n) f_{0n} \tag{21}$$

$$p = \frac{\partial \mathcal{L}}{\partial(\partial_0 \sigma)} = \frac{\partial_0 \sigma}{(1+\frac{g^2}{4\mu^2}\sigma^2)^2} + \mu \frac{1}{1-\frac{ig}{2\mu}\sigma} a_0 \frac{1}{1+\frac{ig}{2\mu}\sigma} \tag{22}$$

The vanishing of the canonical momentum for a_0 field means that this field is not an independent variable and is expressed through other dynamical variables. To obtain this expression we use the time component of Lagrangian equations of motion

$$\partial_\mu f_{\mu\nu} + \mu^2 a_\nu + \mu \Sigma_\nu - ig[a_\mu, f_{\mu\nu}] + j_\nu = 0 \tag{23}$$

Passing to the canonical variables we obtain the equation for a_0

$$D a_0 = \Delta a_0 - 2ig[a_m, \partial_m a_0] - g^2[a_m, \partial_m \Delta^{-1}[a_n, \partial_n a_0]] =$$
$$= j_0 - ig[a_m, p_m] + \mu\left(1 - \frac{ig}{2\mu}\sigma\right) p \left(1 + \frac{ig}{2\mu}\sigma\right) \tag{24}$$

The D operator may be written as

$$D = \Delta\left(1 - ig\Delta^{-1}[a_m, \partial_m \ldots]\right)^2 \tag{25}$$

By general rules we obtain the following expression for Hamiltonian

$$H = H_0(\psi) + \frac{1}{2}\mathrm{Sp}\Big\{\frac{1}{2}p_m p_m + \frac{1}{2}\partial_n a_m \partial_n a_m + \frac{1}{2}\mu^2 a_m a_m -$$
$$- ig a_m [a_n, \partial_m a_n] - \frac{g^2}{4}[a_n, a_m][a_n, a_m] + a_m j_m +$$
$$+ \mu a_m \frac{1}{1 + \frac{ig}{2\mu}\sigma}\partial_m \sigma \frac{1}{1 - \frac{ig}{2\mu}\sigma} - \frac{1}{2}a_0 D a_0 +$$
$$+ \frac{1}{2}p^2\left(1 + \frac{g^2}{4\mu^2}\sigma^2\right)^2 + \frac{1}{2}\frac{\partial_n\sigma\,\partial_n\sigma}{\left(1 + \frac{g^2}{4\mu^2}\sigma^2\right)^2}\Big\} \tag{26}$$

where a_0 is determined from (24)

$$a_0 = D^{-1}\left\{j_0 - ig[a_m, p_m] + \mu\left(1 - \frac{ig}{2\mu}\sigma\right)p\left(1 + \frac{ig}{2\mu}\sigma\right)\right\} \tag{27}$$

For quantization of the fields we set the canonical commutation relations

$$\left[a_m^\alpha(\bar{x}, t), p_n^\beta(\bar{y}, t)\right] = i\delta^{\alpha\beta}\left(\delta_{mn} - \Delta^{-1}\partial_m\partial_n\right)\delta(\bar{x} - \bar{y}) \tag{28}$$

$$[\sigma^{\alpha}(\bar{x},t), p^{\beta}(\bar{y},t)] = i\delta^{\alpha\beta}\delta(\bar{x}-\bar{y}) \tag{29}$$

The indices α, β (α, β = I,2,3) are isotopic ones so that $a_m = \bar{\tau}\bar{a}_m = \tau^{\alpha} a_m^{\alpha}$.

To finish the construction of canonical formalism note that the expression (26) for Hamiltonian must be symmetrized in Bose-operators and antisymmetrized in Fermi-operators.

