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abstract

Charge asymmetry in the angular distribution of muons produced by electron-positron collision is computed. The contribution of radiative corrections to the effect is found to be rather large and even at the energy E=4,5 GeV in every beam exceeds considerably the contribution of hypothetical weak interaction with coupling constant equal to the usual Fermi one.

The interest to the charge asymmetry in the angular distribution of muons produced by electron-positron collision has arisen because the investigation of this effect would allow to decide whether or not a weak interaction of electrons and muons of the kind $(\bar{e}e)(\bar{\mu}\mu)$ exists /1,2/. This experiment will become possible already in the near future, due to the construction of high-energy electron-positron storage rings and also of the equipment capable to determine the sign of the charge of produced particles.

Another qualitative effect indicating the existence of weak interaction would be longitudinal polarization of produced muons. However, the measurement of muon polarization /1-4/ seems to be more complicated experiment than the measurement of charge asymmetry in the angular distribution. On the other hand, the charge asymmetry, in distinction from longitudinal polarization, is induced by usual radiative corrections.

The discussed asymmetry arises because the muons are produced in the state that does not possess definite charge parity. The usual one-photon mechanism (see fig. 1) leads to the negative C -parity

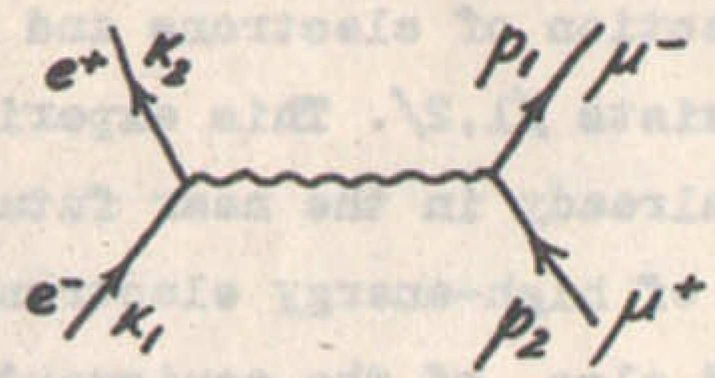


Fig.1

of the final state. The asymmetry in $\mu^+\mu^-$ angular distribution may arise due to the interference of this contribution to the matrix element of the process

$$M_o = \frac{e^2}{s}\bar{u}(k_2)\gamma_\mu u(k_1)\bar{v}(p_1)\gamma_\mu v(p_2) \qquad (1)$$

with that of weak interaction which we write down as

$$M_w = \frac{G}{\sqrt{2}}\bar{u}(k_2)\gamma_\mu(g_v+g_a\gamma_5)u(k_1)\bar{v}(p_1)\gamma_\mu(h_v+h_a\gamma_5)v(p_2) \qquad (2)$$

(The notations are given at fig.1; $s = (k_1+k_2)^2 = 4E^2$.) Other possible variants of weak interaction S,P and T are not considered here since their interference with M_o, due to chirality conservation, is proportional to $m_e m_\mu/E^2$ and is negligibly small in high-energy region we are interested in.

Standard calculation leads to the following differential cross-section of the reaction summed over polarizations of muons

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta - \xi_+\xi_-\sin^2\theta\cos 2\varphi + \frac{G g_a h_a s}{\sqrt{2}\pi\alpha}\cos\theta\right) \qquad (3)$$

Here θ is the angle between e^- and μ^- momenta, φ is the azimuthal angle counted off the storage ring magnetic field direction, ξ_+ and ξ_- are the degrees of radiative polarization of positrons and electrons correspondingly along and against the magnetic field. Weak interaction is taken into account here only to the first order in G and in so far as it leads to the charge asymmetry in the angular distribution. The distinction from the corresponding results of the works /1,2/ is in essence only in that here the possibility of radiative polarization of positrons and electrons in the storage ring /5/ is taken into account. Note that the question of the degree of particles' radiative polarization in real machines with colliding beams is open up to now.

