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# MULTIPLE PRODUCTION IN HIGH ENERGY COLLISIONS

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A b s t r a c t

The model for multiple production of hadrons in the high energy collisions and in $e^+e^-$ annihilation is proposed. The thermodynamics of the excited hadronic systems is obtained. The collective interaction of secondary hadrons is treated with the help of relativistic hydrodynamics. The obtained spectra of secondaries are in good agreement with the data on various inclusive reactions in the energy range 10-70 Gev.

MULTIPLE PRODUCTION IN HIGH ENERGY COLLISIONS

E.V.Shuryak

## 1.Introduction

At high energy of colliding particles the multiple production processes become dominant. The complete dynamical treatment of these reactions encounters difficulties connected with the very complicated character of the interaction of numerous secondary particles. The statistical model originally proposed by E.Fermi /1/ and developed further by I.Ya.Pomeranchuk /2/ describes rather well many features of the phenomenon /3/.

As it was noted by L.D.Landau, the simple statistical model is inconsistent: on one hand, the strong final state interaction is assumed to cause the thermodynamical equilibrium of the secondaries; and, on the other, they are treated as an ideal gas of independent particles. The account of the collective interaction of secondaries can be made with a help of relativistic hydrodynamical equations which can be used if the mean free path is much smaller than the whole system dimensions: the same condition as for the applicability of thermodynamics. The solution of these equations obtained by Landau /4/ gave the description of multiple production in agreement with the data available by that time. Such predictions of this theory as the composition of the secondaries, the transverse momentum distribution and its practical independence of initial energy, anisotropy of angular distribution etc, have been proved by the later experiments. But because of some quantitative disagreement of these results with modern data and the development of new models the hydrodynamical model turns out to be almost forgotten. The thermodynamical model developed by R.Hagedorn and J.Ranft /5/ is in some respects close to Landau theory. The momentum spectra of the secondaries in this model are the "thermal" spectra transformed into the moving with a velocity $V$ coordinate system and averaged over $V$ with the weight function $\varphi(V)$. This function is introduced for the account of the "collective motion" and it is not determined by the model, but by the fit to data. The spectrum in Landau model is of the same kind, the role of the weight function being played by the hydrodynamical velocity distribution. Hence it follows, for example, that the weight function is the same for different particles which was indeed discovered in the fit to the data on $\pi$, $K$, $\bar{p}$ production in the framework of the thermodynamical model /5/.

Let us single out two groups of secondaries. The first group contains the "throughgoing" particles, namely, the fast ones with quantum numbers of colliding particles, for example, leading protons or isobars (and their decay products) in the $pp$ collision. The second one consists of the "new created" particles such as $\pi^-$, $K^-$, $\bar{p}$ in the $pp$ collision. The present paper is devoted only to the "new created" particles whose spectra are assumed to be determined by the intense final state interaction.

The following two points are considered in a new way as compared to Landau's original work:

a) The excited hadronic matter in /4/ was considered as equivalent to the ultrarelativistic ideal gas. In section II the thermodynamical description of this system with the account of the strong interaction is obtained.

b) The solution for the one-dimensional expansion process and estimates for the three-dimensional one were given in /4/. Here (section III) we obtain a more general solution. The obtained spectra of secondaries are compared to the existing experimental data on negative particle production in the $pp$ collision in the energy range of 10 - 70 Gev (sec.IV). In section V the model is applied to the lepton annihilation into hadrons (for example, $e^+e^-$ hadrons). Some general discussion of the model is presented in section VI.

II. <u>The Thermodynamics of the Excited Hadronic Matter</u>

We use the method originally elaborated by E.Beth and G.E.Uhlenbeck /6/ in their treatment of the pair interaction of particles in the nonideal gas (see also /7/). The main idea of this approach is the counting of the number of states inside the normalization volume with the account of the interaction. The wave function of the $\ell$-th partial wave at large distance $r$ between the particles has the form $\Psi_\ell(r) \approx \frac{1}{r}\sin(pr+\delta_\ell(p))$ where $\delta_\ell$ is the scattering phase and $p$ is the relative momentum. The boundary condition $\Psi_\ell(R)=0$ picks out states with $pR+\delta_\ell(p)=n\pi$, (here $R$ is the radius of the normalization volume). If one changes the summation over $n$ to the integration over $p$ one obtains the following expression for the part of the partition function connected with the relative motion:

$$Z=\frac{1}{\pi}\sum_\ell(2\ell+1)\int_0^\infty\left(R+\frac{d\delta_\ell}{dp}\right)e^{-\frac{E(p)}{T}}dp \qquad (1)$$

If there is a resonance in scattering at some relative momentum $p_0$, then $\delta_\ell(p)$ changes rapidly in the vicinity of $p_0$. One can see that its contribution to (1) is the term $\exp[-E(p_0)/T]$, the same as for the bound state with this energy. This leads to the following conclusion: in the study of thermodynamics of excited hadronic matter one has to treat resonances on the same foot as stable particles. This fact has been pointed out by L.D.Landau (see /8/) in 1956 in the discussion of the role of the isobar in the statistical model. But the experimental data available by that time were poor, few resonances were known and this idea was not used for the study of the hadronic matter thermodynamics.

Now there is no theory of strong interaction, but the data clearly shows the resonant character of the low energy hadronic scattering. We assume the scattering phase to be the sum of terms corresponding to different resonances. We also assume that the manibody forces can be taken into account with the help of manibody resonances. Let us add that this approach is close to method of quasiparticles which is very often used in the study of the thermodynamics of various systems.

Let the density of resonance states of mass $m$ (spin and isospin states are also included) of fermions ($\delta=1$) or bosons ($\delta=-1$) be $\rho_\delta(m)$. Then the energy density and pressure are given by ($E=\sqrt{p^2+m^2}$):

$$\varepsilon=\sum_\delta\int dm\,\rho_\delta(m)\int\frac{d^3p}{(2\pi)^3}\,E\left[\exp\left(\frac{E}{T}\right)+\delta\right]^{-1} \qquad (2)$$

$$p=\sum_\delta\int dm\,\rho_\delta(m)\int\frac{d^3p}{(2\pi)^3}\,\frac{p^2}{3E}\left[\exp\left(\frac{E}{T}\right)+\delta\right]^{-1} \qquad (3)$$

With the assumption of the asimptotical equivalence of the number of states of the excited hadronic cluster ("fire-ball") with $\rho(m)$. R.Hagedorn/9/ has obtained the following expression for at high masses $m \to \infty$.

$$\rho(m) \to b\, m^{-5/2} \exp(am) \qquad (4)$$

Here $a$ and $b$ are some parameters.

