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Abstract

Low energy theorems are obtained for cross-sections of pion electroproduction on nucleons in the limit of zero relative momentum of pion and final nucleon and for arbitrary values of the square of virtual photon momentum κ^2, and also for reaction $\pi N \to N e^+ e^-$ for the case of slow pions. When deriving the low energy theorems we use the PCAC hypothesis and expansion of the amplitude in those 4-momenta which are much small than some internal mass. As compared with the known calculations the subsequent terms of expansion are taken into account.

The cross-sections are expressed via nucleon electromagnetic and axial formfactors, radius of the pion electromagnetic formfactor and threshold photoproduction cross-section. The accuracy of results which depends on a range of κ^2 considered and the possibility of determination of the pion radius from experimental data are discussed.

1. Introduction

In this paper we shall derive low-energy theorems for the threshold amplitude of electroproduction of a pion on a nucleon. The derivation of the theorems is based on two main assumptions. The first one is the hypothesis of a partial conservation of axial current and the second assumption is the conjectured smallness of the pion mass μ as compared with some internal mass m_{int}.

The theorems allow to calculate the cross section of electropion production in the limit of vanishing relative 3-momentum of final nucleon and pion. This kinematic constrain does not fix the magnitude of the electron momentum transfer squared, $k^2 = -t$, and t ranges from zero to infinity. The accuracy of the results obtained depends however crucially on t. For this reason we shall distinguish three possible ranges of variation of t, namely, $t \gtrsim m^2_{int}$, $\mu^2 \ll t \ll m^2_{int}$ and $t \lesssim \mu^2$. The internal mass m_{int} is not known exactly, of course, and its magnitude depends on a particular mechanism of the reaction. For practical purposes m^2_{int} may be seemingly thought to be of order (0.3-1.0) $(\mathrm{GeV})^2$.

The cross section of neutral pion production at any t coincides with that calculated in Born approximation with pseudovector πNN-coupling and is ralated in this way to the well known electromagnetic formfactors of nucleon. The order of neglected terms are estimated for different ranges of t.

The cross section of charged pion production for large t, $t \gtrsim m_{int}$ apart from electromagnetic form factors depends also on the axial-vector form factor of nucleon, $g(k^2)$. This dependence allows for determination of $g(k^2)$ through the measurement of electroproduction cross section. At this region of variable t our results coincide in essential with the results of a recent paper /1/ which came to our attention after completion of the present work. The accuracy of results obtained for large t is of order of μ/m_{int}.

For values of t satisfying inequalities $\mu^2 \ll t \ll m^2_{int}$ the accuracy of the predictions of the cross sections of charged pion production improves. The neglected terms are of order of μ^2/m^2_{int} or $\mu t/m^3_{int}$ in the transversal part of the cross section and of order of t^2/m^4_{int} or $\mu t/m^3_{int}$ in the longitudinal part of the cross section. Since the parameter of the expansion of the amplitude, μ/m_{int}, is not very small in fact the account of terms of the next order in μ/m_{int} is quite essential to our mind.

At the region $t \ll m^2_{int}$ two additional parameters enter our final expression for the longitudinal cross section of a charged pion production. One of these parameters is immediately related to the photoproduction cross section and its value may be extracted from the existing experimental data. The longitudinal cross section depends also on the pion electromagnetic radius. Therefore a measurement of the longitudinal cross section would allow for determination of pion radius. It should be kept in mind however that for $t \gg \mu^2$ the longitudinal part of cross section is predicted to be small as compared with the transversal one. Their ratio is of order of μ^2/t and becomes of order of unity only for $t \sim \mu^2$. At such values of t it is sufficient to measure the total cross section to determine the pion radius.

For values of t comparable with μ^2 it is possible to take into account the terms of order of μ^2/m^2_{int}. Therefore, an expected violation of the relations obtained for $t \sim \mu^2$ does not exceed several per cents. Due to the high accuracy of theoretical predictions their comparison with corresponding experimental data could serve as a crucial test of the basic assumptions involved in the derivation of the low-energy theorems.

