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TO THE QUESTION OF THE NONCAUSAL MOTION OF THE PARTICLES OF HIGH SPIN IN AN EXTERNAL FIELD

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The quasiclassical description of the influence of anomalous magnetic and quadrupole moments on the motion of a particle in an external electromagnetic field is constructed. For the vector particle both anomalous interactions may lead to velocities exceeding the speed of light if the classical radiation is not taken into account. The existence of self-consistent description of interaction with an external electromagnetic field for charged particle with spin 2 is pointed.

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1. Introduction

Non-renormalizability of quantum electrodynamics of the particles with spin $\geqslant 1$ is well known. The anomalies arising for the interaction of such particles with external electromagnetic field are less known. In 1961 Johnson and Sudarshan[1] turned attention to the contradictions that occur in this case under the quantization of relativistic wave equation for spin 3/2. Then Velo and Zwanziger [2,3] noted that relativistic wave equations for spin 1 with anomalous quadrupole moment and for spin 3/2 may lead to the motion with velocities exceeding the speed of light in an external field. Of course, the question whether such noncausality is retained when one takes into account the radiative corrections remains open. Moreover, the classical radiation of a particle that turns to infinity when its velocity approaches the speed of light does not allow the particle to reach superluminal velocities even under the classical consideration of motion[4]. Nevertheless, it seems that the problem of the motion of the particle with high spin in the external field is of some methodical interest, at least, as an example of relativistically invariant equations leading to noncausality.

In this paper I consider in quasiclassical approximation the motion in external field of a particle of spin 1/2 with anomalous magnetic moment and a vector particle with anomalous magnetic and quadrupole moments. Such a problem was considered previously in a number of works /5-10/. The equations of motion obtained in this paper differ from those considered in works /5-9/. The essence of these contradictions and their origin are discussed in detail below. As to work /10/ our results overlap partly and in the region of overlapping are in complete agreement.

For calculation I consider anomalous moments of particles to be numerically large, that allows to take them into account even in zeroth quasi-classical approximation. This condition is necessary not only to simplify computation. It is of principal importance because it allows to account for the interaction of multipole moments of a particle with an external field neglecting for all that the wave properties of a particle, that is, retaining the notion of the trajectory of motion.

It appears that the interaction of both magnetic and quadrupole anomalous moments of a vector particle with an external field changes the relation between its energy and momentum in such a way that superluminal velocities become possible. The curious effect found recently in the works /11,12/ which consists in that the solution of a stationary problem for a vector particle with anomalous magnetic moment in a homogeneous magnetic field may lead to an imaginary energy is discussed. The concrete example of motion with superluminal velocity for a particle with anomalous quadrupole moment is constructed.

Then the paper deals with the classical radiation of a particle with anomalous moments moving in an external field. The turning to infinity of radiation intensity when the particle velocity approaches the speed of light makes it impossible to obtain superluminal velocities, at least, in the frame of classical consideration, this result being in general agreement with the conclusions of the work/4/.

The interaction with an external field of particles with spin larger then unity is considered briefly; the question about their magnetic moments is discussed. The relation between quasiclassical approximation and the perturbation theory where the noncausality is absent is considered.

2. The particle with anomalous magnetic moment in an external field

Let us consider the case of a particle with spin 1/2. Dirac equation with anomalous magnetic moment may be written as

$$\left[\gamma_\mu \left(i\hbar\partial_\mu - \frac{e}{c}A_\mu\right) - i\frac{\mu}{2c}\sigma_{\mu\nu}F_{\mu\nu} - mc\right]\psi = 0 \quad (1)$$

(Feynman metric is used, $\sigma_{\mu\nu} = \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$; the other notations are obvious). The quasiclassical solution is being looked for as $\psi = a \exp(iS/\hbar)$. Assumption $g \gg 1$ (g - factor is introduced, as usually, by the relation $\mu = \frac{e\hbar}{2mc} s g$ where s is the spin of a particle) allows, as it was noted in the introduction, to take into account magnetic moment in the zeroth approximation. In this approximation we obtain the following set of equations for the spinor a

$$(\gamma_\mu u_\mu - \frac{i}{2}\sigma_{\mu\nu} f_{\mu\nu} - 1)a = 0$$

in which dimensionless variables are introduced

$$u_\mu = \frac{1}{mc}(-\partial_\mu S - \frac{e}{c}A_\mu), \quad f_{\mu\nu} = \frac{\mu}{mc^2} F_{\mu\nu}.$$

To obtain the characteristic equation of this set it is convenient to use Weil representation for γ-matrices: $\gamma_0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$, $\bar{\gamma} = \begin{pmatrix} 0 & \bar{\sigma} \\ -\bar{\sigma} & 0 \end{pmatrix}$. Some tedious calculations lead to the following equation:

$$(u_0^2 - \bar{u}^2 - 1)^2 - 2\{(u_0^2 + \bar{u}^2)(\bar{e}^2 + \bar{h}^2) - 2[(\bar{u}\bar{e})^2 + (\bar{u}\bar{h})^2]$$
$$- 4u_0\bar{u}\cdot[\bar{e}\times\bar{h}]\} - 2(\bar{h}^2 - \bar{e}^2) + (\bar{h}^2 - \bar{e}^2)^2 + 4(\bar{e}\bar{h})^2 = 0 \qquad (2)$$

or in covariant form

$$(u^2-1)^2-(u^2+1)f_{\mu\nu}f_{\mu\nu}+4u_\mu u_\nu f_{\mu\lambda}f_{\nu\lambda}+ \\ +\frac{1}{4}(f_{\mu\nu}f_{\mu\nu})^2+\frac{1}{4}(f_{\mu\nu}\tilde{f}_{\mu\nu})^2=0 \tag{2a}$$

Here $\bar{e}=\frac{\mu\bar{E}}{mc^2}$, $\bar{h}=\frac{\mu\bar{H}}{mc^2}$, $\tilde{f}_{\mu\nu}=\frac{1}{2}\varepsilon_{\mu\nu\varkappa\lambda}f_{\varkappa\lambda}$.