For the description of hadronic collision with energies available at the modern accelerators there is no need to know the $\rho(m)$ for very high masses. On the other hand, in the region in question 2 Gev the majority of the resonances are already discovered. Thus, we use no assumptions concerning $\rho(m)$, but simple parametrization of the data. The expression (5) fit the data at $m \leq 1{,}2$ Gev but we prefer more simple parametrization

$$\rho(m) \sim m^{\varkappa} \qquad (5)$$

which also describes the experiment in this region of mass with $\varkappa \simeq 3$, but leads to much simplier equation of state - the fact of great importance for the solution of hydrodynamical problems.

Let us note, that as $\rho(m)$ is a rapindly increasing function, so the main contribution to (2) and (3) is given by the region of masses where $\exp(\frac{m}{T}) \gg 1$ and the difference between Bose and Fermi particles can be neglected.



The substitution of (5) into (2) and (3) shows that both $\varepsilon$ and $p$ are proportional to $T^{\varkappa+5}$. Thus, the sound velocity $c$ defined by

$$c^2 = \frac{dp}{d\varepsilon} \qquad (6)$$

turnes out to be temperature independent. Its value can be obtained by the direct calculation of integrals (2), (3), but it is more convenient to use the thermodynamical identities which lead to the following expressions for the energy density, enthropy and pressure: ($\alpha$ is some constant)

$$\varepsilon = \alpha T^{\varkappa+5} \;;\; S = \alpha \frac{\varkappa+5}{\varkappa+4} T^{\varkappa+4} \;;\; p = \frac{\alpha}{\varkappa+4} T^{\varkappa+5} \qquad (7)$$

and thus

$$c^2 = \frac{1}{\varkappa+4} \approx 0{,}14 \qquad (8)$$

The initial energy density in the particle collision with energy $E$ (in the C.M. system) is proportional to $E^2$ because of the lorentz contraction of volume /1/. One can see from (7) that the initial temperature very slowly increases with $E$, so the parametrization (5) can be used up to energies of the order of some hundreds of Gev. At higher energies the $c^2$ becomes temperature dependent and the behaviour of the solution of the hydrodynamical problem changes. We do not consider this energy range in the present paper.

## III. The collective motion of the secondary particles

The general approach to the problem as well as the qualitative features of the results in this paper are the same as in the Landau theory /4/. So we first remind briefly few essential points of this work.

1. The cluster of excited matter created in the collision has an initial form of thin disk. Its transverse dimensions are of the order of the radius of hadrons $m_\pi^{-1}$, while the longitudinal one is $\gamma$ times smaller, where $\gamma$ is the Lorentz factor of colliding hadrons.

2. The strong interaction in this cluster causes very small mean free path, so the hydrodynamical equations are valid.

3. The strong interaction disappears when the distances between particles approach $m_\pi^{-1}$, this corresponds to the fixed final temperature of the order of $m_\pi$.

4. The process of expansion of the cluster of secondaries has predominantly one-dimensional character due to its initial form.

5. The value of transverse momentum of secondaries is determined mainly by the "thermal motion" corresponding to the fixed final temperature.



Let us write down the relativistic hydrodynamical equations which has the form of the energy-momentum conservation lows:

$$\frac{\partial T^{ik}}{\partial x^k} = 0 \tag{9}$$

The energy-momentum tensor $T^{ik}$ is determined by

$$T_{ik} = (\varepsilon + p) u_i u_k - p g_{ik} \tag{10}$$

where $\varepsilon$ and $p$ are energy density and pressure in the rest system of the volume element, $u_i$ is its 4-velocity.

During the expansion the matter is much more strongly accelerated in longitudinal direction than in transverse one. So one can adopt the following way of the solution of (9): to solve firstly the averaged (over transverse coordinates) equations (9) with $i$ = 0,1 (an axis $x$ is taken in the longitudinal direction). In this stage the transverse size of the system $r_\perp(x, t)$ at distance $x$ from the collision place at the moment $t$ is arbitrary, but it is determined in the second stage of the solution from the transverse equations (9) in agreement with the solution of the first stage.

The averaged energy-momentum tensor ($i$=0,I) is

$$\tilde{T}_{ik} = \int T_{ik}\, dy\, dz \approx \pi r_\perp^2 \left[ (\tilde{\varepsilon} + \tilde{p}) \tilde{u}_i \tilde{u}_k - \tilde{p} g_{ik} \right] \tag{11}$$

The equation (10) with $i = 0,1$ then becomes

$$\frac{\partial \tilde{T}^{00}}{\partial t} + \frac{\partial \tilde{T}^{01}}{\partial x} = 0 \qquad \frac{\partial \tilde{T}^{01}}{\partial t} + \frac{\partial \tilde{T}^{11}}{\partial x} = 0 \tag{12}$$

Let us consider longitudinal motion as ultrarelativistic one and neglect the slow transverse one. Then

$$\tilde{u}^0 \approx \tilde{u}^1 \equiv u \ ; \quad \tilde{u}^0 - \tilde{u}^1 \approx \frac{1}{2u} \tag{13}$$

Substituting the equation of state (8) into (12) one obtains in the region $t \gg \xi = t - x$ the following equations:

$$\frac{\partial}{\partial t}\left(\tilde{\varepsilon}\tau_\perp^2 u^2\right) + \frac{1}{2}\frac{1-c^2}{1+c^2}\frac{\partial}{\partial \xi}\left(\tilde{\varepsilon}\tau_\perp^2\right) = 0 \tag{14}$$

$$\frac{\partial}{\partial t}\left(\tilde{\varepsilon}\tau_\perp^2\right) + \frac{1}{2}\frac{1+c^2}{1-c^2}\frac{\partial}{\partial \xi}\left(\frac{\tilde{\varepsilon}\tau_\perp^2}{u^2}\right) = 0 \tag{15}$$