Before going to the perturbation theory we give the expression for Hamiltonian in the case of exponential parametrization (see exp.(I8a) for $\sum_{\mu}$). Just for a change we use now the usual isovector notation, not matrix one

$$\begin{aligned} H = {} & H_0(\psi) + \frac{1}{2}\bar{p}_m\bar{p}_m + \frac{1}{2}\partial_n\bar{a}_m\partial_n\bar{a}_m + \frac{1}{2}\mu^2\bar{a}_m\bar{a}_m + \\ & + 2g\bar{a}_m(\bar{a}_n\times\partial_m\bar{a}_n) + g^2(\bar{a}_n\times\bar{a}_m)(\bar{a}_n\times\bar{a}_m) + \bar{a}_m\bar{j}_m + \\ & + \bar{a}_m\Big[g f_1\bar{\sigma}\times\partial_m\bar{\sigma} + \frac{g^2}{\mu}f_2\bar{\sigma}\times(\bar{\sigma}\times\partial_m\bar{\sigma})\Big] - \frac{1}{2}\bar{a}_0 D\bar{a}_0 + \\ & + \frac{1}{2}\bar{p}^2 + \frac{1}{2}\partial_n\bar{\sigma}\partial_n\bar{\sigma} + \frac{1-f_1}{2f_1\sigma^2}(\bar{\sigma}\times\bar{p})^2 - \\ & - \frac{1-f_1}{2\sigma^2}(\bar{\sigma}\times\partial_m\bar{\sigma})(\bar{\sigma}\times\partial_m\bar{\sigma}) \end{aligned} \tag{30}$$

Here

$$f_1(\xi^2) = \frac{1-\cos 2\xi}{2\xi^2}, \quad f_2(\xi^2) = \frac{1-\frac{\sin 2\xi}{2\xi}}{\xi^2}, \quad \xi^2 = \frac{g^2\sigma^2}{\mu^2} \tag{3I}$$

and a_0 is determined by

$$\begin{aligned} D\bar{a}_0 = {} & \Delta(1+2g\Delta^{-1}\bar{a}_n\times\partial_n)^2\bar{a}_0 = \bar{j}_0 + 2g\bar{a}_m\times\bar{p}_m + \\ & + \mu\bar{p} + g\bar{\sigma}\times\bar{p} + \frac{1}{f_1}\Big(\frac{g^2}{\mu}f_2 - \mu\frac{1-f_1}{\sigma^2}\Big)\bar{\sigma}\times(\bar{\sigma}\times\bar{p}) \end{aligned} \tag{32}$$

The Hamiltonians (26) and (30) lead of course to identical amplitudes on the mass-shell and the choice of (26) and (30) is determined from convenience reasons.

Let us go to interaction representation. In zero order in the coupling constant we obtain from eqs.(2I, 22, 27)

$$p_m^\alpha = \partial_0 a_m^\alpha , \quad p^\alpha = \left(1 - \frac{\mu^2}{\Delta}\right)^{-1} \partial_0 \sigma^\alpha \tag{33}$$

Using free equations of motion and expressions (33) we obtain Green functions of free fields

$$\mathcal{D}_{mn}^{\alpha\beta}(k) = i\int dx e^{ikx} \langle T a_m^\alpha(x) a_n^\beta(0)\rangle_0 = \delta^{\alpha\beta} \frac{\delta_{mn} - \frac{k_m k_n}{k^2}}{\mu^2 - k^2} \tag{34}$$

$$\mathcal{D}^{\alpha\beta}(k) = i\int dx e^{ikx} \langle T \sigma^\alpha(x) \sigma^\beta(0)\rangle_0 = \delta^{\alpha\beta} \frac{1 + \frac{\mu^2}{k^2}}{\mu^2 - k^2} \tag{35}$$

The σ^α field may be substituted for by the new field $\tilde{\sigma}^\alpha = \left(1 - \frac{\mu^2}{\Delta}\right)^{-1/2} \sigma^\alpha$ Green function of which has the usual form.