The degree of charge asymmetry $\eta(\theta) = \frac{d\sigma(\theta)-d\sigma(\pi-\theta)}{d\sigma(\theta)+d\sigma(\pi-\theta)}$ cannot evidently depend on h_v since C -parity of weak muon vector current is negative as well as C -parity of electromagnetic current. Terms proportional to $g_v h_a$ (and also to $g_a h_v$) violate space parity and cannot at all show up in the cross-section under given conditions.

Indicate order of magnitude of the effect. At $G = G_F = 10^{-5} m_p^{-2}$, $g_a = h_a = 1$ and energy $E = 4{,}5$ GeV in every bunch $\eta(\pi/4) = 1{,}3\%$. In the presently popular Weinberg model/6/, independently of the mixing parameter magnitude, $G = -\frac{1}{2}G_F$, $g_a = h_a = 1$, so at the same energy $\eta(\pi/4) = -0{,}7\%$. (The sign of G is negative since we deal here with the local limit of intermediate vector boson theory.) At the luminosity 10^{31} cm^{-2} sec^{-1} the discussed effect observation becomes close to reality.

In this connection it is of interest to compute accurately radiative corrections that lead to charge asymmetry in the muon angular distribution also. Here the effect is due, firstly, to the interference between M_0 and the sum of "two-photon" contributions to the matrix element (see figs. 2,3) that describe

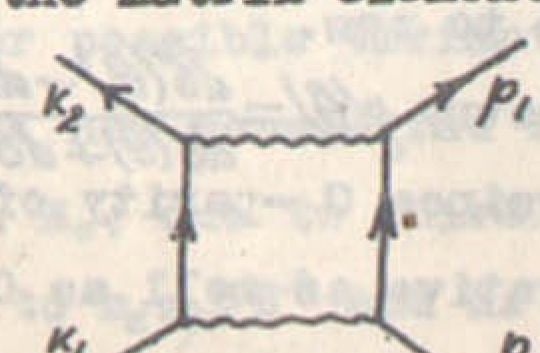


Fig.2

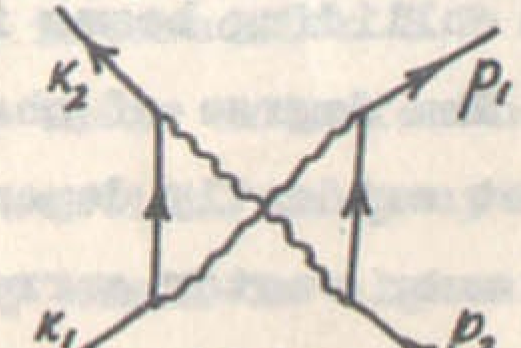


Fig.3

the production of $\mu^+\mu^-$ in the state with positive charge parity. Secondly, the asymmetry takes place in the process with soft quantum emission, this reaction being indistinguishable from the elastic one. In this case it arises due to the interference between the amplitude in which the soft quantum is emitted by initial particles, muons being in C-odd state (figs.4a,b), and the amplitude in which the γ-

Fig.4

quantum is emitted by muons, their final state being consequently C-even (figs.5a,b).

