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A b s t r a c t

Estimates of the cross-sections of the processes $e^-e^+ \to 3\pi, 4\pi$ are obtained. They do not contradict the present experimental data at the energy $\sim 1,2$ GeV and explain in natural way the data in the energy interval 1,6-2 GeV.

Recently the experiments were carried out[1,2,3] which showed that the cross-section of the multiple π-meson production on the colliding electron-positron beams in the energy region 1-2 GeV exceeds essentially the cross-section of the reaction $e^-e^+ \to \pi^+\pi^-$ and is comparable with the point-like one - the cross-section of the muon pair production.

The processes of the multiple hadron production on colliding beams were discussed earlier from the point of view of parton scheme[4] and in the frame of statistical model[5] (see also[6]). It seems to us however that those approaches is hardly applicable in the energy region 1-2 GeV. As for the processes of particle production in higher orders in α with the cross-sections not decreasing with energy widely discussed recently[7,8] (see also[9]), their contribution is certainly small at the energies below 1,5 GeV.

Meanwhile, there are usual mechanisms of the multiple pion production at the e^-e^+ annihilation that certainly give considerable contribution to the total production cross-section in the energy region under discussion. We mean the reactions $e^-e^+ \to \rho\pi$; $\omega\pi$; $A_1\pi$ and $\rho\varepsilon$ with the subsequent decay of the produced resonances ($\rho\pi$, $\omega\pi$ production was discussed earlier in the work [10] in connection with the possible test of the higher symmetries schemes). To estimate the cross-sections we use the vector dominance model, i.e., we suppose that the virtual photon goes to the final state via ρ-meson in all reaction except the first one which goes via ω and φ. The contribution of the intermediate φ-meson may be neglected however since the coupling constant $g_{\varphi\rho\pi}$ is small that follows from the data on the $\varphi \to 3\pi$ decay.

The matrix element of the $\omega\rho\pi$ transition needed to compute the first two processes we write as

$$M_{\omega\rho\pi} = i\,\frac{g_{\omega\rho\pi}}{m}\,\varepsilon_{\mu\nu\varkappa\lambda}\,\omega_\mu \rho_\nu K_\varkappa q_\lambda \qquad (1)$$

where ω_μ and ρ_ν are the polarization vectors of the ω- and ρ-mesons and K and q are their momenta, m is the ρ-meson mass. The total cross-sections of these processes are

$$\sigma_{\rho\pi} = \frac{\pi\alpha^2}{2s}\left(\frac{g_{\omega\rho\pi}}{g_\omega}\right)^2 \frac{m^2}{s}\left(1-\frac{m^2}{s}\right) \qquad (2)$$

$$\sigma_{\omega\pi} = \frac{\pi\alpha^2}{6s}\left(\frac{g_{\omega\rho\pi}}{g_\rho}\right)^2 \frac{m^2}{s}\left(1-\frac{m^2}{s}\right) \qquad (3)$$

We neglected here the π-meson mass, the widths and mass difference of ω- and ρ-mesons. Besides that we took into account in formula (2) three charge states in the $\rho\pi$ system. From colliding beam experiments /11-13/ it is known that $g_\rho^2/4\pi \approx 2$, $g_\omega^2/4\pi \approx 14$.

The constant $g_{\omega\rho\pi}$ may be estimated using the assumption of the ρ-dominance in $\omega \to 3\pi$ and $\omega \to \pi^0\gamma$ decays /14/ that gives for the constant $g^2_{\omega\rho\pi}/4\pi$ the values ≈ 20 and ≈ 10 correspondingly. We shall use the first number since it is obtained in the region of invariants that is nearer to ours.

The matrix element of the $A_1\rho\pi$ transition present as

$$M_{A_1\rho\pi} = \frac{g}{m} A_\mu \rho_\nu \left\{ (q\cdot K)\delta_{\mu\nu} - K_\mu q_\nu + \beta\left[(pK)\delta_{\mu\nu} - K_\mu p_\nu\right] \right\} \quad (4)$$

where A_μ and ρ_ν are the polarization vectors of A_1- and ρ-mesons, q and K are their momenta, p is pion momentum. All the momenta we take as incoming. Then the total cross-section of the $A_1\pi$ production taking into account two possible charge states is

$$\sigma_{A_1\pi} = \frac{2\pi\alpha^2}{3s}\frac{|\vec{p}|}{\sqrt{s}}\left(\frac{g}{g_\rho}\right)^2\left(\frac{m^2}{m^2-s}\right)^2\frac{1}{m^2 s}\left\{\left[s+m_A^2-\right.\right.$$
$$\left.\left.-m_\pi^2+\beta\left(s-m_A^2+m_\pi^2\right)\right]^2+\frac{s}{2m_A^2}\left[2m_A^2+\beta\left(s-m_A^2-m_\pi^2\right)\right]^2\right\} \quad (5)$$

