19 февраля

18

И Н С Т И Т У Т
ЯДЕРНОЙ ФИЗИКИ СОАН СССР

ПРЕПРИНТ И Я Ф 74 - 108

E.V.Shuryak.

The Statistical Theory of Multiple Production of Hadrons.

Новосибирск

1974

The Statistical Theory of Multiple Production of Hadrons.

E.V.Shuryak.

Institute of Nuclear Physics, Novosibirsk.

Abstract.

The following topics are discussed in this paper: 1) the final state interaction of secondaries; 2) inclusive and exclusive spectra; 3) the probability of particular channels production; 4) the composition of secondaries; 5) the $p_\perp$ distribution and composition at large $p_\perp$; 6) the principal questions of the statistical approach; 7) Ericson fluctuations in two and manybody reactions; 8) the interference effects of the identical pions.

1. Introduction.

The multiple production of hadrons is very complicated phenomenon and although it is known for many years its intensive investigations have begun only recently. The first reason for it is, of course, the difficulty of its experimental studies. But there is another important difficulty, namely the need for the adequate language for its description. One crucial point was the transition to inclusive experiments, in which only one (two etc.) particle is detected. The language of the statistical physics, which we explore in the present paper, seems to be quite adequate to such a situation. The inevitable averaging over many unobserved variables becomes an ally on the way to the construction of the description of the phenomenon, both simple and general, while it is an enemy in the detailed dynamical approach.

The history of the statistical theory of multiple production can be traced to works of Heisenberg /1/, in which he first assumed the mixing of the secondaries up to thermodynamical equilibrium. The first particular statistical model has been proposed by Fermi /2/. But as Pomeranchuck has noted, this model is not logically consistent because the interaction of the initial particles is assumed to be strong, while the interaction of secondaries is not considered. The Pomeranchuck model is based on the idea, that the observed properties of the secondaries are determined on the stage of the process, which we call the final stage, just preceeding the end of the interaction and the free propagation of the secondaries from the collision point /3/. The condition of this stage has been given by Landau /4/: the mean free path of secondaries becomes of the order of the system dimensions. This condition is familiar in many physical problems, e.g. it determines the visible temperature of stars.

The interaction between secondaries must cause some collective effects, which according to Landau /4/ can be described by the relativistic hydrodynamics. The most speculative point of the Landau theory is the special assumption about the initial stage of the process, which we discuss together with its observable consequences elsewere /5/. In this paper we deal only with the characteristics, determined at final stage. The reason for such separation of the material is that the present discussion is consistent with the wide range of theories, including the multiperipheral and the parton models. Our treatment of the problem can be considered as a complementary to these models, for they do not consider seriously the final stage interaction.

The Pomeranchuck idea has been shown to be in agreement with data /6-8/ and the present paper moves this comparison a little further. We discuss not only the average properties, but also the deviations from them which can be described by the standard theory of fluctuations. First we discuss the low energy case (E_{LAB} =1÷10 Gev.) and then proceed to high energy reactions. We discuss also some phenomena at large transverse momenta, in which they are considered as a consequence of the particle "leakage" before the final stage. A large part of this paper is devoted to the discussion of the principal questions of the statistical theory, e.g. what the equilibrium and the statistical ensemble means in this case, what is the precision and the applicability limits of the theory etc., which are very little discussed in the literature. Of course, we do not claim to give the final answers to them, but hope that our discussion will clarify them somehow. Our ideas in this direction are very close to those known in the nuclear reaction physics. The last chapter deals with

another question, principal to the theory: to what degree are there the produced pions coherent? The new interference experiments are proposed in order to measure the lifetime of the system created in the collision. In all discussions we pay more attention to the main arguments and concepts, presenting the particular formulae and their comparison with data mainly as an illustration to the text.

One more comment concerning the statistical bootstrap model /9/. There is no direct contradiction between this approach and ours. The decay chain of this model can be considered as some particular mechanism, leading to the discussed final stage. Many particular predictions of these appraaches coincide . But the approach, used in this paper, needs neither rather arbitrary assumptions, like the bootstrap condition /9/, nor the use of the Beth-Uhlenbeck method in the cases, where it can hardly be justified. So it is more general and more simple and thus seems to be more preferable.

2. The Final State Interaction.

Let us begin the discussion of the statistical theory from the most simple case of low energy reactions, that is those with C.M. energy of the order of one Gev. In this energy range most of the secondaries, with the exception of the so called leading ones, have almost isotropic angular distribution. So in this energy range one can assume that the secondaries form some common statistical system. In the case of high energy reactions the equilibrium, if any, takes place only in local sense, as we discuss below.

According to Pomeranchuck /3/, the observed properties of the secondaries are to a large extent determined at final stage, just preceeding their free propagation from the collision point. This is due to the fact that the strong final state interaction "mix" the system so that it "forgets" the preceeding stage except some global integrals of motion . In particular, the kind of the initial particles is not of much importence- the well known factorisation property of the hadronic production. The same property is known for the nuclear reactions which proceed through the compound nuclei formation. Of course, there must be also some nonequilibrium effects, to which we return in the chapter 5.