To obtain the interaction Hamiltonian in interaction representation we are to omit in expressions (26), (30) the terms of the zero order in the coupling constant and replace p_m and p by $\partial_0 a_m$ and $(I - \mu^2 \Delta^{-1})^{-I} \partial_0 \sigma$.

For illustration we present the graphs describing singular in μ parts of the amplitudes of processes $p\bar{n} \to \sigma^- \sigma^+ \sigma^+$ and $p\bar{n} \to \sigma^- \sigma^+ \sigma^+ \sigma^0$ (see Fig.3 and 4 where dotted lines correspond to σ-particles).

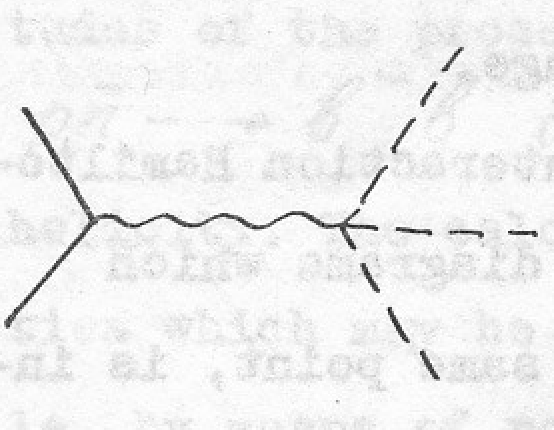

Fig.3

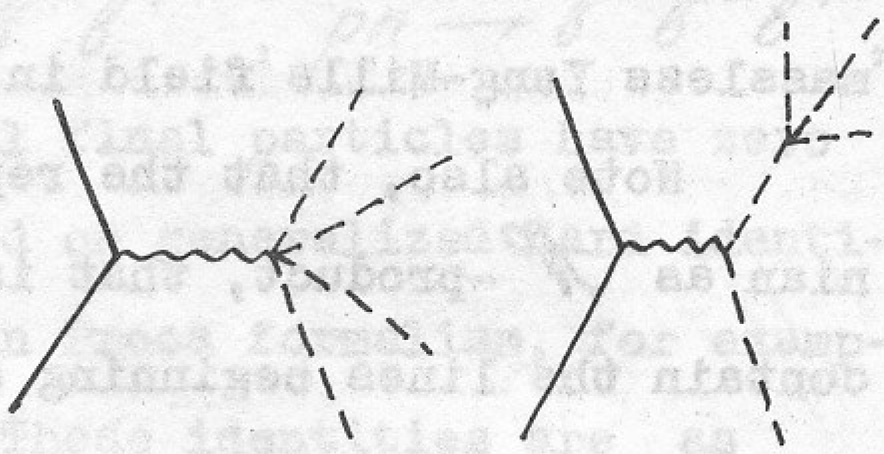

Fig.4

The simple calculations lead to results corresponding in those in the Appendix.

Point out some peculiarities of the perturbation theory. The interaction Hamiltonian is not a polynom in fields and the number of primary vertices is infinite. This fact is due to non-polynomiality of the part of Hamiltonian which contains σ-field as well as to expansion in fields /I7-20/ of operator D^{-1} arising in Hamiltonian after the substitution of a_0.

The noncommutativity of operations of time differentiation and T-ordering leads to the so-called with normals terms. As it is known these terms may be omitted if simultaneously one replaces interaction Hamiltonian by interaction Lagrangian with the opposite sign. We have checked that this statement takes place for self-interaction of σ-field. Besides, if the terms with the normals are not taken into account, then σ-field enters the right-hand side of the eqs.(27), (32) determining a_0 in Σ_0 combination only. Taking into account this remark the most singular in μ terms become covariant in every order of perturbation theory.

After the mentioned modification the Hamiltonian has zero mass limit (in contrast to the original form (26), (30)) and coincides in this limit with the Hamiltonian /I8-20/ of the massless Yang-Mills field in the radiation gauge.