Our results at $t \lesssim \mu^2$ are also applied to the calculation of the cross section of capture of a pion at rest with production of electron-positron pair

$$\pi^- + p \rightarrow n + e^+ + e^- \qquad (1)$$

It will be shown that the cross section of this reaction is rather sensitive to the magnitude of the pion radius and the latter may be consistently determined through measurement of the cross section of reaction (1).

Let us notice that some experimental efforts have been made to determine the pion radius in this way /2/. The Born approximation with pseudoscalar πNN-coupling /3/ was used however to find the dependence of the cross section on the pion radius. Our result for the cross section of reaction (1) differs from this approximation.

Electroproduction of pions within the PCAC hypothesis has been studied by many authors /4-11/. In particularly we use the predictions for form factors which are obtained in paper /5/. Further more as was already mentioned above our results at $t \gtrsim m^2_{int}$ coincide with the results of a recent paper /1/.

Earlier a similar approach to calculate the electroproduction cross section was used in paper /9/. However, the final result of this work is to our mind erroneous (for further comments see discussion after eq.(15)).

Our main purpose in this paper is to discuss expected accuracy of the theoretical predictions and to clarify a related topic on the possibility of determining the pion radius through measurement of electroproduction cross section or cross section of reaction (1). New predictions for the cross section will be also obtained. For values of variable t satisfying condition $t \ll m_{int}^2$ all the terms of order of μ / m_{int} and larger will be kept. For extremely small t, $t \lesssim \mu^2$, terms of order of μ^2 / m_{int}^2 will be also evaluated. We shall not accept any new assumptions to find corrections of order of μ / m_{int} or μ^2 / m_{int}^2. Therefore the predictions obtained are not based on any particular model and their check could serve as a crucial test of the PCAC hypothesis.

2. Low-energy theorems for threshold amplitudes of electroproduction of a pion on a nucleon.

In this section we shall outline the derivation of low-energy theorems for amplitudes of electroproduction of a "soft" pion with 4-momentum q of order μ. A more detailed description of a similar derivation may be found, e.g., in the paper by Adler and Dothan /8/ or in a review article /12/. If apart from q the momentum of the virtual photon is also considered to be small the consequences from the crossing symmetry of the amplitude will be essentially used. Let us consider first the reaction (2)

$$e + p(p_1) \to e + n(p_2) + \pi^+(q) \tag{2}$$

The amplitude of this reaction M has the following form

$$M = \frac{4\pi\alpha}{k^2}\, \bar{v}_2 \gamma_\lambda v_1 M_\lambda$$

$$M_\lambda = -\langle n\pi^+ | j_\lambda(0) | p\rangle \tag{3}$$

where v_1, v_2 are the spinors describing the initial and final leptons respectively, $j_\lambda(0)$ is the operator of the hadronic electromagnetic current.

It is convenient to represent M_λ as a sum of three terms

$$M_\lambda = M_\lambda^{pole} + \Delta M_\lambda + M_\lambda^{\perp} \tag{4}$$

where M_λ^{pole} is the contribution of the nucleons' and pion's pole graphs, ΔM_λ is defined so that $(k_\lambda(M_\lambda^{pole} + \Delta M_\lambda) = 0$, and $M_\lambda^{\perp}$ is by itself divergenless (k is the momentum of virtual quantum).