Let us introduce new variables

$$\alpha = \frac{1}{2}\ln\frac{t+x}{t-x} \ ; \quad \beta = \frac{1}{2}\ln\frac{t^2-x^2}{\Delta^2} \ ; \quad \varphi = \ln\left(\tilde{\varepsilon}\tau_\perp^2\right) \tag{16}$$

where $\Delta$ is the initial longitudinal dimension of the system. Let us follow Landau and assume that

$$u^2 = f\frac{t}{\xi} \tag{17}$$

where $f$ is slowly varying function (which can be proved with the solution obtained later). Excluding $f$ from (14), (15) we obtain equation

$$1 + \frac{\partial \varphi}{\partial \beta} + \frac{c^2}{(1+c^2)^2}\left[\left(\frac{\partial \varphi}{\partial \beta}\right)^2 - \left(\frac{\partial \varphi}{\partial \alpha}\right)^2\right] = 0 \tag{18}$$



It can be solved with the help of two-parameter integral

$$\varphi = \alpha\sqrt{B^2 + (1+B)\left(\frac{1+c^2}{c}\right)^2} + B\beta + A \tag{19}$$

and the initial conditions $\varphi \sim \varphi_0$ at $\alpha, \beta \sim 1$ with the following result:

$$\varphi = \varphi_0 + \frac{1-c^4}{2c^2}\sqrt{\beta^2-\alpha^2} - \frac{(1+c^2)^2}{2c^2}\beta \tag{20}$$

At the beginning of the expansion $\tau_\perp \approx \tau_0$ where $\tau_0$ is the initial transverse dimensions ($\tau_0 \approx m_\pi^{-1}$). At time $t \gg \tau_\perp$ the transverse dimensions can be estimated with the help of transverse equations (9) which we write in the following manner:

$$\frac{T^{02}}{t} \sim \frac{T^{22}}{\tau_\perp} \tag{21}$$

Substituting here $T^{02} = \varepsilon u_0 u_2 \sim \varepsilon u^2 \frac{\tau_\perp}{t}$ ; $T^{22} \sim \varepsilon$ and account for (17) one obtains:

$$\tau_\perp^2 \sim t\xi \tag{22}$$

This result can be used in (20) and the energy density dependence on $x$ and $t$ is given by

$$\tilde{\varepsilon} = \varepsilon_0 \frac{\tau_0^2}{\Delta^2} \exp\left[\frac{1-c^4}{2c^2}\sqrt{\beta^2-\alpha^2} - \frac{(1+c^2)^2}{2c^2}\beta - 2\beta\right] \tag{23}$$

As it was mentioned above when some low density is reached the hydrodynamical expansion comes to an end and particles fly out independently. At this moment energy density is also fixed, $\varepsilon_f \sim T_f^{\frac{1+c^2}{c^2}}$.

Putting $\varepsilon = \varepsilon_f$ in (23) we obtain the trajectory of the end of the hydrodynamical expansion. We consider the case $\alpha \ll \beta$ and expand the result up to the terms of the order of $(\alpha/\beta)^2$ .

$$\ln\left(\frac{\varepsilon_0 \tau_0^2}{\varepsilon_f \Delta^2}\right) = (3+c^2)\beta + \frac{1-c^4}{2c^2}\frac{\alpha^2}{2\beta} \tag{24}$$

The expressions $(\varepsilon_0/\varepsilon_f)\sim\gamma^2; (\tau_0/\Delta)\sim\gamma$ can be substituted here. The particle distribution is given by

$$dN = \pi n u \tau_\perp^2 \, d\xi \tag{25}$$

where $n$ is the density of particles (in the rest system of the volume element) and it is fixed at the end of hydrodynamical expansion. The (17), (22), (25) lead to

$$dN \sim e^{3\beta} d\alpha \tag{26}$$

and putting here (24) we obtain the final result

$$dN \sim \frac{1}{\sqrt{2\pi L}} e^{-\frac{\alpha^2}{2L}} d\alpha \equiv \varphi(\alpha) d\alpha \tag{27}$$

where

$$L = \frac{8c^2 \ln\gamma}{3(1-c^4)} \tag{28}$$

Due to simple connection between $\alpha$ and hydrodynamical velocity * (17), this expression

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*In order to account the symmetry for the exchange $x \leftrightarrow -x$ we write $u_0 \approx ch\alpha$ ; $u_1 \approx sh\alpha$ .



is the distribution of the particles over the hydrodynamical velocities. Let us add that $\varphi(\alpha)$ (27) corresponds to the phenomenological weight function of the paper /5/.

It is interesting to compair the proposed solution with that in /4/ which has been obtained in the following way: one- and three-dimensional stages of the expansion process were treated separately and the found solution were "sewed" together. The exact solution of one-dimensional problem has been found by Khalatnikov /10/, but only estimates remaind for the three-dimensional problem. The numerical solution for $c^2 = 1/3$ where obtained by G.A.Milekhin /11/.

For $t\xi \ll \tau_0^2$ $\tau_\perp \approx \tau_0$ and (14), (15) coincide with Landau one-dimensional equation as well as the solution. If one takes $\tau_\perp = t\theta$ (where $\theta$ is some fixed angle) than one obtains the equations for "konical" expansion. The estimates in /4/ for this case $\varepsilon u^2 t^2 \sim const$; $s u t^2 \sim const$ corresponds to the neglect of the second term in (14) and in the enthropy conservation low following from (14),(15). The found solution shows that they can not be neglected, it causes some differences in the results. The numerical solution /11/ give $\varphi(\alpha)$ of the form (27) with $L \approx 1{,}12 \ln\gamma$ which agree rather well with (29) ( at $c^2 = 1/3$ ) $L = \ln\gamma$.