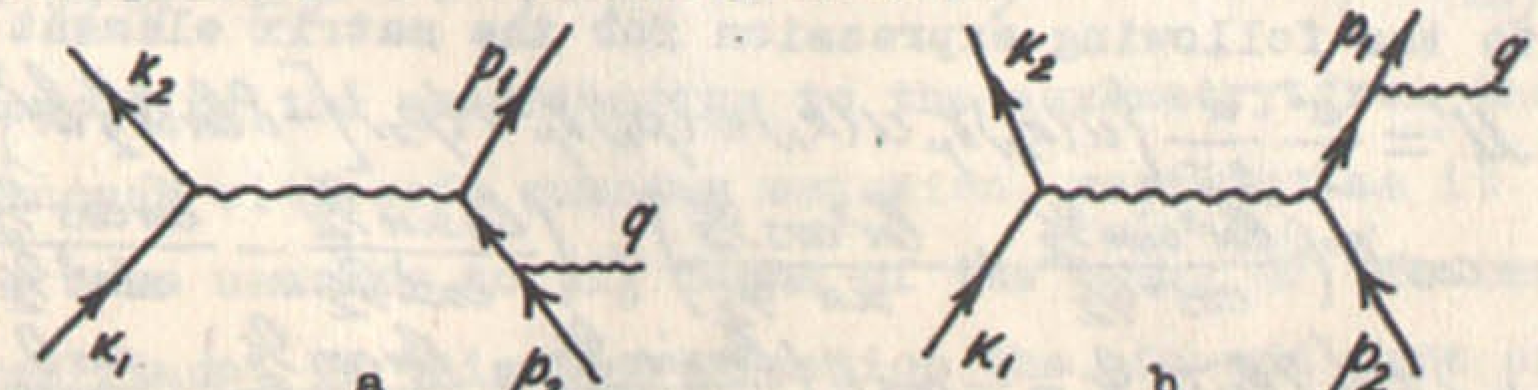


Fig.5

Begin with the computation of the "two-photon" amplitude. The matrix element corresponding to diagrams 2,3 is

$$\mathcal{M}' = \frac{ie^4}{(2\pi)^4}\int \frac{d^4q}{(q^2-\lambda^2)[(\kappa-q)^2-\lambda^2]}\bar{u}(k_2)\frac{\gamma_\mu(\hat{q}-\hat{k}_1)\gamma_\nu}{q^2-2k_1q}u(k_1)\times$$
$$\times\bar{v}(p_1)\left[\frac{\gamma_\mu(\hat{q}-\hat{p}_2)\gamma_\nu}{q^2-2p_2q}-\frac{\gamma_\nu(\hat{q}-\hat{p}_1)\gamma_\mu}{q^2-2p_1q}\right]v(p_2) \tag{4}$$

In connection with infra-red divergence the photon mass λ is introduced here. We restrict to the range of scattering angles θ not too close to 0 or π. At these angles the experimental observation of produced particles is perhaps more simple. This restriction allows us to neglect in the final expression for M′ all the masses (at intermediate stages of calculations it is sometimes convenient to retain them). There are no more approximations in the computation of M′.

Quite standard, also tedious, calculations lead to the following expression for the matrix element

$$\mathcal{M}' = \frac{e^2}{s}\frac{\alpha}{2\pi}\Big\{\bar{u}(k_2)\gamma_\mu u(k_1)\bar{v}(p_1)\gamma_\mu v(p_2)\Big[-8\ln tg\tfrac{\theta}{2}\ln\tfrac{2E}{\lambda}-$$
$$-\cos\theta\left(\frac{\ln^2\sin\theta/2}{\cos^4\theta/2}+\frac{\ln^2\cos\theta/2}{\sin^4\theta/2}\right)+\left(\frac{\ln\sin\theta/2}{\cos^2\theta/2}-\frac{\ln\cos\theta/2}{\sin^2\theta/2}\right)+$$
$$+2\pi i\left(2\ln tg\frac{\theta}{2}-\frac{1}{2}\cos\theta\left(\frac{\ln\sin\theta/2}{\cos^4\theta/2}+\frac{\ln\cos\theta/2}{\sin^4\theta/2}\right)-\frac{\cos\theta}{\sin^2\theta}\right)\Big]+ \tag{5}$$

$$+\bar{u}(k_2)\gamma_\mu\gamma_5 u(k_1)\bar{v}(p_1)\gamma_\mu\gamma_5 v(p_2)\Big[-\cos\theta\left(\frac{\ln^2\sin\theta/2}{\cos^4\theta/2}-\frac{\ln^2\cos\theta/2}{\sin^4\theta/2}\right)+$$
$$+\left(\frac{\ln\sin\theta/2}{\cos^2\theta/2}+\frac{\ln\cos\theta/2}{\sin^2\theta/2}\right)+2\pi i\left(-\frac{1}{2}\cos\theta\left(\frac{\ln\sin\theta/2}{\cos^4\theta/2}-\frac{\ln\cos\theta/2}{\sin^4\theta/2}\right)+\frac{1}{\sin^2\theta}\right)\Big]\Big\}$$