Now we prosesd to the derivation of the quantitative condition which determines the properties of the system at final stage. The argumentation in /3.6/ is the following: the particles become free when their spacing reaches the maximum range of the strong interaction, something like m_π^{-1}*. In other words, the final stage is characterized by the particle density

$$n \equiv \frac{N}{V} \sim \frac{1}{V_o} \qquad V_o = \frac{4\pi}{3} m_\pi^{-3} \tag{1}$$

where N is the total particle number and V is the volume of the system at final stage. We can not agree with these arguments. First, the range of the interaction between pions and kaons is not m_π^{-1} but rather something like m_ρ^{-1}, for due to the negative parity these particles can not exchange pions. But the condition itself is not also correct. According to Landau /4/ the final stage is determined by the following condition: the mean free path l is of the order of the minimal system dimensions L_{min}. Indeed, the case $l \ll L_{min}$ implies that the particles are "closed up" by the interaction and can not leave the system, while in the opposite limit $l \gg L_{min}$ they can be considered as free particles **. But it turns out, that

* We use units $\hbar = c = 1$

**The Landau condition implies the slow system expansion and neglects the surface effects. The more exact determination of the final stage parameters needs the discussion of the kinetics of the process, which remains to be done in the future.

numerically this condition practically coincides with (1) since the cross section of the interaction and L_{min} are of the order m_π^{-2} and m_π^{-1} respectively. With this warning we use below the condition (1). Note the principal difference with the Fermi model and their modifications: it is not the volume which is fixed, but the intensive quantity- density.

The evident consequence of our discussion is the thermal distribution of the secondaries in inclusive spectra .

$$dN \doteq \left[\exp\left(\frac{E}{T}\right) \pm 1\right]^{-1} \frac{g\, d^3p V}{(2\pi)^3} \tag{2}$$

where E and p are the energy and momentum of the particle, $g = 2S+1$, S is the particle spin, V is the volume at final stage. The value of the temperature according to the discussion above is of the order of m_π. As an illustration of the validity of (2) we show at Fig.1a the data /10/ for Kp collisions at 12Gev. and at Fig. 1b the data /11/ for $p\bar{p}$ annihilation. Interesting, that the accuracy of the data is so high, that the difference between (2) and simple exponential can be seen. Note, that the temperature value is the same for pions and kaons. The data on other low energy production look very similar.

In order to make the calculations of the properties of the system at final stage one needs to take the interaction into account. In our approach, based on the Pomeranchuck idea, this problem is not so difficult as in other statistical models because we are interested only in the final stage, at which the density and temperature is not high and the properties of the strong interaction is rather well known.

As it has been proposed by Belenky and Landau /12/ one can use the Beth-Uhlenbeck method /13/ of the nonideal gas theory. Its main idea lies in the counting the number of the states in the statistical sum as with the account of the scattering phase caused by the interaction. The most simple result is obtained in the case of purely resonant interaction and when the resonance width is much smaller than the temperature. In this case the problem is reduced to the simple case of the mixture of noninteracting gases of stable particles together with resonances. Note, that the Boltsmann factor $exp(-m/T)$ makes the strong cutoff of the heavy resonance contribution*, so practically only ρ, ω give the noticeable contribution to the number of pions and φ to the number of kaons observed.

This method is also used in the statistical bootstrap model /9/. Some criticism of this model can be found in /5,6/. Let us mention here only one point concerning the applicability limits of the Beth-Uhlenbeck method. This method gives the second coefficient of the virial expansion, which is the expansion on the following parameter: the range of the interaction r_{int} divided over the particle spacing $n^{-1/3}$. The rough estimate of this parameter value at the final stage is according to the discussion above $r_{int} \sim m_\rho^{-1}$, $n^{-1/3} \sim m_\pi^{-1}$, so the expansion parameter is m_π/m_ρ. Thus, although it is not very small and high accuracy can not be claimed, its use is justified. In the statistical bootstrap model this method is essentially used up to very high density, where its even qualitative validity is doubtful.

Finally, in order to give the idea what the hadronic plasma at final stage looks like, we note that it is the nonideal gas rather

---

* Unless the resonance spectrum grows too strongly, as in the statistical bootstrap model. See the detailed discussion of this in /5/.

than liquid with particle separation of the order of m_π^{-1} and intensive interaction at distances m_ρ^{-1}. This gas is partly quantum one, since the wave length is of the order of particle spacing. The main interaction is the intensive production and dissociation of the resonances. The known methods of the statistical mechanics are sufficient for the determination of the properties of such media, to which we proceed in the next chapter.