## IV. Secondary particle spectra

The account of "thermal" motion inside the moving volume elements is also necessary for the description of the momentum spectra of the secondaries. Let us define $\beta$ and $\zeta$ by the following expressions

$$E = m\,ch\beta\,ch\zeta\ ;\ p_{\parallel} = m\,sh\beta\,ch\zeta\ ;\ p_{\perp} = m\,sh\zeta \tag{29}$$

One can see that transition into moving coordinate system with a velocity $V = th\alpha$ (in the longitudinal direction) changes $\beta$ and $\zeta$ as follows:

$$\beta \to \beta + \alpha\ ;\ \zeta \to \zeta \tag{30}$$

The density of particles with some quantum numbers $i$ and momentum $\vec{p}$ (in the rest system of the volume element) is

$$dn_i = f_\delta(E_i)\frac{g_i\,d^3p}{(2\pi)^3}\ ;\ f_\delta(E_i) = \left[exp\left(\frac{E_i}{T_f}\right) + \delta\right]^{-1} \tag{31}$$

Here $\delta$ = 1 for fermions and –1 for bosons, $g_i$ is the number of internal states of particle $i$, $E_i = \sqrt{p^2+m_i^2}$. The application of variables (29) helps us to reach the final expression: ($f_\delta(\alpha, E) = f_\delta(m\,ch(\alpha-\beta)\,ch\zeta)$ )

$$dN_i = \left\{\int \varphi(\alpha) f_\delta(\alpha, E)\frac{ch(\beta-\alpha)}{ch\beta}\,d\alpha\right\}\frac{g_i\,d^3p\,V}{(2\pi)^3} \tag{32}$$

In this formula $\varphi(\alpha)$ is determined by (27). The



factor $\frac{ch(\beta-\alpha)}{ch\beta}$ originates from the transformation of $d^3p$. $V$ is the total volume of different elements taken at the moment of the end of hydrodynamical expansion. As far as these moments are different for different elements, $V$ has no direct geometrical definition. The total energy of "through-going" particles (see introduction) is unknown, so one can not normalize (32) and $V$ remains the free parameter of the model. Integration of (32) lead to the following expressions for the average particle number and their average energy (for Bose statistics)

$$\bar{N} = V\,\frac{gTm^2}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}K_2\left(\frac{mn}{T_f}\right) \tag{33}$$

$$\bar{E} = T\,exp\left(\frac{L}{2}\right)\frac{\sum\frac{1}{n^2}\left[3K_2\left(\frac{mn}{T_f}\right) + \frac{mn}{T_f}K_1\left(\frac{mn}{T_f}\right)\right]}{\sum_{n=1}\frac{1}{n}K_2\left(\frac{mn}{T_f}\right)} \tag{34}$$

where $K_\nu$ are Bessel functions. At $T_f = m_\pi$ one obtains $\bar{E} \approx 0{,}43$ Gev which agree to the data at accelerator energies.

Particles with "rare" quantum numbers such as $K^-$ and $\bar{p}$ can be created only together with one-ther particle due to conservation lows. This fact decreases the rate of their production. We treat this point with the help of the phenomenological method of the account of this effect proposed in /5/. It is based on the idea that the probability of the reaction in question is the product of the probabilities of their separate production.

Let the probability of pair production (particlea have quantum numbers $i$ and $K$ ) be

$$dW_{iK} = \left\{ \int \varphi(\alpha_1) f(\alpha_1 E_i) \varphi(\alpha_2) f(\alpha_2 E_K) K(\alpha_1 \alpha_2)\, d\alpha_1\, d\alpha_2 \right\} \times \frac{g_i V d^3p}{(2\pi)^3} \times \frac{g_K V\, d^3p}{(2\pi)^3} \tag{35}$$

where $K(\alpha_1 \alpha_2)$ represents correlation between particles. As far as strangeness (and barion number) are concerved locally, $K(\alpha_1 \alpha_2)$ is to have maximum at $\alpha_1 = \alpha_2$ /5/ and can for the simplicity be taken equal to $\delta(\alpha_1 - \alpha_2)$. Performing an integration over $d^3p_K\, d\alpha_2$ one obtains:

$$dW_i = \left( \sum_K n_K \right) \left\{ \int \varphi^2(\alpha) f(\alpha, E_i)\, d\alpha \right\} \frac{g_i\, d^3p_i V}{(2\pi)^3} \tag{36}$$

where

$$n_K = \int f(E_K) \frac{g_K V\, d^3p_K}{(2\pi)^3} \tag{37}$$

The sum is taken over all particles $K$ which can be produced together with $i$. The conservation of the charge is not to be taken into account for it can be transfered to accompanied pions.

At low collision energies $\gamma \to 1$ and according to (28) $L \to 0$. Then $\varphi(\alpha) \to \delta(\alpha)$ and the spectrum (32) coincides with the results of the statistical model. It is known to describe rather well



the data at low energies /3/, so although the solution found above were obtained for $\gamma \gg 1$, but the result turns out to be true for $\gamma$ close to 1. So one can assume that the results (32), (36) approximately holds for all energies*.

As at high energies $L$ is large the maximum of the integrand in (32) moves to the point $\alpha = \beta$ and the spectrum is mainly determined by the collective velocities.

Let us remind that (27) has been obtained for $\alpha \ll \beta$ and (32,36) does not hold for the high energy end of the spectrum. This region close to the kinematical limit is connected with rare processes of small multiplicity which are not described by the proposed model.

We have chosen three groups of data for the comparison of the model with experiment: the detailed spectra of various particles produced in 19,2 Gev $pp$ collision /12/ recently obtained in Serpukhov the momentum spectra of negative particles produced in 70 Gev $p$-$Al$ collision in forward direction /13/ and the buble chamber data on the processes $\pi p \to \pi + ...$ /17/.

No adjustible parameters have been used in the calculation of the form of momentum spectra,

* With the restrictions discussed in section 2.

the final temperature $T_f$ has been taken equal to pion mass. The normalization of the curves has been done as follows: (32) and (36) has been multiplied by the total absorption cross-section which has been taken to be equal to 31 mb for $pp$ collision. An additional factor $A^{2/3}$ has been introduced for the $p$-nucleus collisions ( $A$ is the nucleon number of the nucleus). Besides, the temperature variations have been used for the normalization of the heavy particle spectra for which the factor $\exp\left[2m\left(\frac{1}{m_\pi}-\frac{1}{T}\right)\right]$ has been inserted in (36). The following values of $T$ have been obtained in the fit: $T_{K^-} = 0,90\, m_\pi$, $T_{\bar p} = 0,97\, m_\pi$ at 19,2 Gev and $T_{K^-} = 1,13\, m_\pi$ and $T_{\bar p} = 1,11\, m_\pi$ at 70 Gev*. Thus, the deviations from the expected value $m_\pi$ are not large. The parameter $V$ is $8V_0$ and $9V_0$, respectively, where $V_0 = \frac{4\pi}{3} m_\pi^{-3}$.