The quantities g and β may be estimated using the hard-pion technique /15,16/ from the A_1-meson width which we take 130 MeV. Then $g^2/4\pi = 1{,}3$, $\beta = 1{,}6$. To estimate the cross-section of π-meson production via this mechanism somewhat below the threshold of $A_1\pi$ production one may as before use the formula (5) taking into account for computation of $|\vec{p}|$ the Breit-Wigner distribution of A_1-meson mass.

At last, the matrix element of $\rho\rho\varepsilon$ transition we take as

$$M_{\rho\rho\varepsilon}=\frac{g_{\rho\rho\varepsilon}}{m}\left[(KK_1)\delta_{\mu\nu}-K_{1\mu}K_{\nu}\right]\rho_{\mu}\rho_{1\nu} \quad (6)$$

where ρ_{μ} and $\rho_{1\nu}$ are the polarization vectors of ρ-mesons and K_{ν} and $K_{1\mu}$ are their momenta. Then the total cross-section of real ρ- and ε-meson production with neglect of their mass difference is

$$\sigma_{\rho\varepsilon}=\frac{\pi\alpha^2}{6s}\left(\frac{g_{\rho\rho\varepsilon}}{g_{\rho}}\right)^2\left(\frac{m^2}{m^2-s}\right)^2\frac{s}{m^2}\left(1+\frac{2m^2}{s}\right)\sqrt{1-\frac{4m^2}{s}} \quad (7)$$

But first of all we are interested in the pion production by this mechanism below the threshold of real ρ and ε production. Using the matrix element of ρ and 2π production via the virtual ε-meson, we get the following approximate expression for the total cross-section of this process near the threshold

$$\sigma_{\rho 2\pi}=\frac{\pi\alpha^2}{2\sqrt{2}\,s}\left(\frac{g_{\rho\rho\varepsilon}}{g_{\rho}}\right)^2\frac{m_{\varepsilon}\Gamma_{\varepsilon}}{m^2}\left(\frac{m^2}{m^2-s}\right)^2\left(\frac{m^2}{s}\right)^{3/4}\times$$

$$\times\frac{a_-^2}{\sqrt{a_+}}\left[\left(\frac{m_{\varepsilon}^2}{m^2}-\frac{1}{2}a_+\right)^2+\frac{m_{\varepsilon}^2\Gamma_{\varepsilon}^2}{m^4}\frac{a_-}{a_+}\right]^{-1} \quad (8)$$

where $a_{\pm}=\left(\frac{\sqrt{s}}{m}-1\right)^2\pm 4\left(\frac{m_{\pi}}{m}\right)^2$. Following the analogy between the matrix elements (1) and (5)(one of them is the product of two field strengths and the second is the product of the field strength and the dual field strength), assume that $g_{\rho\rho\varepsilon} \approx g_{\rho\omega\pi}$. Note, that the assumption of the universal interaction of the ε-meson with the trace of the hadronic energy-momentum tensor leads to considerably larger value of the constant $g_{\rho\rho\varepsilon}$. Take also m_ε=700 MeV, Γ_ε = 400 MeV.

The estimate using the formula (2), (3), (5), (7) in the energy region 1,6-2 GeV gives the summary cross-section of pion production constituting more than a half of the observed one[2,3]. Therefore, the explanation of the experimental data[2,3] by order of magnitude causes no difficulties. Although the results based on the vector dominance model are not reliable in this region, the obtained estimate of the total cross-section is hardly overestimated since there are many channels (including quasi-two-particle ones) not taken into account by us.

In this energy region one may use for estimates other considerations. As was noted by V.E.Balakin, it is natural to expect that the average multiplicity of pions produced in colliding beams at energies 2 GeV is the same as for the $p\bar{p}$ annihilation at rest. The point is that $p\bar{p}$ annihilation at rest takes place in the states 0^- and 1^-, the state 1^-

giving large contribution to this process, and the e^-e^+ annihilation goes via 1^- state only. By means of this analogy one can estimate also the total cross-section of e^-e^+ annihilation at energy $\sim$ 2 GeV

$$\sigma_{e^+e^-} \sim \frac{v}{2c}\,\alpha^2\left(\frac{m}{2m_p}\right)^2 \sigma_{p\bar{p}} \qquad (9)$$

Using the value [17] $v\sigma_{p\bar{p}} = 1{,}5 \cdot 10^{-15}\,cm^3/sec$, we get $\sigma_{e^+e^-} \sim 3 \cdot 10^{-32}\,cm^2$ in agreement with experiment [2,3].