For M_λ^{pole} and ΔM_λ we have the following expressions

$$M_\lambda^{pole} = -if\sqrt{2}\, \bar{u}_2 \Big\{ \hat{q}\gamma_5 \frac{1}{\hat{p}_2 + \hat{q} - m} \Gamma_\lambda^p + \Gamma_\lambda^n \frac{1}{\hat{p}_1 - \hat{q} - m} \hat{q}\gamma_5 + (\hat{q} - \hat{k})\gamma_5 \frac{1}{(k-q)^2 - \mu^2} \Gamma_\lambda^\pi \Big\} u_1 \tag{5}$$

$$\Delta M_\lambda = -if\sqrt{2}\, \bar{u}_2 \{ -\gamma_\lambda \gamma_5 - k_\lambda \hat{q}\gamma_5 G_1^V \} u_1 \tag{6}$$

where u_1, u_2 are the spinors describing the initial and final nucleons, f is the πNN pseudovector coupling constant, $f=g_r/2m=1.01\,\mu^{-1}$ and we have introduced notations $\Gamma_\lambda^p, \Gamma_\lambda^n, \Gamma_\lambda^\pi$ for electromagnetic vertices of proton, neutron and pion, respectively

$$\Gamma_\lambda^{p,n} = \gamma_\lambda F_1^{p,n}(k^2) - \frac{1}{2m}\sigma_{\lambda\xi}k_\xi F_2^{p,n}(k^2)$$

$$\Gamma_\lambda^\pi = (2q-k)_\lambda + (g_{\lambda\nu}k^2 - k_\lambda k_\nu)(2q-k)_\nu G^\pi(k^2) \qquad (7)$$

where $F_{1,2}^{p,n}(k^2)$, $G^\pi(k^2)$ are form factors. The quantity which appears in formula (6) is related to $F_1^{p,n}$ by equation

$$G_1^V = \frac{F_1^p - F_1^n - 1}{k^2}$$

Let us notice that the pion electromagnetic vertex is often put in a slightly different form $\tilde{\Gamma}_\lambda^\pi$

$$\tilde{\Gamma}_\lambda^\pi = (2q-k)_\lambda F^\pi(k^2) \qquad (8)$$

Form factor $F^\pi(k^2)$ is easily seen to be equal to $1 + k^2 G^\pi(k^2)$.

The remaining part of the amplitude, M_λ^L, may be expanded in the following form /13/

$$M_\lambda^L = -if\sqrt{2}\sum_{i=1}^{6} V_i\, \bar{u}_2 O_\lambda^i u_1 \qquad (9)$$

where

$$O_\lambda^1 = \gamma_5 \sigma_{\lambda\xi}k_\xi, \qquad \eta_1 = +1$$

$$O_\lambda^2 = \frac{1}{2m}(\hat{q}-\hat{k})\gamma_5[(p_1+p_2)_\lambda(kq) - q_\lambda(k,p_1+p_2)], \qquad \eta_2 = +1$$

$$O_\lambda^3 = \gamma_5[\gamma_\lambda(qk) - q_\lambda \hat{k}], \qquad \eta_3 = -1$$

$$O_\lambda^4 = -i\epsilon_{\lambda\gamma\rho\sigma}\gamma_\gamma k_\rho q_\sigma, \qquad \eta_4 = +1$$

$$O_\lambda^5 = \frac{1}{2m}(\hat{q} - \hat{k})\gamma_5[k_\lambda(kq) - q_\lambda k^2], \qquad \eta_5 = -1$$

$$O_\lambda^6 = \gamma_5[k_\lambda \hat{k} - \gamma_\lambda k^2], \qquad \eta_6 = -1$$

and form factors V_i are the functions of $\nu = (k+q, p_1+p_2)/4m$, k^2 and kq. Numbers η_i indicate the parity of form factors $V_i^{\pi^0 p}$, $V_i^{\pi^0 n}$ and $(V_i^{\pi^+ n} + V_i^{\pi^- p})$ under the crossing transformation $\nu \to -\nu$, $kq \to kq$, $k^2 \to k^2$, while the parity of form factor $(V_i^{\pi^+ n} - V_i^{\pi^- p})$ is equal to $(-\eta_i)$.