In Figs. 1,2 the $\pi^-$ and $K^-$ spectra are compared with the data /12/ represented as $\frac{d\sigma}{2\pi p_\perp dp_\perp dp_\parallel}$ in the C.M.system as a function of $p_\perp$ and $p_\parallel$. The invariant quantity $F = E\frac{d^3\sigma}{d^3p}$ as a function of $\beta$ (see (29)) for $p_\perp = 0$ is compared to the data /13/ in Fig.3. The experimental data on reactions $\pi^- p \to \pi^\pm$ /17/ are represented in Fig.4 and are compared with the predictions of the model

* In this case the $T$ value corresponds only to some effective normalization, because for $p$-nucleus collisions the model needs modifications. Let us also note, that the experimental precision of the normalization is not high.



(full curve). The $\pi^-$ spectra clearly shows the "through-going" contribution. The model is also in agreement with other data on reactions $pp, Kp, \bar{p}p \to \pi, K$ studied in bubble chambers. The detailed comparison with those data and the discussion will be presented later.

## V. Lepton's annihilation into hadrons

An example of such process is one photon annihilation of electron and positron, the reaction studied by $e^+e^-$ colliding beams. The excited hadronic system (is in this case produced by the decay of a virtual photon) can be treated in the frame-work of the same model. It is the geometry of the problem which is different here: it is not the "thin disk explosion" as in the case of hadronic collisions but rather an isotropic "explosion from a

* The cross section of this process falls with energy not slowly than $(\text{energy})^{-2}$, while the reaction $e^+e^- \to e^+e^- +$ hadrons where hadrons are produced in two photons collision, has even increasing cross section. One may assume that the two photon collision does not differ much from the hadronic collision.

**The virtual photon polarization cause some unisotropy but as many secondaries with large angular momenta are produced, this unisotropy can be neglected.

point". This approach to the process $e^+e^- \rightarrow$ hadrons has been suggested by the author /14/ and the solution has been found for the equation of state $c^2=1/3$. Here we present the more general solution for any temperature independent $c^2$.

According to /15/ another mechanism of one photon $e^+e^-$ annihilation into hadrons is possible, namely, the subsequent decay of the virtual photon into the pair of pointlike particles ("partons"), and their decay into secondaries with the two jets production in the direction of primary "partons". This case is similar to the hadronic scattering studied above, so we shall not discuss it here.

The equations of spherically symmetrical expansion process in the variables $t, \zeta = t - r$ at $t \gg \zeta$ are similar to (14), (15)

$$\frac{\partial}{\partial t}\left(\varepsilon t^2 u^2\right) + \frac{1}{2}\frac{1-c^2}{1+c^2}\frac{\partial}{\partial \zeta}\left(\varepsilon t^2\right) = 0 \qquad (38)$$

$$\frac{\partial}{\partial t}\left(\varepsilon t^2\right) + \frac{1}{2}\frac{1+c^2}{1-c^2}\frac{\partial}{\partial \zeta}\left(\frac{\varepsilon t^2}{u^2}\right) = 0 \qquad (39)$$

With the help of the solution obtained in section III one has the result

$$u \sim \exp\alpha \, ; \; \varepsilon = \varepsilon_0 \exp\left[\frac{1-c^4}{2c^2}\sqrt{\beta^2-\alpha^2} - \frac{(1+c^2)^2}{2c^2}\beta - 2(\alpha+\beta)\right] \qquad (40)$$



where

$$\alpha = \frac{1}{2}\ln\left(\frac{t+r}{t-r}\right) ; \; \beta = \frac{1}{2}\ln\left(\frac{t^2-r^2}{\Delta^2}\right) ; \; x = \ln\frac{\varepsilon_i}{\varepsilon_f} \qquad (41)$$

and $\frac{4\pi}{3}\varepsilon_i\Delta^3$ is the total $e^+e^-$ energy in the C.M.system.

The condition of the end of hydrodynamical expansion is the following (for $\alpha \ll \beta$ up to terms of the order of $(\alpha/\beta)^2$ )

$$x = (3+c^2)\beta + 2\alpha + \frac{1-c^4}{2c^2}\frac{\alpha^2}{2\beta} \qquad (42)$$

The substitution of (40) into the expression of the particle number

$$dN = n u t^2 d\zeta \sim \exp(3\beta + 2\alpha)\, d\alpha \qquad (43)$$

leads to*

$$dN \sim \exp\left[\frac{2c^2}{3+c^2}\alpha - \frac{3(1-c^4)}{4c^2 x}\alpha^2\right] d\alpha \sim \varphi(\alpha)\, d\alpha \qquad (44)$$

where $\varphi(\alpha)$ is normalized by

$$\int_0^\infty d\alpha \, \varphi(\alpha) = 1 \qquad (45)$$

The momentum spectra of the secondaries is given by

$$\frac{dN}{dp} = \sum_{n=1}^{\infty}\frac{gVp^2T}{2\pi^2 En}\int d\alpha\, \varphi(\alpha) e^{-\frac{nE}{T}\mathrm{ch}\alpha}\left[\left(1+\frac{nE}{T}\mathrm{ch}\alpha\right)\frac{\mathrm{sh}\left(\frac{np}{T}\mathrm{sh}\alpha\right)}{\frac{np}{T}\mathrm{sh}\alpha} - \mathrm{ch}\left(\frac{np}{T}\mathrm{sh}\alpha\right)\right] \qquad (46)$$

where $E = \sqrt{p^2+m^2}$ . The modification for the case

* The solution given in /14/ (expression (7)) for $c^2=1/3$ coincides with (40). But there is a numerical error in expression (8) of /14/, the right one is given by (44).

of production of the particles with rare quantum numbers can be done in the same way as in the preceeding section.