To computs the integrals the next parametrization is convenient to use

$$\frac{1}{abcd}=6\int_0^1 dx\int_0^1 dy\, y(1-y)\int_0^1 dz\,\big[(1-x)(1-y)a+$$
$$+y(1-z)b+x(1-y)c+yzd\big]^{-4} \tag{6}$$

Besides, the following identities, valid if one neglects particles' masses, proved to be useful

$$\bar{u}(k_2)\hat{p}_1u(k_1)\bar{v}(p_1)\hat{k}_1v(p_2)+\bar{u}(k_2)\hat{p}_1\gamma_5u(k_1)\bar{v}(p_1)\hat{k}_1\gamma_5v(p_2)=$$
$$=(p_1k_1)\big[\bar{u}(k_2)\gamma_\mu u(k_1)\bar{v}(p_1)\gamma_\mu v(p_2)+\bar{u}(k_2)\gamma_\mu\gamma_5u(k_1)\bar{v}(p_1)\gamma_\mu\gamma_5v(p_2)\big] \tag{7}$$

$$\bar{u}(k_2)\hat{p}_1u(k_1)\bar{v}(p_1)\hat{k}_1v(p_2)-\bar{u}(k_2)\hat{p}_1\gamma_5u(k_1)\bar{v}(p_1)\hat{k}_1\gamma_5v(p_2)=$$
$$=-(p_2k_1)\big[\bar{u}(k_2)\gamma_\mu u(k_1)\bar{v}(p_1)\gamma_\mu v(p_2)-\bar{u}(k_2)\gamma_\mu\gamma_5u(k_1)\bar{v}(p_1)\gamma_\mu\gamma_5v(p_2)\big] \tag{8}$$

Computing the contribution to the asymmetry from the process with soft quantum emission, restrict as it is done usually to the terms of the order ω^{-1} in the amplitude. In this approximation the C-odd part of the squared amplitude of the process, summed over the photon polarization and integrated over its momentum, is

$$|M_0|^2 \frac{\alpha}{\pi^2}\int \frac{d\vec{q}}{q_0}\left[\frac{(p_1k_1)}{(p_1q)(k_1q)} - \frac{(p_1k_2)}{(p_1q)(k_2q)}\right] \qquad (9)$$

Here $q_0 = \sqrt{\vec{q}^2+\lambda^2}$ is the γ-quantum energy; the integration range is restricted by the condition $0 \leqslant |\vec{q}| \leqslant \Delta E$ where ΔE is the maximal permissible energy of Bremsstrahlung quantum for which the process with its emission is still indistinguishable experimentally from the elastic one.

The integral is computed in the same way as it is done in the book /7/ to calculate soft quantum Bremsstrahlung. Then after some additional transformation (9) is reduced to the next form

$$|M_0|^2 \frac{4\alpha}{\pi}\left\{2\ln tg\frac{\theta}{2}\ln\frac{2\Delta E}{\lambda} + \ln^2\sin\frac{\theta}{2} - \ln^2\cos\frac{\theta}{2} - g(\theta)\right\} \qquad (10)$$

Values of the function

$$g(\theta) = \sin\frac{\theta}{2}\int_{\cos\frac{\theta}{2}}^{1}\frac{dv}{\sqrt{v^2-\cos^2\frac{\theta}{2}}}\left(\frac{\ln\frac{1+v}{2}}{1-v} - \frac{\ln\frac{1-v}{2}}{1+v}\right) - \cos\frac{\theta}{2}\int_{\sin\frac{\theta}{2}}^{1}\frac{dv}{\sqrt{v^2-\sin^2\frac{\theta}{2}}}\left(\frac{\ln\frac{1+v}{2}}{1-v} - \frac{\ln\frac{1-v}{2}}{1+v}\right) \qquad (11)$$

are presented in the table. The integral through which it is expressed was investigated in connection with the calculation of radiative corrections to electron scattering by external field /8/.