Note also, that the representation of interaction Hamiltonian as $\mathcal{N}$-product, that is the omission of diagrams which contain the lines beginning and ending at the same point, is incorrect as it leads to the contradiction with the current conservation /I9/.

But it may be shown that the summing of all mentioned diagrams with virtual σ-particles leads to the function of the type $\exp(-g^2\Lambda^2/\mu^2)$ (at exponential parametrization) that is to vanishing quantity. From our point of view this circumstance is one more argument supporting the suggestion of the continuity in mass at $\mu = 0$ and renormalizability out of the frame of perturbation theory.

In conclusion the authors sincerely thank B.L.Ioffe and V.V.Sokolov for the invariable interest to the work and discussions.

A P P E N D I X

Here we shall calculate in the limit $\mu \to 0$ the amplitudes of the processes $p\bar{p} \to b^- b^+$, $p\bar{n} \to b^- b^+ b^+$, $p\bar{n} \to b^- b^+ b^+ b^0$ where all final particles have zero helicity. The calculations are based on generalized Ward identities which may be obtained easily in Proca formalism, for example, by means of reduction formula. These identities are as follows

$$k_{i\mu_i} \mathcal{M}^{\alpha_1 \dots \alpha_i \dots \alpha_j \dots \alpha_n}_{\mu_1 \dots \mu_i \dots \mu_j \dots \mu_n} (k_1, \dots, k_i, \dots, k_j, \dots k_n) =$$
$$= 2g \sum_{j \neq i} \varepsilon^{\alpha_i \alpha_j \beta_j} \mathcal{D}^{-1}_{\mu_j \nu_j}(k_j) \mathcal{D}_{\nu_j \lambda_j}(k_i + k_j) \times$$
$$\times \mathcal{M}^{\alpha_1 \dots \beta_j \dots \alpha_n}_{\mu_1 \dots \lambda_j \dots \mu_n} (k_1, \dots, k_i + k_j, \dots, k_n) \tag{A.I}$$

where tensor $\mathcal{M}^{\alpha_1 \dots \alpha_i \dots \alpha_j \dots \alpha_n}_{\mu_1 \dots \mu_i \dots \mu_j \dots \mu_n}(k_1, \dots, k_i, \dots, k_j, \dots, k_n)$ determines the process with n vector quanta (not necessary physical). Index α_i shows the charge state of the i-th particle with momentum k_i. The tensor in the right-hand side corresponds to the amplitude of the process with n-I quanta which is obtained out of original one by means of substituting of one particle with momentum $k_i + k_j$ for particles i, j. All the momenta are considered as outgoing. Propagator $\mathcal{D}_{\mu\nu}(k)$ is determined by eq.(3); the tensor $\mathcal{D}^{-1}_{\mu\nu}(k)$ which is inverse to it is

$$\mathcal{D}^{-1}_{\mu\nu}(k) = g_{\mu\nu}(k^2 - \mu^2) - k_\mu k_\nu \tag{A.2}$$

The amplitude of the process in which the Ist and the 2nd quanta are longitudinal is expressed by eq.(8). Let for definiteness the first quantum have negative charge and the second one have positive charge. By means of Ward identity (A.I) one may transform the term in $\mathcal{M}$ containing $k_{2\mu_2}$

$$\mathcal{M} = 2g \frac{\omega_1 \omega_2}{|k_1||k_2|} k_{1\nu} \mathcal{D}_{\nu\lambda}(k_1+k_2) \mathcal{M}_{\lambda\mu_3\ldots\mu_n} \varepsilon_{\mu_3}\ldots\varepsilon_{\mu_n} +$$

$$+ \frac{\mu^2}{|k_1||k_2|} n_{\mu_1} n_{\mu_2} \mathcal{M}_{\mu_1\mu_2\mu_3\ldots\mu_n} \varepsilon_{\mu_3}\ldots\varepsilon_{\mu_n} \qquad \text{(A.3)}$$

We have taken into account that $\mathcal{D}^{-1}_{\mu_2\nu}(k_2) = -k_{2\mu_2} k_{2\nu}$ since $k_2^2 = \mu^2$.