The mechanism of the production of primary hadronic cluster in the virtual photon decay is not quite clear. So its characteristics size $\Delta$ (and $X$ ) is not determined in the model and remains free. No such parameter appears in the case od hadronic scattering because here it is determined by the dimensions of the colliding hadrons. Some estimate for the value of $\Delta$ is suggested in /14/. May be $\Delta^{-1}$ is some characteristic energy of strong interaction, something of the order of 1 Gev.

## VI. Summary and Discussion

Let us first ennumerate the most important points of the present work. This is the derivation of the thermodynamics of the excited hadronic matter with the account of strong interaction. Then it is the solution of the hydrodynamical problem describing the collective motion of secondaries. The results obtained are compared with experiment and the good agreement is found.

In contrast to dynamical models of multiple production in the proposed approach only very limited and available information about hadronic interaction is needed: namely, the resonance



spectrum. On the other hand, this model predicts only some averaged behaviour of the system, while dynamical treatment can, in principle, give all the details. But in the comparison of the existing dynamical models with the data many free parameters are used and the predictive power of the model is not always clear.

Let us note, that the proposed model has some very essential point in common with the multiperipheral model: there exists an interaction only between particles whose velocities do not differ much. It causes some similarity in the results: for example, the approximate $d\alpha = dp_{\parallel}/E$ behaviour of the momentum spectrum of secondaries at small $\alpha$ which is known to be the general consequence of the multiperipheral approach.

The very simplified treatment of the thermodynamics of the excited hadronic matter is presented in this paper, for it is very appropriate for the solution of the hydrodynamical problem. In papers to follow this point will be treated in more detail with the account of separate resonances and conservation laws.

It is not clear now whether the "throughgoing" particle production is caused by the peripherical* collisions (and in this case the combined

*This word is often used in various meanings. Here it does not mean just the process with small inelastisity, but that corresponding to a diagram with one particle exchange.

peripheral-hydrodynamical model like /16/ is needed) or they take part in the intensive final state interaction similar to "newcreated" ones. So the study of the region close to the expansion front is of interest because the fastest particles are produced there. The solution in this region is not important for the spectra of the majority of secondaries and is not treated in this paper.

For the application of the model above the energy limit pointed out in section 2 it is necessary to obtain the solution of the hydrodynamical problem for more complicated than (8) equation of state. This problem can probably be solved only by the numerical calculations**.

The solution obtained shows that by the end of the hydrodynamical expansion the total size of the system is large and grows with energy. Thus, as it was pointed out by L.D.Landau the $p$-nucleus collisions are the collective processes similar to two-particle reactions rather than the internuclear cascade. The hydrodynamical model seems to be the most appropriate for the treatment of these reactions.

** If there appears many collision centres at superhigh energies, the model can be used for the description of separate centres ("fireballs"). They correspond to the collision energy of the order of 10-30 Gev for which the model gives rather good description of the data.



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R e f e r e n c e s


1. E.Fermi. Progr.Theor.Phys. 5, 510 (1950).
2. I.Ya.Pomeranchuck. "Doklady Akademii Nauk", 78, 889 (1951).
3. E.L.Feinberg. "Uspekhi fiz.nauk", 104, 539 (1971).
4. L.D.Landau. "Izvestia Akademii Nauk, ser.phys.", 17, 51 (1953).
5. R.Hagedorn, J.Runft. Suppl.Nuovo Cim. 6, 169 (1968).
6. E.Beth, G.E.Uhlenbeck. Physica 3, 729 (1936).
7. L.D.Landau, F.M.Lifshitz. Statistical physics, § 77 (Moscow 1964).
8. S.Z.Belenkij. "Uspekhi fiz.nauk" 62, 1 (1956).
9. R.Hagedorn. Suppl.Nuovo Cim. 3, 147 (1965).
10. I.M.Khalatnikov. JETP 26, 529 (1954).
11. G.A.Milekhin. JETP 35, 1185 (1958).
12. I.V.Allaby et al. High energy particle spectra from proton interactions at 19,2 Gev/c. Preprint CERN 70-12 (1970).
13. Yu.M.Antipov et al. Phys. Let. 34B, 164 (1971).
14. E.V.Shuryak. Phys.Letters. 34B, 509 (1971).
15. J.Bjorken, S.Brodsky. Phys.Rev. 1D, 1416 (1970).
16. E.L.Feinberg, D.S.Chernavsky. "Uspechi fiz.nauk", 82, 1(1964).
17. N.N.Biswas et al. Phys. Rev. Let. 26, 1589(1971).