Eqs.(2) and (2a) are the Hamilton-Jacobi equations for a particle with spin 1/2 and large magnetic moment. By the way, it follows from them that u_μ does not coincide with 4-dimensional velocity the square of which is equal to unity by definition; i.e., not only the canonical momentum $-\partial_\mu S$, but also the kinetic one $-\partial_\mu S-\frac{e}{c}A_\mu$ are not parallel to 4-velocity.

In the approximation $|f_{\mu\nu}|\ll 1$ it is easy to express u_0 through $\bar{u}$, $\bar{e}$, $\bar{h}$ from eq.(2). Small addition to the usual expression $u_0=\sqrt{\bar{u}^2+1}$ corresponds to the interaction of the magnetic moment with an external field. It gives the Lagrangian of this interaction if being taken with opposite sign and written in usual units

$$\Delta L=\pm\mu\sqrt{\bar{H}^2-\left(\frac{\bar{v}}{c}\bar{H}\right)^2+\frac{\bar{v}^2}{c^2}\bar{E}^2-\left(\frac{\bar{v}}{c}\bar{E}\right)^2-2\frac{\bar{v}}{c}[\bar{E}\times\bar{H}]} \tag{3}$$

Here, $\bar{v}$ is the three-dimensional velocity of the particle.

The obtained approximate Lagrangian (3) has a simple interpretation. In the particle's rest frame the Lagrangian of interaction of its magnetic moment with an external field is well known:

$$\Delta \mathcal{L}' = \pm \mu |\bar{H}'| \tag{4}$$

Here and further the values that refer to the rest frame are denoted by primes. The signs $\pm$ correspond to the states with the spin projection $\pm 1/2$ on the direction of the magnetic field $\bar{H}'$. The large value of the magnetic moment ($g \gg 1$) allows to neglect at writing $\Delta \mathcal{L}'$ the non-inertiality of the rest frame which is due to the interaction of a particle charge with an external field, that is, to the effect of Thomas precession /13,14/. Lagrangian $\Delta \mathcal{L}$ is obtained from eq.(4) if $\bar{H}'$ is expressed through fields $\bar{E}$ and $\bar{H}$ in the laboratory frame and if the factor $\gamma^{-1} = (1 - v^2/c^2)^{1/2}$, taking into account the transition to own time from the laboratory one, is introduced.

Note, that if proper time is retained, that is, factor γ^{-1} is not inserted and expression for $|\bar{H}'|$ through $\bar{E}$ and $\bar{H}$ is written in 4-dimensional notations, then covariant Lagrangian of interaction

$$\Delta \tilde{\mathcal{L}} = \pm \mu \sqrt{\frac{1}{2} \mathcal{F}_{\mu\nu} \mathcal{F}_{\mu\nu} - u_\mu u_\nu \mathcal{F}_{\mu\lambda} \mathcal{F}_{\nu\lambda}} \tag{3a}$$

is obtained in which u_μ may be considered with the taken accuracy as 4-dimensional velocity.

Thus, the bunch of particles with spin 1/2 and with the large magnetic moment in an external field is divided into two bunches with opposite polarizations which are moving along different trajectories and the transitions between these two states in the considered approximation are absent. In the case of non-relativistic particles and inhomogeneous magnetic field it is usual Stern-Gerlach effect which becomes observable for the charged particles due to the condition $g \gg 1$.

The description of the effect of the magnetic moment of a particle with spin 1/2 on its trajectory which is given by approximate Lagrangian (3) coincides with the description of this effect obtained in the work /10/ in another way if Thomas precession is neglected. But these results contradict the equations of motion suggested in /5-9/. E.g. in our description the trajectory of relativistic particle is effected by the interaction of its magnetic moment even with a constant homogeneous field; it is evident from the Lagrangian (3) non-linear dependence on a particle velocity. However, in the equations of motion given in works /5 / the force corresponding to magnetic moment depends only on the field derivatives and for the constant fields turns to zero. The relativistic equations of motion are obtained in works /5 / by means of covariant generalization of certainly correct non-relativistic equations. But it is clear that the equations of motion resulting from (3) or (3a) may be obtained in the same way. Thus, it is obvious that such generalization is not unique. The defect of classical relativistic equations of motion obtained in /5-9/ is that they do not correspond to quasiclassical limit of any relativistic wave equation.

In some special cases Hamilton-Jacobi equation (2) may be exactly solved with respect to u_0 and closed expression for the Hamiltonian of a particle may be obtained. E.G., for the motion in magnetic field the Hamiltonian is equal to

$$\mathcal{H} = \sqrt{\left(c\sqrt{\bar{\bar{\pi}}_\perp^2 + m^2c^2} \mp \mu|\bar{H}'|\right)^2 + c^2\bar{\bar{\pi}}_\parallel^2}, \quad \bar{\pi} = \bar{p} - \frac{e}{c}\bar{A} \tag{5}$$