Pass now to the comparison with experimental data [1] at energies 1,18; 1,26; 1,34 GeV. The results of comparison present as a table.

$E=\sqrt{S}$ ГэВ	1,18	1,26	1,34
1 $\sigma_{\rho\pi} \cdot 10^{32}\,см^2$	0,7	0,6	0,5
2 $\sigma_{\omega\pi} \cdot 10^{32}$	1,6	1,4	1,2
3 $\sigma_{A_1\pi} \cdot 10^{32}$	0,4	1,0	1,4
4 $\sigma_{\rho 2\pi} \cdot 10^{32}$	0,2	0,3	0,4
$\sigma \cdot 10^{32}$	2,9	3,3	3,5
$\sigma_{\mu^+\mu^-} \cdot 10^{32}$	6,2	5,5	4,8

Here σ is the sum of computed cross-sections of pion production; the cross-section of $\mu^+\mu^-$ production $\sigma_{\mu^+\mu^-}$ is given for comparison. Note, that even in this energy region the average pion multiplicity approaches 4.

The estimate gives 3-5 non-collinear events under the conditions of the experiment /1/ at each energy. Some remarks on this estimate. The effective solid angle of counters in the experiment /1/ was ⅛ of total solid angle, pairs of charged particles with relatively small angle of non-collinearity being registrated. Since the matrix element of the process $e^-e^+ \rightarrow \pi^+\pi^-\pi^0$ for collinear pions turns to zero, the registration effectivity for the first process is very small and we neglected its contribution. The angular distribution of π-mesons in the processes 2 and 3 we take isotropic. Then the registration effectivity for the 2nd process where two charged and two neutral particles are produced constitutes ⅛ in respect to the two-particle process. For the third process where in the half of cases 4 charged particles are created the registrating effictivity constitutes 7/16 accounting for combinatorics. When estimating the contribution of the fourth process, we took into account that the ρ -meson is produced in it with small momentum, so that the average angle of non-collinearity of pions from its decay equals approximately the angular size of counters, and that in 2/3 of cases four charged pions are produced. So the $\rho\, 2\pi$ production is essential for explanation of the experimental results /1/. If one takes into account that some of registrated non-collinear events correspond evidently to the production of non-collinear e^-e^+ and $\mu^+\mu^-$ pairs, then our estimate does not contradict experimental data. But the discrepance between the estimates and the average experimental points is large. The persistence of this discrepance for improved experimental accuracy would indicate possibly the existence of some new effects of pion interaction in this energy region.

One more, rather exotic one, mechanism of the non-collinear particles production is possible. We mean the hypothetical heavy lepton, produced in colliding beams with point-like cross-section and decaying quickly due to weak interaction. The usual assumption of the universal structure of weak interactions leads to the following expression for the rate of this lepton decay. into pion and neutrino

$$\Gamma_{\lambda\to\pi\nu} = \frac{g^2 f_\pi^2 m_\lambda^3}{8\pi}\left(1-\frac{m_\pi^2}{m_\lambda^2}\right)^2;\ g=10^{-5} m_p^{-2},\ f_\pi = 0{,}95\, m_\pi \qquad (10)$$

At $m_\lambda \sim 700$ MeV this rate exceeds by order of magnitude, the rate of leptonic decay ($\Gamma_{\lambda\to e(\mu)\nu\nu} = \frac{G^2 m_\lambda^5}{192\pi^3}$) and leads to the range $\sim$ 0,5 cm. To explore such a possibility one needs to increase somewhat the statistics and the resolution in the intersection of non-collinear tracks of the registration apparatus /1/. Note, that when taking into account the dominance of the decay $\lambda \to \pi\nu$, the experiment /18/ gives no limitation on m_λ.

After this work was finished we became aware of the articles /19-20/ where the same questions were treated. Our results somewhat differ from the results of these articles. In particular, we got essentially larger estimate of the production cross-section of four charged π-mesons, essentially due to the $\rho 2\pi$ production. This process is not discussed in the articles /19, 20/. The value of $g_{\rho\varepsilon\varepsilon}$ taken in the work /21/ is much smaller, than ours. But the contribution of the final $\rho 2\pi$ state in the $p\bar{p}$ annihilation at rest is large. It is reasonable, by our opinion, to expect that this channel will be important also in the e^+e^- annihilation.

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