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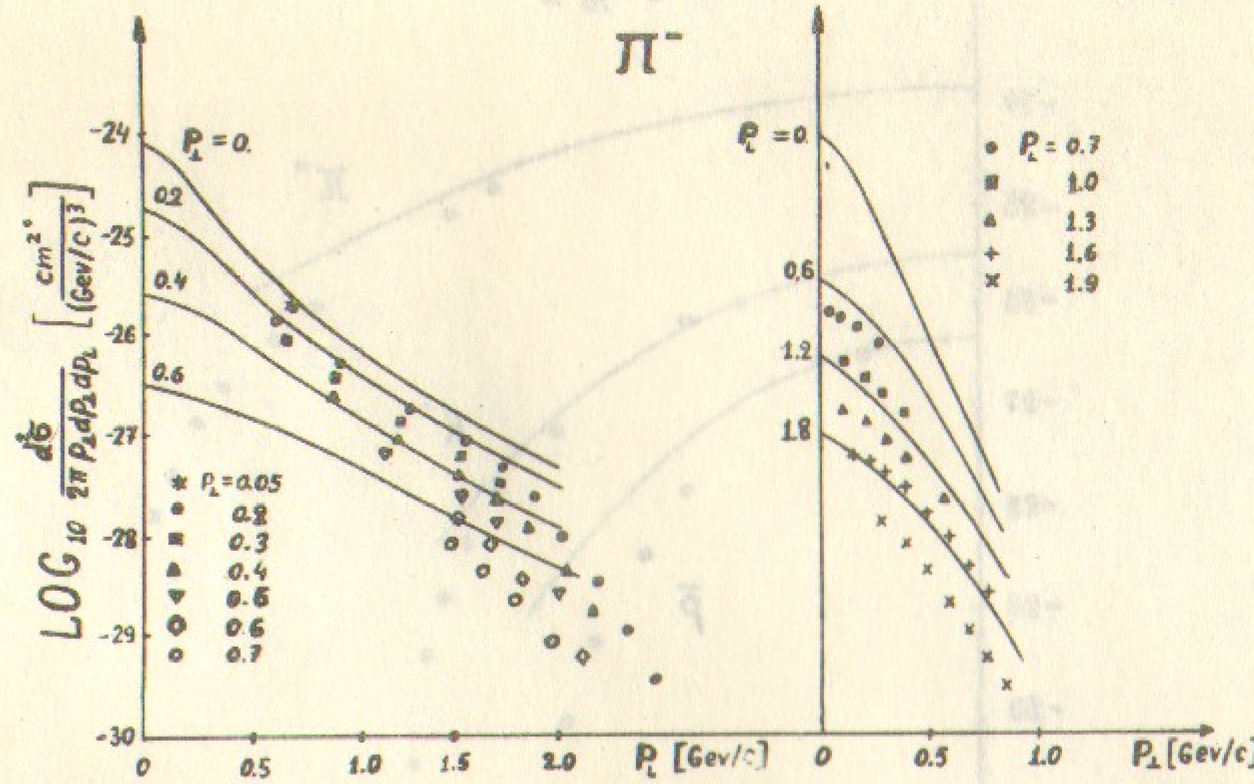
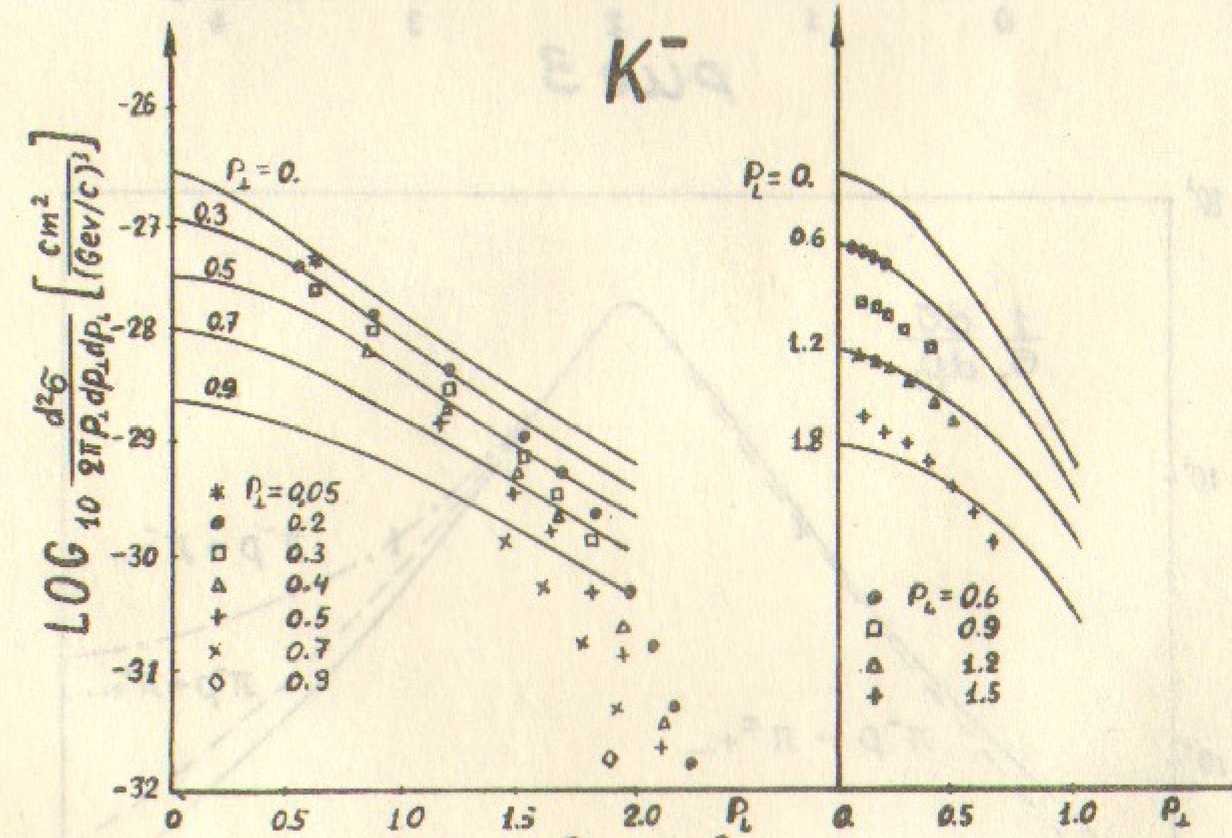