Here indices $\perp$ and $\parallel$ mark the components of vectors perpendicular and parallel to magnetic field $\bar{H}$, respectively. It is curious that at $\bar{\bar{\pi}}_\parallel \neq 0$ and $\mu|\bar{H}'| = c\sqrt{\bar{\bar{\pi}}_\perp^2 + m^2c^2}$ the velocity of the particle with one of polarizations is equal to the speed of light. In the special case of motion in the plane orthogonal to the homogeneous magnetic field the Hamiltonian (5) is reduced to

$$\mathcal{H} = c\sqrt{\bar{\bar{\pi}}^2 + m^2c^2} \mp \mu|\bar{H}| \tag{5a}$$

Let us consider now the effect of a large magnetic moment on a trajectory of a vector particle. The quasiclassical solution of Proca equation with anomalous magnetic moment

$$-\pi_\mu \pi_\mu b_\nu + \pi_\mu \pi_\nu b_\mu + m^2c^2 b_\nu + 2im\mu F_{\nu\mu} b_\mu = 0 \qquad (6)$$

$$\pi_\mu = i\hbar\partial_\mu - \frac{e}{c}A_\mu$$

is being looked for as $b_\nu = a_\nu \exp(iS/\hbar)$. In the zeroth approximation we obtain the following set of linear equations for vector a_μ

$$(1-u^2)a_\nu + u_\nu u_\mu a_\mu + 2i f_{\nu\mu} a_\mu = 0 \qquad (7)$$

The characteristic equation of this set, that is, Hamilton -Jacobi equation, is as follows

$$(u_0^2-\bar{u}^2-1)^3 - 4(u_0^2-\bar{u}^2-1)\{(u_0^2-1)\bar{e}^2+(\bar{u}^2+1)\bar{h}^2-$$

$$-(\bar{u}\bar{e})^2-(\bar{u}\bar{h})^2-2u_0\bar{u}\cdot[\bar{e}\times\bar{h}]\} + 16(\bar{e}\bar{h})^2 = 0, \qquad (8)$$

or in covariant form

$$(u^2-1)^3+2(u^2-1)\{2u_\mu u_\nu f_{\mu\lambda} f_{\nu\lambda} - f_{\mu\nu}f_{\mu\nu}\} + (f_{\mu\nu}\tilde{f}_{\mu\nu})^2 = 0 \qquad (8a)$$

In the approximation $|f_{\mu\nu}| \ll 1$ one may, as well as in the case of spin 1/2, obtain explicitly the Lagrangian of the interaction of magnetic moment with an external field. For the particles with spin projections ± 1 on the direction of a magnetic field in its rest frame it coincides with (3) or (3a). For the particles with zero projection of spin it is equal to zero. The interpretation of these results is the same, of course, as in the case of spin 1/2.

Again in some special cases the exact expression for the Hamiltonian of a particle may be obtained from Hamilton-Jacobi equation (8). In particular, the motion in the plane orthogonal to a homogeneous magnetic field is described now by Hamiltonian

$$\mathcal{H}_s = \left[c^2(\bar{\bar{\pi}}^2 + m^2c^2) + 2s\mu Hc\sqrt{\bar{\bar{\pi}}^2 + m^2c^2}\right]^{1/2} \qquad (9)$$

where spin quantum number $s = -1, 0, 1$, depending on a particle polarization. This Hamiltonian at $s = \pm 1$ coincides with the corresponding Hamiltonian (5a) for spin 1/2 but only in the first order in μ.

The velocity of a particle described by Hamiltonian (9) is

$$\bar{v}_s = \frac{\partial \mathcal{H}_s}{\partial \bar{p}} = \frac{\partial \mathcal{H}_s}{\partial \bar{\pi}} = \frac{c\bar{\bar{\pi}}}{\sqrt{\bar{\bar{\pi}}^2 + m^2c^2}} \, \frac{\sqrt{\bar{\bar{\pi}}^2 + m^2c^2} + s\mu H/c}{\sqrt{\bar{\bar{\pi}}^2 + m^2c^2 + 2s\mu \frac{H}{c}\sqrt{\bar{\bar{\pi}}^2 + m^2c^2}}} \qquad (10)$$

Its module

$$v_s = c\left[\left(1 - \frac{m^2c^2}{\bar{\bar{\pi}}^2 + m^2c^2}\right)\left(1 + \frac{s^2\mu^2H^2/c^2}{\bar{\bar{\pi}}^2 + m^2c^2 + 2s\mu H/c\sqrt{\bar{\bar{\pi}}^2 + m^2c^2}}\right)\right]^{1/2} \qquad (10a)$$

for $\varsigma = \pm 1$ in the case of sufficiently strong fields may exceed the speed of light. Note, that superluminal velocity becomes possible in spite of the relativistic invariance of both initial wave equation (6) and quasiclassical approximation to it, that is, the Hamilton -Jacobi equation (8a).

Besides, at $\varsigma = -1$ and $2\mu H > c\sqrt{\overline{\overline{\pi}}^2 + m^2c^2}$ the Hamiltonian (9) becomes imaginary at all. This corresponds to the effect found in works /11,12/. But, as it is seen from (10), with the increase of the magnetic field the velocity will exceed the speed of light even before it, passing the infinite value, becomes imaginary together with energy if $\overline{\overline{\pi}}$ does not vanish. If we neglect noncausality then the appearance of imaginary energy is not something without precedent in quantum mechanics. Consider, for example, the inverted oscillator, that is a non-relativistic particle in the potential $-\frac{1}{2}m\omega^2x^2$. If, in spite of the infiniteness of the stationary motion, we shall look for the solution of Schroedinger equation infinity then we obtain evidently the imaginary proper value of energy $-i\hbar\omega(n+\frac{1}{2})$. The meaning of this result is clear: the localized wave packet exponentially decreases in time due to the infiniteness of the stationary motion. In the problem under discussion at $2\mu H > c\sqrt{\overline{\overline{\pi}}^2 + m^2c^2}$ the motion becomes also infinite; one can easily ascertain it by consideration, e.g., the quasiclassical equations of motion (under this condition the Larmor frequency becomes imaginary). Therefore, here the solution at infinity is a wave packet decreasing exponentially in time.