Up to now we have exploited only the transversality condition $k_\lambda M_\lambda = 0$. Conservation of axial current in the limit of $\mu = 0$ gives the following relation

$$M_\lambda\Big|_{q=0,\mu=0} = ic\, g(k^2)\, \bar{u}_2\left[\gamma_\lambda\gamma_5 - \frac{1}{k^2}k_\lambda \hat{k}\gamma_5\right]u_1 \qquad (10)$$

where $g(k^2)$ is axial form factor of nucleon and constant c is equal to $f\sqrt{2}/g_A$, $g_A = g(k^2=0)$. Eq.(10) results in the following constraints on form factors $V_{1,6}$

$$V_1\Big|_{\nu=kq=\mu^2=0} = 0,$$

$$V_6\Big|_{\nu=kq=\mu^2=0} = -\frac{(g(k^2)/g_A) - 1}{k^2} \qquad (11)$$

so that for the matrix element M_λ we get finally

$$M_\lambda = -if\sqrt{2}\,\bar{u}_2\Big\{\hat{q}\gamma_5\frac{1}{\hat{p}_2+\hat{q}-m}\Gamma_\lambda^p + \Gamma_\lambda^n\frac{1}{\hat{p}_1-\hat{q}-m}\hat{q}\gamma_5 -$$

$$-\kappa_\lambda\hat{q}\gamma_5\frac{F_1^p-F_1^n-1}{\kappa^2} + (\hat{q}-\hat{\kappa})\gamma_5\frac{1}{(\kappa-q)^2-\mu^2}\Gamma_\lambda^\pi -$$

$$-\gamma_\lambda\gamma_5 - \frac{g(\kappa^2)/g_A - 1}{\kappa^2}(\kappa^2\gamma_\lambda\gamma_5 - \kappa_\lambda\hat{\kappa}\gamma_5) +$$

$$+O_\lambda^1(\mu^2 a_1 + \nu b_1 + \kappa q c_1) + O_\lambda^6(\mu^2 a_6 + \nu b_6 + \kappa q c_6) +$$

$$+\sum_{i=2}^{5} O_\lambda^i a_i\Big\}u_1 \qquad (12)$$

where a_i, $b_{1,6}$, $c_{1,6}$ are the functions of ν, kq, t. Factors $(\mu^2 a_{1,6} + \nu b_{1,6} + \kappa q c_{1,6})$ arise because of the condition of vanishing at q=0 of form factor V_I and of the sum $V_6 + g_A^{-I}\kappa^{-2}(g(k^2) - g_A)$.

Formula (12) is a basic one to calculate the cross section. The r.h.s. of eq.(12) contains some unknown terms proportional to O_λ^i. Definite predictions for the cross section arise if these terms are neglected. The foundation for such neglegence is the appearance of factors q or μ^2 which result in the suppression of the corresponding contributions to the cross section by factor μ/m_{int} or μ^2/m_{int}^2. Up to the very end of the calculation we shall keep all the contributions to the amplitude to make explicit dependence of the neglected terms on variable t. As was already mentioned in the introduction at different ranges of variable t it proves to be possible to keep terms of different order in μ/m_{int}.

It is worth noticing that proportionality of some term to μ may not lead to suppression of its contribution to the cross section if the internal mass is itself of order μ. For this reason one should consider separately the contribution of isobar N*(1236). When calculating the threshold cross section this contribution may be disregarded however. Indeed, the s-channel resonant contribution isobar N*(1236) goes to zero for q 0 because of the p-wave nature of the resonance. As for the contribution of the graph with N*(1236) in the u-channel it does not vanish at q = 0 but turns out to be extremely small numerically. Therefore the contribution of N*(1236) may be neglected and hereafter we accept that factor μ/m_{int} results in a real smallness of the corresponding term.

Let us formulate now the results for amplitudes of other possible reactions of electroproduction of pions. To obtain the matrix element of the reaction $en \to ep\pi^-$ one should make in eq.(12) the following replacements: $F^p_{I,2} \to (-F^n_{I,2})$, $F^n_{I,2} \to (-F^p_{I,2})$, $\nu \to -\nu$, $O^i_\lambda \to -\eta_i O^i_\lambda$; the total sign of the amplitude should be also changed.