In the case of $p\bar{p} \to b^- b^+$ process the general eq.(A.3) leads in $\mu = 0$ limit to matrix element

$$\mathcal{M}^{-+} = g^2 \frac{1}{(k_1+k_2)^2} (k_1 - k_2)_\lambda \bar{u}_2 \gamma_\lambda u_1 \qquad \text{(A.4)}$$

The second term in (A.3) does not contribute to $\mathcal{M}^{-+}$ at $\mu = 0$ as $\mathcal{M}^{-+}_{\mu_1\mu_2}$ tensor describing the interaction of two vector quanta and fermions is non-singular in mass. The "tail" of vector propagator (see diagram b, Fig.2) may be dropped out because of the transversality of the nuclon vertex.

Let now in eq.(A.3) the polarization vector of the third quantum (for definiteness, positively charged one) be three-dimensionally longitudinal. Again using Ward identities for the transformation of the terms which are proportional to $k_{3\mu_3}/\mu$, we get

$$\mathcal{M} = -4g^2 \frac{1}{\mu} \frac{\omega_1\omega_2\omega_3}{|\bar{k}_1||\bar{k}_2||\bar{k}_3|} k_{1\nu} \mathcal{D}_{\nu\lambda}(k_1+k_2+k_3) \mathcal{M}_{\lambda\mu_4\ldots\mu_n} \varepsilon_{\mu_4}\ldots\varepsilon_{\mu_n} -$$
$$-2g \frac{\mu\,\omega_1\omega_2}{|\bar{k}_1||\bar{k}_2||\bar{k}_3|} \eta_{\mu_3} k_{1\nu} \mathcal{D}_{\nu\lambda}(k_1+k_2) \mathcal{M}_{\lambda\mu_3\mu_4\ldots\mu_n} \varepsilon_{\mu_4}\ldots\varepsilon_{\mu_n} -$$
$$-2g \frac{\mu\,\omega_1\omega_3}{|\bar{k}_1||\bar{k}_2||\bar{k}_3|} \eta_{\mu_2} k_{1\nu} \mathcal{D}_{\nu\lambda}(k_1+k_3) \mathcal{M}_{\lambda\mu_2\mu_4\ldots\mu_n} \varepsilon_{\mu_4}\ldots\varepsilon_{\mu_n} -$$
$$-\frac{\mu^3}{|\bar{k}_1||\bar{k}_2||\bar{k}_3|} \eta_{\mu_1}\eta_{\mu_2}\eta_{\mu_3} \mathcal{M}_{\mu_1\mu_2\mu_3\mu_4\ldots\mu_n} \varepsilon_{\mu_4}\ldots\varepsilon_{\mu_n} \qquad \text{(A.5)}$$

Let us apply this expression to $p\bar{n} \to b^- b^+ b^+$ process. Taking into account the transversality of the nucleon vertex, it is not difficult to show that the functions $\mathcal{D}_{\nu\lambda}(k_1+k_2+k_3)\mathcal{M}^+_\lambda(k_1+k_2+k_3)$, $\mathcal{M}^{o+}_{\lambda\mu_3}(k_1+k_2,k_3)$, $\mathcal{M}^{o+}_{\lambda\mu_2}(k_1+k_3,k_2)$ are regular in μ and the singularity of tensor $\mathcal{M}^{-++}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)$ does not exceed $\frac{1}{\mu^2}$. Therefore the first, the second and the third terms contribute to the singular part of amplitude under consideration, in the last two ones only the "tails" of propagators $\mathcal{D}_{\nu\lambda}(k_1+k_{2(3)})$ being essential. Using Ward identity for the calculation of the contribution of these "tails", we find the final expression for the amplitude of the process $p\bar{n} \to b^- b^+ b^+$ accurate up to terms $\sim\mu$.