I present also the exact Hamiltonian for the motion of a vector particle along the magnetic field (e.g., along the axis of solenoid of finite length)

$$\mathcal{H}_s = c\sqrt{p^2 + m^2c^2 + 2s\mu H(x)m} \quad (9a)$$

At $2\mu H > mc^2$ for $s = -1$ the velocity $v = \frac{\partial \mathcal{H}_-}{\partial p}$, apparently, exceeds c.

Note in conclusion of this section that as the noncausality arises for the interaction of the magnetic moment only in the second order in an external field then it may disappear after inclusion of the terms of higher order in the external field into the initial wave equation. Just in such a way the real energy /12/, that is, the finiteness of motion, may be retained for a vector particle.

3. Vector particle with anomalous quadrupole moment in an external field

Proca equation for a particle with anomalous quadrupole moment

$$-\pi_\mu \pi_\mu b_\nu + \pi_\mu \pi_\nu b_\mu + m^2c^2 b_\nu - \tag{11}$$

$$-\frac{Q}{2c}\left[\pi_\mu b_\lambda \partial_\nu \mathcal{F}_{\mu\lambda} + \pi_\mu (b_\lambda \partial_\lambda \mathcal{F}_{\mu\nu})\right] = 0$$

after substitution $b_\nu = a_\nu \exp(iS/\hbar)$
leads in quasiclassical approximation to the following set of equations

$$(1-u^2)a_\nu + u_\nu u_\mu a_\mu + \varphi_{\nu\mu} a_\mu = 0 \tag{12}$$

where $\varphi_{\nu\mu} = \frac{Q}{2mc^2}(\partial_\nu \mathcal{F}_{\mu\lambda} + \partial_\mu \mathcal{F}_{\nu\lambda})u_\lambda$. The characteristic equation of this set

$$(u^2-1)^3 + (u^2-1)(u_\mu u_\nu \varphi_{\mu\lambda}\varphi_{\nu\lambda} - \frac{1}{2}\varphi_{\mu\nu}\varphi_{\mu\nu}) +$$
$$+(u_\mu \varphi_{\mu\nu}\varphi_{\nu\lambda}\varphi_{\lambda\varkappa}u_\varkappa - \frac{1}{3}\varphi_{\mu\nu}\varphi_{\nu\lambda}\varphi_{\lambda\mu}) +$$
$$+\frac{1}{8}\left[2\varphi_{\mu\nu}\varphi_{\nu\lambda}\varphi_{\lambda\varkappa}\varphi_{\varkappa\mu} - (\varphi_{\mu\nu}\varphi_{\mu\nu})^2\right] = 0 \tag{13}$$

is the Hamilton-Jacobi equation for a vector particle with anomalous quadrupole moment. While deducing this equation it is supposed for simplicity that in the considered region the sources of the external field are absent, so that $\varphi_{\mu\mu} = 0$.

If limiting oneself to the terms leading in $\varphi_{\mu\nu}$, that is, in Q, and taking also $|u_\mu| \gg 1$ we reduce eq.(13) to

$$(u_0^2-1)^3+(u_0^2-1)u_\mu u_\nu \varphi_{\mu\lambda}\varphi_{\nu\lambda}=0 \qquad (13a)$$

Solving in this approximation eq.(13a) in respect to u_0 one may construct the approximate Hamiltonian of a particle

$$\mathcal{H}_s = c\sqrt{\bar{\bar{\pi}}^2+m^2c^2}\left\{1+\frac{sQ}{4mc^3}\left(\frac{d\bar{E}}{dt}-\bar{\nu}\left(\bar{\nu}\frac{d\bar{E}}{dt}\right)+\bar{\nu}\times\frac{d\bar{H}}{dt}\right)\right\} \qquad (14)$$

Here

$$\bar{\nu}=\bar{\bar{\pi}}/|\bar{\bar{\pi}}|, \quad \frac{d}{dt}=\frac{\partial}{\partial t}+c(\bar{\nu}\cdot\bar{\nabla})$$

The obtained quadrupole addition to Hamiltonian permits clear interpretation. In the proper system of a particle the Hamiltonian of interaction of its quadrupole moment with an external field is

$$\Delta\hat{\mathcal{H}}' = -\frac{1}{6}\hat{Q}_{mn}\partial'_m E'_n = -\frac{Q}{4}\left(\hat{S}_m\hat{S}_n+\hat{S}_n\hat{S}_m-\frac{2}{3}\delta_{mn}\hat{S}^2\right)\partial'_m E'_n \qquad (15)$$

For a vector particle using the explicit form of the angular momentum operator $\hat{S}_m=-i\varepsilon_{m\kappa l}$ (indices κ, l = 1,2,3 refer to the space of spin wave functions)and supposing that, as well as in eq.(13), $\partial'_m E'_m$ = 0, the expectation value of interaction energy may be written as:

$$\psi_\kappa^*\Delta\mathcal{H}'_{\kappa l}\psi_l=\frac{Q}{4}\psi_\kappa^*\psi_l\left(\partial'_\kappa E'_l+\partial'_l E'_\kappa\right) \qquad (16)$$

Diagonalize the matrix $\partial'_k E'_l + \partial'_l E'_k$ retaining in it only those components which being expressed through the quantities in laboratory frame will contain the largest degree of γ. Such a degree is γ^2 occuring in derivatives along the velocity of the field components orthogonal to velocity. In this approximation the diagonalization is carried out in the closed form and gives

$$\Delta \mathcal{H}'_s = \frac{sQ}{4} \left| (\bar{v}\cdot\bar{\nabla}')(\bar{E}' - \bar{v}(\bar{v}\bar{E}')) \right| \qquad (16a)$$

Passing in (16a) to the laboratory fields, coordinates and time (the latter leads, in particular, to common factor γ^{-1}) we obtain the following expression for the energy of quadrupole moment interaction

$$\Delta \mathcal{E}_s = \frac{sQ}{4c} \gamma \left| \frac{d\bar{E}}{dt} - \bar{v}(\bar{v}\frac{d\bar{E}}{dt}) + \bar{v} \times \frac{d\bar{H}}{dt} \right| \qquad (17)$$

Here we remain in the space of spin functions determined in the proper system. It is easy to see that eq.(17) coincides with the accepted accuracy with quadrupole correction in the Hamiltonian (14).