For the matrix element of π^0-meson production on proton one gets the following expression

$$M_\lambda(ep \to ep\pi^0) = -if\bar{u}_2\Big[\hat{q}\gamma_5 \frac{1}{\hat{p}_2+\hat{q}-m}\Gamma^p_\lambda + \Gamma^p_\lambda \frac{1}{\hat{p}_2-\hat{q}-m}\hat{q}\gamma_5 + O^1_\lambda(\mu^2\tilde{a}_1 + \kappa q\tilde{c}_1) + O^2_\lambda\tilde{a}_2 + O^3_\lambda \nu\tilde{b}_3 + O^4_\lambda\tilde{a}_4 + O^5_\lambda\nu\tilde{b}_5 + O^6_\lambda\nu\tilde{b}_6\Big]u_1 \qquad (13)$$

where form factors $\tilde{a}_i$, $\tilde{b}_i$, $\tilde{c}_i$ are the functions of ν, κq, t and eq.(13) satisfies the requirements of the crossing symmetry of the amplitude. Proportionality of some terms to ν plays an important rôle at small values of k.

The amplitude of π^0-meson production on neutron may be obtained from eq.(13) by replacing Γ_λ^p by Γ_λ^n and introducing some new independent functions $\bar{a}_i$, $\bar{b}_i$, $\bar{c}_i$. As for the structure of terms proportional to it remains the same as in eq.(13).

If the terms proportional to $\mathcal{O}_\lambda^i$ are neglected as in the case when calculating the cross section then the amplitude of π^0-meson production coincides with the Born approximation with PS-PV πNN-coupling. The amplitude of charged pion production differs from the corresponding Born term.

3. <u>Threshold cross-section of a charged pion production</u>.

The cross-section may be represented as usually as a sum of two terms corresponding to interaction of transversally and longitudinally polarized quanta

$$\frac{t}{|\vec{q}|}\frac{d\sigma}{dO\,d\epsilon_2}(eN\to e'N'\pi)=\frac{\alpha^2}{32\pi^2}\frac{\epsilon_2}{\epsilon_1}\frac{1}{m\sqrt{m^2+2m(\epsilon_2-\epsilon_1)-t}}$$

$$\cdot\left[|M_T|^2 f_1(\epsilon_1,\epsilon_2,\theta)+|M_L|^2 f_2(\epsilon_1,\epsilon_2,\theta)\right],$$

$$f_1=\frac{4\epsilon_1\epsilon_2\cos^2\frac{\theta}{2}}{(\epsilon_1-\epsilon_2)^2+t}+2,\quad f_2=\frac{4\epsilon_1\epsilon_2\cos^2\frac{\theta}{2}}{(\epsilon_1-\epsilon_2)^2+t}\cdot\frac{2t}{(\epsilon_1-\epsilon_2)^2} \tag{14}$$

where ϵ_1, ϵ_2, θ are the energies of the initial and final electrons and the scattering angle measured in the lab.system, $\vec{q}$ is the pion's 3-momentum in the c.m. system of final hadrons, $t=-k^2=4\epsilon_1\epsilon_2\sin^2\frac{\theta}{2}$

$$|M_T|^2=|M_X|^2+|M_Y|^2 \tag{15}$$

$$|M_L|^2=\frac{(\epsilon_1-\epsilon_2)^2}{k_0^2}|M_z|^2=\frac{(\epsilon_1-\epsilon_2)^2}{\vec{k}^2}|M_0|^2\equiv(\epsilon_1-\epsilon_2)^2|U_L|^2 \qquad (16)$$

the z-axis being directed along the momentum of the virtual photon and $M_{0,x,y,z}$ are the components of M_λ in the c.m. system of final hadrons.