$$\mathcal{M}^{-++} = -\frac{2\sqrt{2}g^3}{\mu} \frac{1}{(k_1+k_2+k_3)^2} k_{1\lambda} \bar{u}_2 \gamma_\lambda u_1 + O(\mu) \qquad \text{(A.6)}$$

For computation of the amplitude of the process $p\bar{n} \to b^- b^+ b^+ b^o$ with longitudinal quanta it is necessary to put into eq.(A.5) the explicit expression for $\varepsilon^{(3)}_{\mu_4}(k_4)$ and to use again Ward identities. The final result is

$$\mathcal{M}^{-++0} = \frac{2\sqrt{2}g^4}{\mu^2}\frac{1}{(k_1+k_2+k_3+k_4)^2}\times$$
$$\times\bar{u}_2\Big\{\hat{k}_4 - \frac{(k_2+k_3)^2}{(k_1+k_2+k_3)^2}\hat{k}_4 - \frac{(k_1+k_3)^2}{(k_1+k_3+k_4)^2}\hat{k}_2 -$$
$$- \frac{(k_1+k_2)^2}{(k_1+k_2+k_4)^2}\hat{k}_3\Big\}u_1 + O(\mu^0)$$

(A.7)

Notice, that at the concrete calculations we operate in the other succession than at the general proof of singularity of amplitudes.

R E F E R E N C E S

I. Sh.L.Glashow. Nucl.Phys., IO, IO7, I959.

2. A.Komar, A.Salam. Nucl.Phys., 2I, 624, I960.

3. H.Umezawa, S.Kamefuchi. Nucl.Phys., 23, 399, I96I.

4. S.Kamefuchi, L.O'Raifeartaigh, A.Salam. Nucl.Phys., 28,529, I96I.

5. P.A.Ionides. Nucl.Phys., 28, 662, I96I.

6. A.Salam. Phys.Rev., I27, 33I, I962.

7. M.Veltman. Nucl.Phys., B7, 637, I968.

8. A.A.Slavnov. Preprint ITP-69-20, Kiev, I969.

9. E.S.Fradkin, I.V.Tyutin. Phys.Lett., 30B, 562, I969.

IO. R.P.Feynman. Acta Physica Polonica, 24, 697, I963.

II. B.S.DeWitt. Phys.Rev., I62, II95, I967.

I2. L.D.Faddeev, V.N.Popov. Phys.Lett., 25B, 30, I967.

I3. V.N.Popov, L.D.Faddeev. Preprint ITP, Kiev, I967.

I4. S.Mandelstam. Phys.Rev., I75, I580, I968.

I5. J.Reiff, M.Veltman. Nucl.Phys., BI3, 545, I969.

I6. R.Kallosh. Preprint N I8, Lebedev Phys.Inst., Moscow,I970.

I7. J.Schwinger. Phys.Rev., I25, IO43, I962.

I8. J.Schwinger. Phys.Rev., I27, 324, I962.

I9. I.B.Khriplovich. Yadernaya Fizika, IO, 409, I969.

20. A.M.Altukhov, I.B.Khriplovich, Yadernaya Fizika, II,902,I970.

2I. D.G.Boulware, W.Gilbert. Phys.Rev., I26,I563,I962.

22. D.G.Boulware. Annals of Phys., 56,I40,I970.

23. A.A.Slavnov, L.D.Faddeev. Teoreticheskaya i Matematicheskaya Fizika, 3,I8,I970.

24. E.S.Fradkin, I.V.Tyutin. Preprint N 27, Lebedev Phys.Inst., Moscow,I970.

25. S.Weinberg. Phys.Rev.Lett., I8,I88,I967.

26. G.V.Efimov. Zh.Eksp.i Teor.Fiziki, 44,2I07,I963.

27. E.S.Fradkin. Nucl.Phys., 49,624,I963.

28. A.Salam. Preprint IC/70/7, Trieste,I970.

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