Note, that the transformation of interaction (16) to the laboratory system is not unique operation. For instance, the presence of anomalous magnetic moment of a vector particle leads to the appearance of quadrupole moment /15/* in non-relativistic approximation. Meanwhile, the addition to the relativistic Hamiltonian of a particle from the interaction of a magnetic moment obtained in the previous section does not at all resemble the eq.(17). This ambiguity is connected with the question of transformation of 4-dimensional divergence of a vector field which differs from zero in the presence of interaction and time component of the field arising due to the interaction even in the rest frame. Just these quantities, being expressed through independent space components and external field, in the presence of anomalous magnetic moment lead to the appearance of quadrupole interaction in non-relativistic approximation.

The velocity of a vector particle described by the Hamiltonian (14) $\vec{v}_s = \frac{\partial H_s}{\partial \vec{p}}$ may exceed the speed of light for $s = \pm 1$ at sufficiently large momenta and field derivatives. Here it takes place

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* The method by which the non-relativistic approximation is obtained in the work /15/ seems rather complicated. It is more simple to eliminate b_0 and $\pi_\mu b_\mu$ from Proca equation and then to make an expansion in v/c taking into account the change of normalization of wave functions as it is done in the book /16/ in order to obtain the non-relativistic approximation to Dirac equation.

in the first order in the interaction of anomalous moment with an external field. For illustration consider a particle motion in the electric external field $\bar{E}=(E_x(x,y), E_y(x,y), 0)$, $\bar{H}=0$ in the first order in Q. It is convenient to proceed directly from the Hamilton-Jacobi equation (13a). For simplicity suppose also that the particle is not charged. The solution is looked for in the form

$$S(x,y,t)=-\varepsilon t+p_x x+p_y y+\frac{sQ}{4mc^2}f(x,y) \tag{18}$$

where $\varepsilon^2=c^2(p_x^2+p_y^2)+m^2c^4$. Taking into account Maxwell equation for the external field

$$\partial_x E_x+\partial_y E_y=0, \quad \partial_x E_y-\partial_y E_x=0 \tag{19}$$

and retaining the terms of the first order in Q, we find from (13a) after substituting (18) the following equation for $f(x,y)$:

$$(p_x\partial_x+p_y\partial_y)f(x,y)=[2p_xp_y\partial_x-(p_x^2-p_y^2)\partial_y]E_x(x,y) \tag{20}$$

Its solution is

$$f(x,y)=p_y E_x(x,y)-p_x E_y(x,y) \tag{21}$$

The trajectory of a particle is determined, as usually, by eqs. $\frac{\partial S}{\partial \bar{p}}=0$. The explicit form of these equations with the accepted accuracy in Q is

$$x = v_{ox}t + \frac{sQ}{4mc^2}E_y(x,y) \approx v_{ox}t + \frac{sQ}{4mc^2}E_y(v_{ox}t, v_{oy}t)$$

$$y = v_{oy}t - \frac{sQ}{4mc^2}E_x(x,y) \approx v_{oy}t - \frac{sQ}{4mc^2}E_x(v_{ox}t, v_{oy}t) \qquad (22)$$

Here $v_{oi} = \frac{\partial \varepsilon}{\partial p_i} = \frac{p_i}{\varepsilon}$. The square of velocity is equal to

$$v^2 = \dot{x}^2 + \dot{y}^2 = v_o^2 - \frac{sQ}{2mc^2}\left[2v_{ox}v_{oy}\partial_x - (v_{ox}^2 - v_{oy}^2)\partial_y\right]E_x(x,y)\Big|_{\substack{x = v_{ox}t \\ y = v_{oy}t}} \qquad (23)$$

in the first order in Q. It is evident from (23) that for the given state with $s = 1$ or $s = -1$ one may choose the vector of unperturbed velocity $\bar{v}_o$ sufficiently close by modulus to the speed of light, so that the total velocity of the particle exceeds c. Again the velocity of the particle may exceed the speed of light in spite of the clear relativistic invariance of the corresponding Hamilton-Jacobi equation.

Does the considered effect take place for a composite particle with quadrupole moment? If the quadrupole moment is due to a finite size of the particle then there are no grounds to expect for this effect to arise. Indeed, in such a case the interaction Hamiltonian of quadrupole moment with an external field, before averaging over the internal motion, is equal in the rest frame to

$$\Delta \mathcal{H}' = -\frac{1}{2} \sum e x'_m x'_n \partial'_m E'_n \tag{24}$$

The sum is taken over the internal particles, e is a charge of such a particle, x'_m is its coordinate; it is supposed that $\partial_m E'_m = 0$. The interaction energy in the laboratory frame obtained by transformation of the formula (24) does not increase with increasing γ. The point is that now due to Lorentz contraction of x'_m the factor γ^{-1} arises which is extra one in comparison with the transition from (15) to (17). It is clear that the averaging over the internal motion can not change the situation. Thus, the Hamiltonian of the quadrupole interaction in this case differs qualitatively from the Hamiltonian that follows from the Proca eq.(11). Since now the ratio of the quadrupole Hamiltonian to the main one tends to zero at $v \to c$ then there are no grounds to expect superluminal velocities in this case.