рис 1

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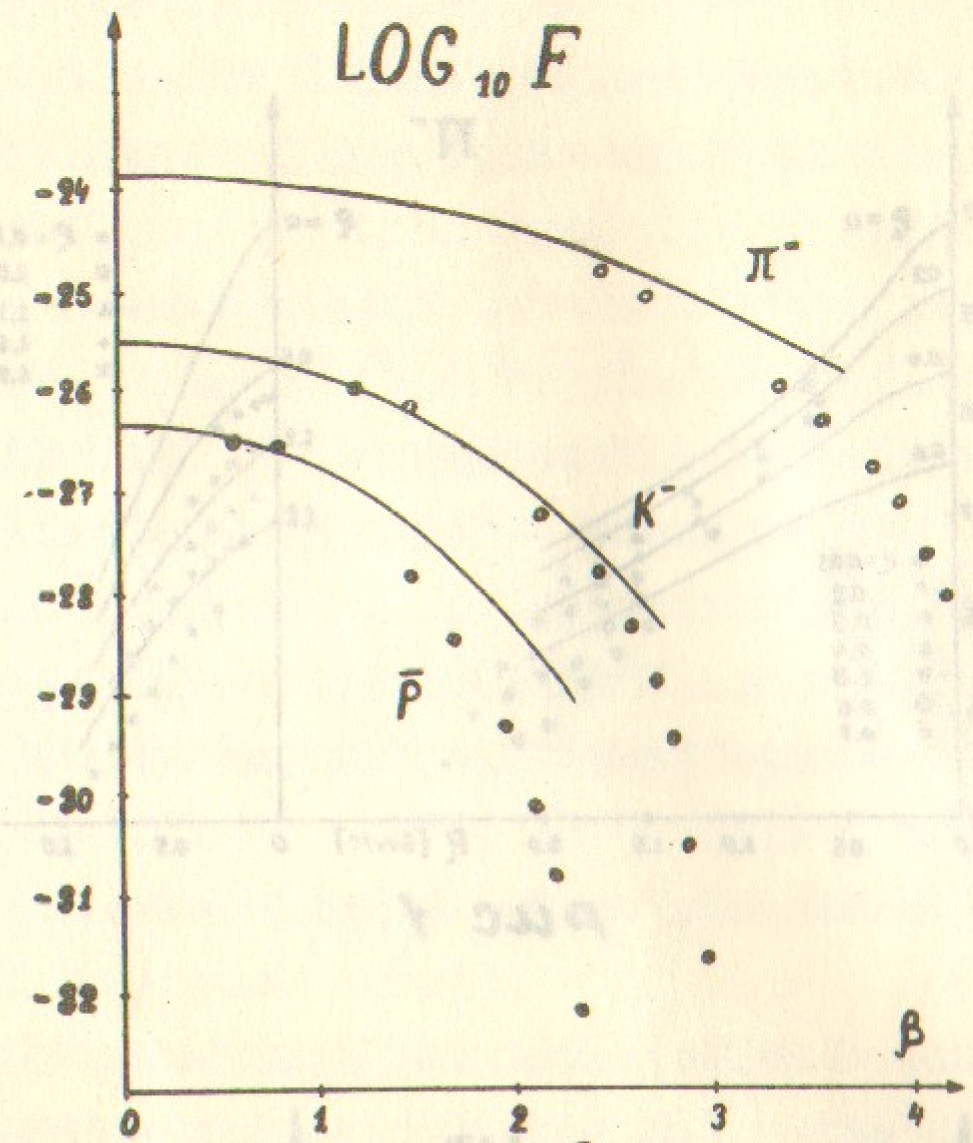
LOG 10 F
-24
-25
-26
-27
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π⁻
K⁻
p̄
β
0
1
2
3
4

рис 3

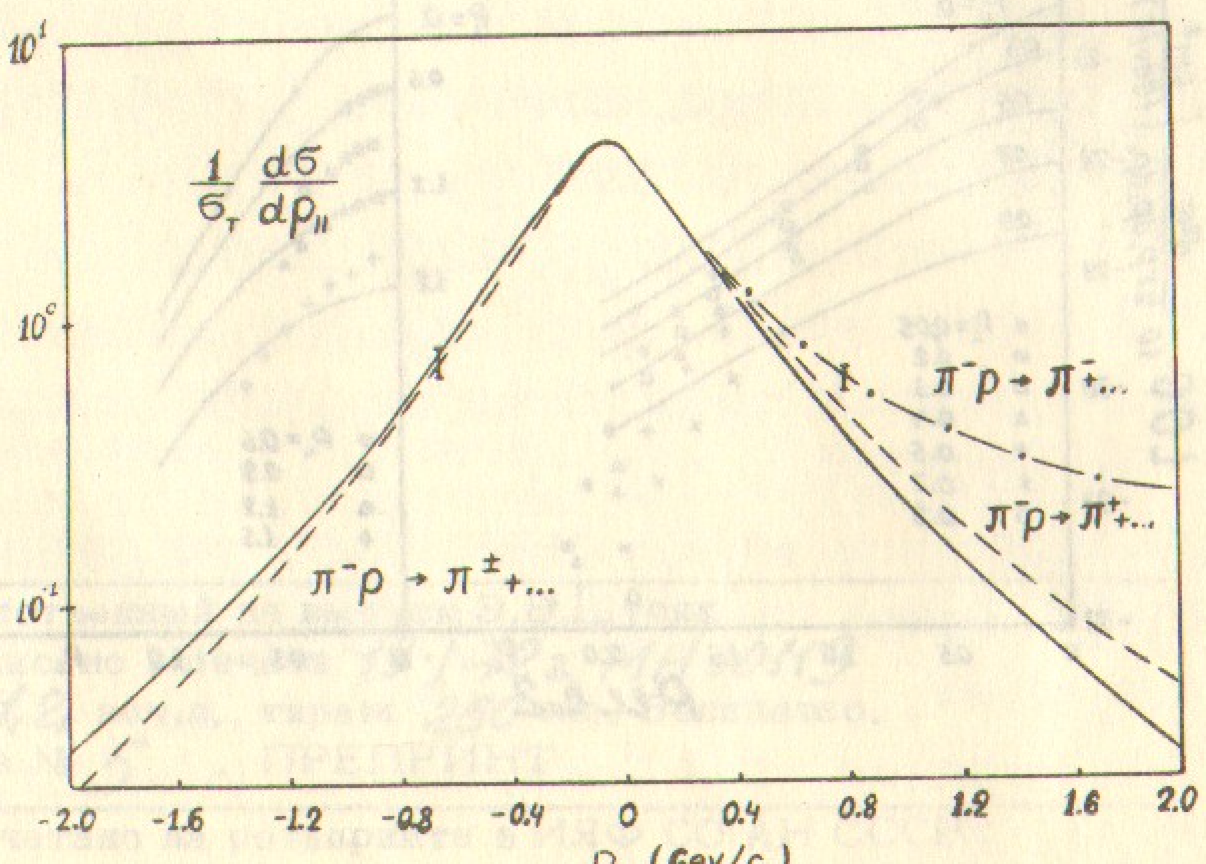
10¹
10⁰
10⁻¹
1/σ_T dσ/dp_∥
π⁻p → π⁻+...
π⁻p → π⁺+...
π⁻p → π±+...
-2.0
-1.6
-1.2
-0.8
-0.4
0
0.4
0.8
1.2
1.6
2.0
P_∥ (Gev/c)

рис 4