We have introduced in eq.(16) the quantity U_L instead of M_z for the following reason. The zeroth component of vector k at threshold is equal to

$$k_0=\mu-\frac{\mu^2+t}{2(m+\mu)} \qquad (17)$$

and goes to zero at $t_0=0{,}28$ $(\mathrm{GeV/c})^2$. Simultaneously goes to zero the matrix element M_z so that the longitudinal matrix element M_L which is proportional to M_z/k_0 remains a smooth function in the vicinity of $t=0.28$ $(\mathrm{GeV/c})^2$. But if the longitudinal matrix element is expressed in terms of M_z the latter should be calculated with a special care to avoid a mistake. In paper /9/ the longitudinal cross section was expressed in terms of M_z. The pion mass μ was neglected however when calculating M_z but it was not neglected in the calculation of k_0. For this reason the final results of this work are incorrect to our mind even by order of magnitude for values of t close to 0.3 $(\mathrm{GeV/c})^2$. There are some other inconsistencies between our results and those of paper /9/ but we shall not discuss this point here.

Let us present now the final results for $|M_T|^2$.

$$|M_T|^2 = 16m^2 f^2\left(1+\frac{t}{4m^2}\right)\left\{\frac{g(-t)}{g_A} + G_{Mn}(-t)\frac{t/2m^2}{1+t/2m^2} - \frac{\mu}{2m} + O\left(\frac{\mu t}{m_{int}(m_{int}^2+t)}, \frac{\mu^2}{m_{int}^2}\right)\right\}^2 \tag{18}$$

where G_{Mn} is the Sachs magnetic form factor of neutron and $O\left(\frac{\mu t}{m_{int}(m_{int}^2+t)}, \frac{\mu^2}{m_{int}^2}\right)$ indicates the order of magnitude of the neglected contributions.

Eq.(18) is valid for all possible values of t. It is seen that for $t \gtrsim m_{int}^2$ the accuracy of the calculation of the cross section is of order μ/m_{int}. For such values of t the term $-\frac{\mu}{2m}$ is of order of the neglected contributions and should be omitted for this reason.

For smaller values of t, $t \ll m_{int}^2$, the accuracy of the calculation of $|M_T|^2$ improves and terms of order μ/m may be consistently kept. For extremely small t, $t \leq \mu^2$, it turns out to be possible to take into account terms of order μ^2/m_{int}^2. All the terms of this order undetermined theoretically may be expressed phenomenologically through the photoproduction cross section. Omitting the details of the derivation we give only the final result for $t \leq \mu^2$

$$|M_T|^2 = |M_\gamma|^2\left[1 - 2t\left(\frac{g'(0)}{g_A} - \frac{\varkappa^n}{2m^2} - \frac{1}{8m^2}\right)\right]\left(1 + O\left(\frac{\mu^3}{m_{int}^3}\right)\right) \tag{19}$$

where $\varkappa^n$ is the anomalous magnetic moment of neutron and

$$\lim_{\vec{q}\to 0}\frac{k}{|\vec{q}|}\frac{d\sigma}{dO}(\gamma p\to n\pi^+)=\frac{\alpha}{32\pi}\frac{1}{(m+\mu)^2}|M_\gamma|^2$$

$$g'(0)\equiv\left.\frac{d\,g(k^2)}{dk^2}\right|_{k^2=0}$$

Expected accuracy of eq.(19) is not worse than several percents.

The cross section of interaction of longitudinally polarized quanta is determined by $|U_L|^2$ where for $|U_L|^2$ we have

$$|U_L|^2=8m^2f^2\left(1+\frac{\mu^2+t}{4m^2}\right)\Big\{-\frac{G_{En}}{m(1+t/2m^2)}+$$
$$+\frac{\mu(m+\mu)}{\mu^2(2m+\mu)+mt}\left[1-\frac{\mu}{m}-(\mu^2+t)\left(2F'_\pi(0)+\frac{1}{4m^2}\right)\right]+$$
$$+\mu\left(\frac{g'(0)}{g_A}+\gamma\right)+\frac{\mu}{4m^2}(1+x^p+x^n)+O\left(\frac{\mu t}{m^2_{int}(m^2_{int}+t)},\frac{\mu^3}{m^4_{int}}\right)\Big\}^2 \qquad (20)$$