4. <u>The influence of radiation on the motion of the particles with anomalous moments in an external field</u>

As it is known, the intensity of charge radiation at $v \to c$ increases, generally speaking, as γ^2.

Here $\bar{k}$ is the photon momentum. Using the condition that the radiation is the classical one $k \ll p$, we write $\varepsilon(\bar{p}-\bar{k}) \simeq \varepsilon(\bar{p}) - \bar{k}\bar{v}$ so that eq.(25) gives the relation

$$v \cos\theta = c \tag{26}$$

where θ is the angle between the particle velocity and the photon momentum. It is clear that at $v < c$ there is no radiation, and at $v > c$ the radiation of Cherenkov type arises* The intensity of usual Cherenkov radiation is finite due to the fact that the dispersion of a medium limits from above the radiative frequencies. In our case the intensity of Cherenkov radiation is infinite, since the spectral density increases as ω, ω^3 and ω^5 respectively for the charge, magnetic and quadrupole radiation, and the cut-off in frequencies under classical consideration is absent. In a variable field the radiation intensity remains infinite since with increasing frequency the radiation becomes more and more local and feels weakly and weakly the deviation of the field from the constant one. Thus, the classical radiation of Cherenkov type makes superluminal

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* The idea of this consideration is contained in essence in the Nobel lecture of I.E.Tamm /17/.

velocities impossible.

Let us return to the velocities lower then c. We are interested just in the variable fields in which a particle velocity increases and for which the radiation is not forbidden due to (26). What is the behaviour of radiation when $v \to c$?

According to the approximation being used in the work for the description of the particles' motion, first of all let us consider the radiation without changing s, that is the polarization of a particle. By the way, for s corresponding to the lowest energy such radiation dominates.

The magnetic moment of a particle with the given s in our approximation is a usual vector $\bar{\mu} = s\mu \bar{H}'/|\bar{H}'|$. The intensity of a magnetic radiation calculated in the rest frame by usual formulae /18/ $I = \frac{2}{3c^3}|\ddot{\bar{\mu}}|^2$ in the laboratory frame is

$$I = \frac{2\mu^2}{3c^3}\left|\frac{d^2}{dt^2}\frac{\bar{H}'}{|\bar{H}'|}\right|^2 \gamma^4, \tag{27}$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{v}\cdot\bar{\nabla}, \quad \bar{H}' = \gamma\left[\bar{H} - \frac{\bar{v}(\bar{v}\bar{H})}{v^2} - \bar{v}\times\bar{E}\right] + \frac{\bar{v}(\bar{v}\bar{H})}{v^2}$$

When $v \to c$ it tends to infinity as γ^4.

Let us turn to the quadrupole radiation. Taking the variation of the Hamiltonian (16a) in vector-potential $\bar{A}$ we find the current density in the rest frame

$$\bar{j}'(\bar{r},t') = \frac{Q}{4c}\frac{\partial}{\partial t'}(\bar{v}.\bar{\nabla}')\left\{\frac{(\bar{v}.\bar{\nabla}')(\bar{E}'-\bar{v}(\bar{v}\bar{E}'))}{|(\bar{v}.\bar{\nabla}')(\bar{E}'-\bar{v}(\bar{v}.\bar{E}'))|}\delta(\bar{r}'-\bar{r}'(t'))\right\} \quad (28)$$

Hence we find in the usual way /18/ the radiation intensity

$$I = \frac{Q^2}{80c^5}\left|\frac{\partial^3}{\partial t'^3}\frac{(\bar{v}.\bar{\nabla}')(\bar{E}'-\bar{v}(\bar{v}\bar{E}'))}{|(\bar{v}.\bar{\nabla}')(\bar{E}'-\bar{v}(\bar{v}\bar{E}'))|}\right|^2 \quad (29)$$

In the laboratory frame it is as follows

$$I = \frac{Q^2\gamma^6}{80c^5}\left|\frac{d^3}{dt^3}\frac{\frac{d\bar{E}}{dt}-\bar{v}(\bar{v}\frac{d\bar{E}}{dt})+\bar{v}\times\frac{d\bar{H}}{dt}}{\left|\frac{d\bar{E}}{dt}-\bar{v}(\bar{v}\frac{d\bar{E}}{dt})+\bar{v}\times\frac{d\bar{H}}{dt}\right|}\right|^2 \quad (30)$$

and tends to infinity when $v \to c$ as γ^6.

For completeness describe the radiation with the change of polarization. It is convenient to find its intensity in the rest frame by the standard quantum-mechanical calculation and then to pass to the laboratory frame. For spin-flip radiation of a vector particle we find

$$I = \frac{4\mu^6}{3c^3}\left(\frac{H'}{\hbar}\right)^4 \quad (31)$$

The increase of magnetic radiation $\sim \gamma^4$ at $v \to c$ is a well known fact (see, e.g., /19/). The intensity of quadrupole radiation with the polarization change of a vector particle is

$$I = \frac{Q^8 \gamma^{12}}{5 \cdot 2^{14}} \left| \frac{d\bar{E}}{dt} - \bar{v}\left(\bar{v}\frac{d\bar{E}}{dt}\right) + \bar{v} \times \frac{d\bar{H}}{dt} \right|^6 (\Delta s)^6 \quad (32)$$

Here, alongside with transitions $(s=0) \to (s=-1)$ and $(s=1) \to (s=0)$ there is transition $(s=1) \to (s=-1)$ with $\Delta s = 2$ and the intensity of 64 times as large.

Therefore, in all the cases under consideration the radiation intensity tending to infinity at $v \to c$ does not allow to reach the speed of light.