TABLE

θ	$-g(\theta) = g(\pi-\theta)$
0.0	0.82
0.1	0.81
0.2	0.79
0.3	0.76
0.4	0.72
0.5	0.67
0.6	0.62
0.7	0.57
0.8	0.51
0.9	0.45
1.0	0.39
1.1	0.32
1.2	0.26
1.3	0.19
1.4	0.12
1.5	0.05
$\pi/2$	0.00

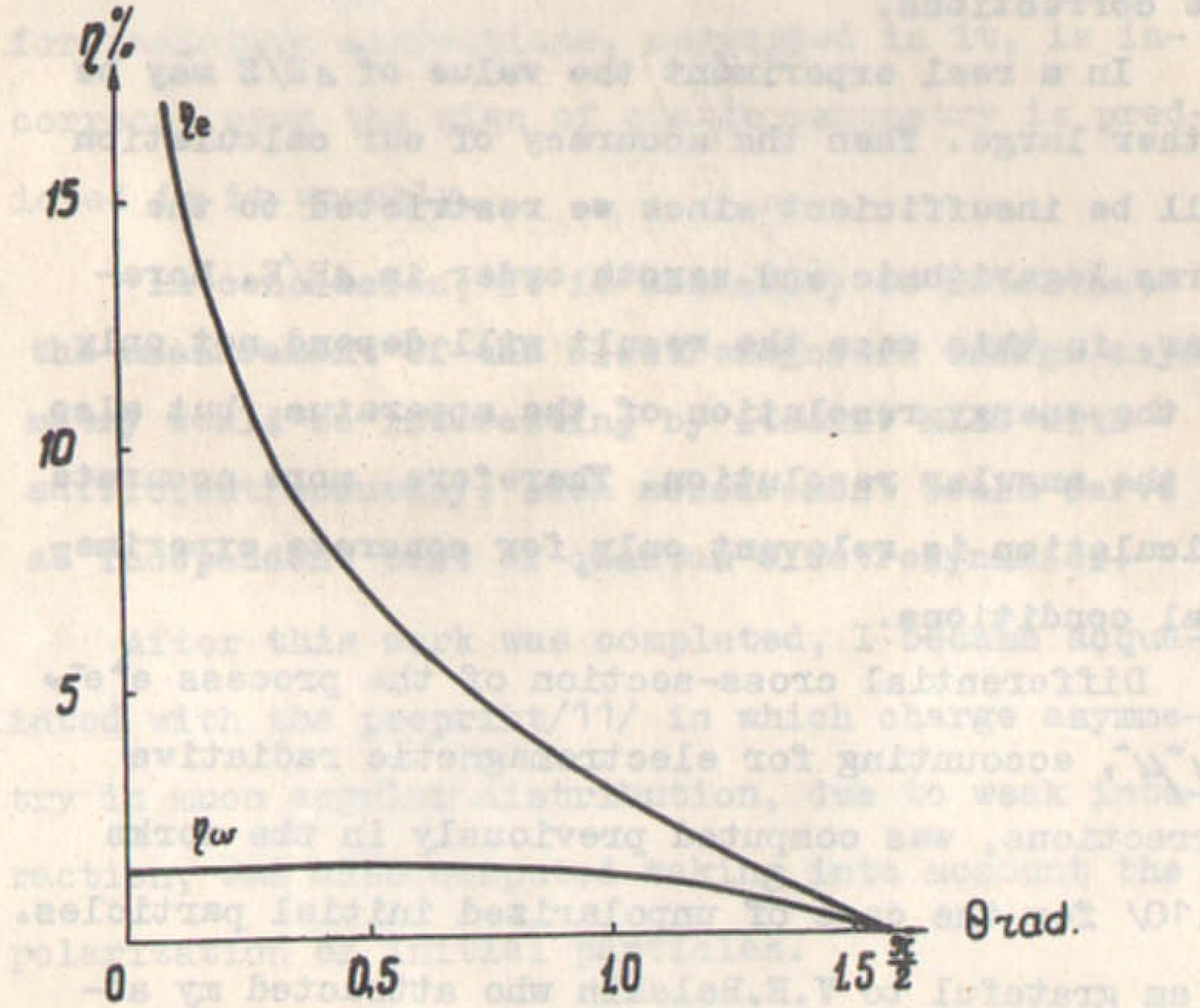
Taking into account the asymmetry induced both by radiative corrections and by weak interaction, the differential cross-section of the process $e^+e^- \to \mu^+\mu^-$ is