where G_{En} is the Sachs electric form factor of neutron, $F'_\pi(0)$ is equal to $dF^\pi(k^2)/dk^2$ at $k^2=0$ and parameter γ is related to the photoproduction matrix element in the following way

$$|M_\gamma|^2=16m^2f^2\left[1-\frac{\mu}{m}+\frac{\mu^2}{2m^2}(2+x^p+x^n)+2\mu^2\gamma\right] \qquad (21)$$

From the existing experimental data /15/ it may be deduced that γ is rather small, $\mu^2\gamma=(0.01\pm 0.01)$.

It follows from eq.(20) that at small t, $t\lesssim\mu^2$, the contribution of the neglected terms is of order of μ^3/m^3_{int} and therefore should be very small. The same is true as was shown above for $|M_\pi|^2$. High accuracy attained in the calculation of the total cross section for $t \lesssim \mu^2$ represents one of the principal results of the present paper.

With t growing the accuracy of calculation of the cross section falls. At large t, $t \gtrsim m_{int}^2$, all the contributions to the longitudinal cross section except for the term containing the neutron electric form factor G_{En} are proportional to μ/m_{int} as well as neglected terms. On the other hand the form factor G_{En} has been found experimentally to be small and therefore the total longitudinal cross section is expected to be negligible at large t as compared with the transversal cross section. If for the sake of estimation we put G_{En} to be identically zero, then the ratio σ_L/σ_T is of order μ^2/m_{int}^2 at large t and is of order $\mu^2/(t+2\mu^2)$ at $t \ll m_{int}^2$.

It is worth emphasizing that the threshold cross section depends on the axial form factor of nucleon and on the pion radius. If these quantities are considered to be given experimentally the relations obtained for the electroproduction cross section provide an opportunity to check the theoretical assumptions involved in the derivation of the low-energy theorems. On the other hand eqs.(18-20) may be considered as providing a possibility of extracting $g(k^2)$ and $F_\pi'(0)$ from experimental data in an independent way. The implications of the measurements of the electroproduction cross section for the determination of $g(k^2)$ have been already studied in paper /1/.

As for the contribution of the pion radius it may be consistently kept at all t satisfying condition $t \ll m_{int}^2$. Moreover the longitudinal cross-section depends rather heavily on $F_\pi'(0)$. If $F_\pi'(0)$ is equal to m_ρ^{-2}, m_ρ being the ρ-meson mass, the account of the term proportional to $F_\pi'(0)$ changes the longitudinal cross section by three times at $t = 0.1$ $(\mathrm{GeV/c})^2$ and by 25 % at $t = \mu^2$. However at $t = 0.1 (\mathrm{GeV/c})^2$ the longitudinal cross section is rather small and becomes comparable with the total cross section only at $t \sim \mu^2$. For such values of t it is not necessary to distinguish between the total and longitudinal cross sections to determine $F_\pi'(0)$.

In conclusion of this section we shall give recipe to calculate the cross-section of reaction $en \to ep\pi^-$. To find the cross-section of this reaction the terms G_{Mn} and $\mu/2m$ in eq.(18) should be replaced by $(-G_{Mp})$ and by $(-\mu/2m)$ respectively; the term $\varkappa^n/2m$ in eq.(19) should be replaced by $(\varkappa^p+1)/2m^2$ and the factor $|M_\gamma|^2$ in the r.h.s. of this equation denotes now the matrix element of the corresponding photoprocess $\gamma n \to p\pi^-$; and form factor G_{En} in the r.h.s. of eq.(20) should be changed to $(-G_{Ep})$.