In principle, the cases are possible when the velocity of a particle increases and the radiation is absent while $v < c$. The example is the motion along the axis of a solenoid of the particle with a magnetic moment directed along a magnetic field (see (9a)). The spin-flip radiation here is forbidden due to the energy conservation and the magnetic moment $\bar{\mu} = \mu \frac{\bar{H}'}{|\bar{H}'|}$ does not at all depend on time. Nevertheless, in the real problem taking into account the velocity spread and non-ideality of an external field leads to the infinite radiation and impossibility to reach the light velocity.

5. The particle with spin 2 in an external field. General remarks.

Why for a vector particle does the interaction of a magnetic moment with an external field lead to superluminal velocities, while for a spinor particle such effect is absent? What is a spinor particle distinguished by in comparison with a vector one? The point is that for the particles with spin larger than 1/2 the number of components of relativistic wave function is larger than the number of independent degrees of freedom $2s + 1$. The elimination of extra components leads to the interaction for independent polarizations which allows for the superluminal velocities. It is the cause of difficulties in the description of particles with spin 3/2 in works /1,2/.

But the existence of interaction leading to superluminal velocities may be also unconnected with extra components of a field. For quadrupole interactions it is evident from the considerations by means of which the interaction (17) is obtained out of the non-relativistic Hamiltonian (15).

One should expect, therefore, that the interaction with an external field of particles with spin larger than 3/2 will lead to superluminal velocities also.

But we can not agree with the statement /3/ according to which there does not exist a consistent description of interaction with electromagnetic field for charged massive field with spin 2. First of all, due to the fact that such a description was constructed in the classical work of Fierz and Pauli /20/. The Lagrangian proposed in it

$$\mathcal{L} = \frac{1}{2}(\bar\pi_\lambda^+ \psi_{\mu\nu}^+ - \bar\pi_\mu^+ \psi_{\lambda\nu}^+)(\bar\pi_\lambda \psi_{\mu\nu} - \bar\pi_\mu \psi_{\lambda\nu}) - \bar\pi_\mu^+ \psi_{\mu\nu}^+ \bar\pi_\lambda \psi_{\lambda\nu} -$$
$$- m^2c^2 \psi_{\mu\nu}^+ \psi_{\mu\nu} - \frac{3}{8} \bar\pi_\mu^+ \psi^+ \bar\pi_\mu \psi + \frac{3}{4} m^2c^2 \psi^+ \psi -$$
$$- \frac{1}{2}(\bar\pi_\mu^+ \psi_{\mu\nu}^+ \bar\pi_\nu \psi + \bar\pi_\nu^+ \psi^+ \bar\pi_\mu \psi_{\mu\nu}) \qquad (33)$$

where $\psi_{\mu\nu}$ is symmetric tensor with zero trace, ψ is subsidiary scalar, leads just to 10 necessary constraint equations even if the terms corresponding to the change of magnetic moment are included. The question of constraints in this theory has been considered in /20/ exhaustively. Now on the formalism used in /3/. The proof given in this paper that such a formalism contains eight constraints only, cannot be correct because it refers as well to the case of a free field where there are undoubtedly ten constraints. Moreover, I succeeded to find the ninth constraint equation for this formalism. If, nevertheless, the formalism used in /3/ appeared to be inconsistent this would mean only that the Fierz-Pauli description should be preferred.

Note, that the magnetic moment of a particle being described by the Lagrangian (33) is $e\hbar/2mc$. It can be easily seen if one omits in (33) after integrating by parts the terms with charge e in a degree higher than the first one. Then all the terms containing ψ and $\pi_\mu \psi_{\mu\nu}$ drop out; for a free field these terms are equal to zero, and they enter always quadratically. After that the Lagrangian (33) is reduced to

$$\mathcal{L} = \pi_\lambda^+ \psi_{\mu\nu}^+ \pi_\lambda \psi_{\mu\nu} - m^2c^2 \psi_{\mu\nu}^+ \psi_{\mu\nu} + i\frac{e\hbar}{c} F_{\mu\nu} \psi_{\nu\lambda}^+ \psi_{\mu\lambda} \tag{33a}$$

The Hamiltonian of interaction with an external magnetic field $\bar{H} = (0,0,H)$ for non-relativistic particle with $S_z = 2$ which can be easily obtained by means of simple transformations out of the last term in (33a) taken with an opposite sign, is equal to $-\frac{e\hbar}{2mc}H$. It is clear from it that $\mu = \frac{e\hbar}{2mc}$. The natural generalization of Proca Lagrangian for spin 1 and Fierz-Pauli one for spin 2 to the case of arbitrary integer spin is

$$\mathcal{L} = (-1)^s \Big\{ \frac{1}{2} (\pi_\lambda^+ \psi_{\mu\nu\varkappa\ldots}^+ - \pi_\mu^+ \psi_{\lambda\nu\varkappa\ldots}^+)(\pi_\lambda \psi_{\mu\nu\varkappa\ldots} - \pi_\mu \psi_{\lambda\nu\varkappa\ldots}) - \\ -(s-1)\pi_\mu^+ \psi_{\mu\nu\varkappa\ldots}^+ \pi_\lambda \psi_{\lambda\nu\varkappa\ldots} - m^2c^2 \psi_{\mu\nu\varkappa\ldots}^+ \psi_{\mu\nu\varkappa\ldots} \Big\} + \ldots \tag{34}$$

Here, the terms are omitted which depend upon the subsidiary fields; these terms are not interesting to us since they do not contribute to the magnetic moment. Such a choice of a Lagrangian is preferable because it allows one to obtain the constraint equa -

tions in the simplest way [20]. The "magnetic" term in (34) is $(-1)^s i \frac{e\hbar}{c} F_{\mu\nu} \psi^+_{\nu\lambda\varkappa\ldots} \psi_{\mu\lambda\varkappa\ldots}$, that leads to magnetic moment $\mu = \frac{e\hbar}{2mc}$ at any spin s.