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\Big\{\Big(1+\cos^2\theta - \xi_+\xi_-\sin^2\theta\cos 2\varphi\Big)\Big[1 - \frac{4\alpha}{\pi}\Big(2\ln tg\frac{\theta}{2}\ln\frac{E}{\Delta E} + \\
+ g(\theta) - \ln^2\sin\frac{\theta}{2}\Big(1-\frac{\cos\theta}{4\cos^4\frac{\theta}{2}}\Big) + \ln^2\cos\frac{\theta}{2}\Big(1+\frac{\cos\theta}{4\sin^4\frac{\theta}{2}}\Big) - \\
- \frac{1}{4}\Big(\frac{\ln\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}} - \frac{\ln\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}\Big)\Big)\Big] + \cos\theta\Big[\frac{G g_a h_a s}{\sqrt{2}\pi\alpha} - \\
- \frac{2\alpha}{\pi}\Big(\cos\theta\Big(\frac{\ln^2\sin\frac{\theta}{2}}{\cos^4\frac{\theta}{2}} - \frac{\ln^2\cos\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\Big) - \Big(\frac{\ln\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}} + \frac{\ln\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}\Big)\Big)\Big]\Big\}
\end{aligned}
\tag{12}
$$

This expression simplifies considerably if one either neglects the polarization of initial particles, or integrates it over the azimuthal angle φ:

$$
\begin{aligned}
\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\Big\{1+\cos^2\theta + \frac{4\alpha}{\pi}\Big[-(1+\cos^2\theta)\Big(2\ln tg\frac{\theta}{2}\ln\frac{E}{\Delta E} + \\
+ g(\theta)\Big) + (1-\cos\theta+\cos^2\theta)\ln^2\sin\frac{\theta}{2} - \\
- (1+\cos\theta+\cos^2\theta)\ln^2\cos\frac{\theta}{2} + \cos^2\frac{\theta}{2}\ln\sin\frac{\theta}{2} - \\
- \sin^2\frac{\theta}{2}\ln\cos\frac{\theta}{2}\Big] + \frac{G g_a h_a s}{\sqrt{2}\pi\alpha}\cos\theta\Big\}
\end{aligned}
\tag{13}
$$

The angular dependence of the degree of charge asymmetry $\eta_e(\theta)$ caused by radiative corrections at $\Delta E/E = 0.1$ is presented at the diagram. The effect turns out to be rather large numerically and depends



on the energy only via ratio $\Delta E/E$. For comparison at the same figure the charge asymmetry $\eta_w(\theta)$ caused by weak interaction at $E = 4.5$ GeV and $Gg_a h_a = G_F$ is presented. The weak interaction effect turns out to be considerably smaller than that of radiative corrections.

In a real experiment the value of $\Delta E/E$ may be rather large. Then the accuracy of our calculation will be insufficient since we restricted to the terms logarithmic and zeroth order in $\Delta E/E$. Moreover, in this case the result will depend not only on the energy resolution of the apparatus, but also on the angular resolution. Therefore, more accurate calculation is relevant only for concrete experimental conditions.

Differential cross-section of the process $e^+e^- \to \mu^+\mu^-$, accounting for electromagnetic radiative corrections, was computed previously in the works /9,10/ for the case of unpolarized initial particles. (I am grateful to V.E.Balakin who attracted my attention to these papers.) Our results do not agree with those of the article/9/. It cannot be excluded however that the discrepance is caused by misprints

in the paper/9/. It is impossible to compare the numerical results since in the work/9/ they are presented only for energies comparable with muon mass at which our formulae are inapplicable. As for the article/10/, the very structure of the expression for radiative corrections, suggested in it, is incorrect; even the sign of charge asymmetry is predicted in it wrongly.

In conclusion, it is necessary to note that the measurement of the electromagnetic charge asymmetry would be interesting by itself. Made with sufficient accuracy, such measurement would serve as independent test of quantum electrodynamics.

After this work was completed, I became acquainted with the preprint/11/ in which charge asymmetry in muon angular distribution, due to weak interaction, was also computed taking into account the polarization of initial particles.

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