4. Cross section of the reaction $\pi^- p \to n e^+ e^-$.

The results obtained in the previous section for $t \leq \mu^2$ may be also applied to the calculation of the cross section of the reaction $\pi^- p \to n e^+ e^-$ in the case of the stopping initial π^--meson

$$\frac{d\sigma}{d\kappa^2}(\pi^- p \to n e^+ e^-) = \frac{\alpha^2}{12\pi\, m(m+\mu)} \frac{|\vec{p}_2|}{\mu} \cdot \frac{1}{v} \cdot \frac{1}{\kappa^2} \left\{ |M_T|^2 + \kappa^2 |M_L|^2 \right\} \quad (22)$$

where v is the π-meson velocity in the laboratory system, $k^2=-t$ is the invariant mass squared of the lepton pair, $\vec{p}_2$ is the neutron 3-momentum and $|M_T|^2$, $|U_L|^2$ are given by eqs.(19), (20).

Let us compare the result obtained with the Born approximation with pseudoscalar πNN-coupling, which used for the description of experimental data/2/. We have taken into account all the terms of order μ^2/m^2_{int} in the amplitude and have shown that apart from magnetic moments of nucleons these terms are parametrized by three coefficients, namely, $F_\pi'(0)$, $g'(0)$ and γ. The Born approximation with pseudoscalar πNN-coupling /3/ differs from our result by replacing $g'(0)\to(dF_1^p/dk^2-dF_1^n/dk^2)|_{k^2=0}$ if we take into account the smallness of γ (see discussion after eq.(21)). This difference is essential for determination of pion radius from experimental data.

5. <u>Threshold cross-section of neutral pion production.</u>

In the case of reaction $ep\to ep\pi^0$ the following relations for the transversal and longitudinal parts of the matrix element may be obtained

$$|M_T(\pi^0 p)|^2=8\mu^2 f^2\left(1+\frac{t}{4m^2}\right)\left\{G_{Mp}\frac{t/2m\mu}{1+t/2m^2}-1+\frac{\mu}{2m}(1+\varkappa^p)+O\left(\frac{t}{m^2_{int}+t},\frac{\mu^2}{m^2_{int}}\right)\right\}^2$$

$$|U_L(\pi^0 p)|^2=4f^2\left(1+\frac{t}{4m^2}\right)\left\{\frac{G_{Ep}}{1+\frac{t}{2m^2}}+O\left(\frac{\mu t}{m_{int}(m^2_{int}+t)},\frac{\mu^2}{m^2_{int}}\right)\right\}^2 \qquad (23)$$

It follows from eq.(23) that the cross section of neutral pion production is expressed in terms of the well known electromagnetic form factors. For large t the accuracy of the calculation of the cross section is μ/m_{int}. For smaller t the magnitude of neglected terms decreases but the cross section diminishes. It is comparable with the cross section of charged pion production only for $t \gtrsim 4m^2$.

As for the ratio of the longitudinal and transversal cross sections σ_L/σ_T it is not small for any t. Moreover the transversal cross section is predicted to vanish near $t \sim \frac{\mu m}{1+\varkappa^p}$ and for this reason the ratio σ_T/σ_L is expected to be smaller than unity for a some range of t.

In conclusion we give predictions for the cross section of reaction $en \to en\pi^0$

$$|M_T(\pi^0 n)|^2 = 8\mu^2 f^2\left(1+\frac{t}{4m^2}\right)\left\{G_{Mn}\frac{t/2m\mu}{1+t/2m^2} + \frac{\mu}{2m}\varkappa^n + O\left(\frac{t}{m_{int}^2+t}, \frac{\mu^2}{m_{int}^2}\right)\right\}^2$$

$$|M_L(\pi^0 n)|^2 = 4f^2\left(1+\frac{t}{4m^2}\right)\left\{\frac{G_{En}}{1+t/2m^2} + O\left(\frac{\mu t}{m_{int}(m_{int}^2+t)}, \frac{\mu^2}{m_{int}^2}\right)\right\}^2 \qquad (24)$$

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