The particles with arbitrary half-integer spin described by the Rarita-Schwinger Lagrangian [21]

$$\mathcal{L} = (-1)^{s-\frac{1}{2}} \bar{\psi}_{\mu\ldots} \Big[(\hat{\pi} - mc)\delta_{\mu\nu} + a(\gamma_\mu \pi_\nu + \gamma_\nu \pi_\mu) +$$
$$+ \left(\tfrac{3}{2}a^2 + a + \tfrac{1}{2}\right)\gamma_\mu \hat{\pi} \gamma_\nu + (3a^2 + 3a + 1) mc \gamma_\mu \gamma_\nu \Big] \psi_{\nu\ldots} \tag{35}$$

(a is arbitrary real number not equal to -1/2, $\hat{\pi} = \pi_\mu \gamma_\mu$) have also the magnetic moment $e\hbar/2mc$. Indeed, since $\pi_\mu \psi_{\mu\ldots}$ and $\gamma_\mu \psi_{\mu\ldots}$ are equal to zero for a free field then the terms with these quantities in (35) are $\sim e^2$. Theref re, the interaction Lagrangian to the first order in the charge e is $(-1)^{s+\frac{1}{2}} \bar{\psi}_{\mu\ldots} \gamma_\nu A_\nu \psi_{\mu\ldots}$. The spin part of the interaction Hamiltonian in a non-relativistic approximation is: $-\frac{e\hbar}{2mc} \varphi^+_{m\ldots} \bar{\sigma}\bar{H} \varphi_{m\ldots}$. For the magnetic field $\bar{H} = (0, 0, H)$ and the particle with $s_z = s$ this Hamiltonian is $-\frac{e\hbar}{2mc} H$. Thus, it is clear that the magnetic moment $\mu = e\hbar/2mc$.

The hypothesis that the magnetic moment of a particle with an arbitrary spin is equal to $e\hbar/2mc$ was made by Belinfante [22] and was proved at first for half-integer spin [21,23], then for spin 2 [24] and, at last, quite recently for arbitrary spin

/25/*. All these proofs seem to me to be too complicated. Besides, it is necessary to stress all the convention of the obtained result: it is clear that one can always add to the Lagrangian the terms depending upon $\mathcal{F}_{\mu\nu}$ which may lead to any value of the magnetic moment.

In conclusion note one more curious circumstance. Solving any wave equation leading to superluminal velocities by iterations we shall not obtain the noncausality in any order of the perturbation theory; causality is conserved due to the very structure of retarded Green function /2/. It does not mean, however, that in the perturbation theory everything is all right. For the non-renormalizable interactions the increasing divergences arise here and the scattering amplitudes increasing with energy lead to the violation of the unitarity condition. Repeatedly the hopes were expressed that the summation of perturbation theory series would allow to get rid of these difficulties. The used approximations (quasiclassics, the expansion action in the coupling constant) are equivalent to the partial summation of the perturbation theory series. But having got rid of the divergences and unitarity

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* The paper /25/ became known to me only after this work had been over.

violation we come across not less serious difficulty, that is the noncausality. Here it is necessary to note that, strictly speaking, the applicability of the external field approximation in the case of nonrenormalizable interactions is not proved.

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Литература

1. K.Johnson, E.C.G.Sudarshan. Annals of Physics, 13, 126, 1961.
2. G.Velo, D.Zwanziger. Phys.Rev., 186, 1337, 1969.
3. G.Velo, D.Zwanziger. Phys.Rev., 188, 2218, 1969.
4. P.C.Aichelburg, G.Ecker, R.U.Sexl. Nuovo Cim., 2B, 63, 1971.
5. R.H.Good. Phys.Rev., 125, 2112, 1962.
6. R.Schiller. Phys.Rev., 128, 1402, 1962.
7. P.Nyborg. Nuovo Cim. 31, 1209, 1964; 32, 1131, 1964.
8. E.Plahte. Suppl. Nuovo Cim., 4, 246, 291, 1966.
9. R.Schiller. Phys.Rev., 169, 1282, 1968.
10. Я.С.Дербенев, А.М.Кондратенко, ЖЭТФ, в печати.
11. W.Tsai, A.Yildiz. Phys.Rev., 4D, 3643, 1971.
12. T.Goldman, W.Tsai. Phys.Lett., 36B, 467, 1971. Phys.Rev., 4D, 3648, 1971.
13. L.H.Thomas. Nature, 117, 514, 1926; Phil.Mag. 3, 1, 1927.
14. Я.С.Дербенев, А.М.Кондратенко, А.Н.Скринский, ДАН, 192, 1255, 1970.
15. J.A.Young, S.A.Bludman. Phys.Rev., 131, 2326, 1963.

16. А.И.Ахиезер, В.Б.Берестецкий. Квантовая электродинамика, изд. "Наука", 1969.

17. И.Е.Тамм. УФН, 68, 387, 1959.

18. Л.Д.Ландау, Е.М.Лифшиц. Теория поля, изд."Наука", 1967.

19. В.Л.Любошиц. ЯФ, 4, 269, 1966.

20. M.Fierz, W.Pauli. Proc.Roy.Soc., A173, 211, 1939.

21. P.A.Moldauer, K.M.Case. Phys.Rev. 102, 279, 1956.

22. F.J.Belinfante. Phys.Rev., 92, 997, 1953.

23. C.Fronsdal. Suppl. Nuovo Cim., 9, 416, 1958.

24. В.С.Туманов, ЖЭТФ, 46, 1755, 1964.

25. В.С.Туманов, ТМФ, 9, 388, 1971.

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