3. The Thermodynamics at Final Stage.

All average characteristics of the hadronic plasma at final stage can be found with the help of the integration of the thermal distribution (2) and with the account of the resonancies as it has been explained above. It is useful to introduce the following standard functions /4/:

$$n_i(T) = \int \frac{g_i\, d^3p}{(2\pi)^3} \left[exp\left(\frac{E_i}{T}\right) \pm 1 \right]^{-1} = \frac{g_i T^3}{2\pi^2} F_\pm\left(\frac{m_i}{T}\right) \quad (3)$$

$$\varepsilon(T) = \sum_i \int \frac{g_i\, d^3p}{(2\pi)^3} E_i \left[exp\left(\frac{E_i}{T}\right) \pm 1 \right]^{-1} = \sum_i \frac{g_i T^4}{2\pi^2} \Phi_\pm\left(\frac{m_i}{T}\right) \quad (4)$$

$$\chi_i(T) = T \cdot \Phi\left(\frac{m_i}{T}\right) / F\left(\frac{m_i}{T}\right) \quad (5)$$

The physical meaning of these quantities are the particle and energy density and the average particle energy respectively. Their analitical representation and tables can be found in /4b/. Note that in the nonrelativistic limit $T \ll m_i$ $\chi_i = m_i + \frac{3}{2}T$ and in the ultrarelativistic case $T \gg m_i$ $\chi_i = 3T$. At final stage $T \approx m_\pi$ so the average pion energy $\chi_\pi(m_\pi) = 420$ Mev. This results, for example, in the fact that the average multiplicity at $p\bar{p}$ annihilation at rest is close to 5. Other predictions, following from these formulae, are discussed in /6-8/.

Now we proceed to the discussion of the deviations from the average values, described as thermodynamical fluctuations. We consider the example of the particular reaction channel "a" realisation for there are a lot of data of this kind. The channel means the production of the definite set of secondaries $N_i^{(a)}$ of the kinds "i". The distribution of the secondaries over momenta is, of course, also thermal

$$dN_i^{(a)} = \frac{g_i d^3p V_a}{(2\pi)^3}\left[exp\left(\frac{E_i-\mu_i^{(a)}}{T}\right)-1\right]^{-1} \qquad (6)$$

but with the set of chemical potencials $\mu_i^{(a)}$. It is important, that the values of $\mu_i^{(a)}, T_a, V_a$ all depend on the channel in question. There are two normalising conditions

$$N_i^{(a)} = \int dN_i^{(a)}, \qquad E_{tot} = \sum_i \int E_i\, dN_i^{(a)} \qquad (7)$$

which can be used for $\mu_i^{(a)}$ and T_a determination. In order to determine the last parameter V_a some physical condition must be introduced prescribing the character of the fluctuations discussed. According to the considerations of chapter 2 this condition is the fixed density of particles, or in other words

$$V_a = V_0 \cdot \sum_i N_i^{(a)} \qquad (8)$$

So everything is fixed and (6) can be compared with data.

The more important point is that we can find the probability of this channel production. It is the ratio of the phase space, corresponding to this channel Γ_a to the total one $\Gamma_{tot} = \sum_a \Gamma_a$. The thermodynamical expression for the phase space is

$$\Gamma_a = \left\{\prod_{k=1}^{4}\left(2\pi\Delta p_k^2\right)^{-1/2}\right\} exp\left[S\left(\mu_i^{(a)}, T_a, V_a\right)\right] \qquad (9)$$

where p_k are the four coordinates of the total momentum, their

average fluctuation $\Delta p_k^2 \equiv \langle p_k^2\rangle - \langle p_k\rangle^2$ can be found from the integration of the thermal spectrum (6). The quantity $S(\mu_i, T, V)$ in the exponent is the entropy of the system, which in our approximation can be taken as that of the mixture of gases:

$$S(\mu_i, T, V) = \sum_i \int \frac{g_i d^3p V}{(2\pi)^3}\cdot\frac{1}{T}\cdot\frac{\left(\frac{p^2}{3E_i} + \mu_i - E_i\right)}{\left[exp\left(\frac{E_i-\mu_i}{T}\right) \pm 1\right]} \qquad (10)$$

Note that the formulae (9,10) are very similar to those used for the statistical calculation of the level density of nuclei in the Fermi gas model.

In order to make these formulae much simpler the following approximation can be made. For all particles but pions the term $exp\left(\frac{E}{T}\right) \gg 1$, so the unity in the denominator of (3,4,6,7,10) can be neglected. Moreover, it can also be made for pion term because due to factor p^2 the main contribution to the integral is given by the region $p = 200 \div 400$ Mev/c in which the exponent is several times larger than unity. In other words, we proceed to Boltsmann statistics (which we discuss in the chapter 8).

In this approximation the equation for $\mu_i^{(a)}$ (7) can be easily solved and substituting the result into (10) one has

$$W_a = const\cdot\left[\prod_{k=1}^{4}\left(2\pi\Delta p_k^2\right)^{-1/2}\right]\cdot\left\{\prod_i \frac{\left[V_a n_i(T_a)\right]^{N_i^{(a)}}}{N_i^{(a)}!}\right\}\cdot exp\left(\frac{E_{tot}}{T_a}\right) \qquad (11)$$

Note that the similarity between this formula and the Poisson distribution is misleading for T_a is determined from (7) and itself depends on the channel.

The comparison of the predictions of this formula with data for $p\bar{p}$ annihilations into pions and kaons is presented at Fig. 2. The only parameter is V_0, taken just to be $\frac{4\pi}{3} m_\pi^{-3}$. The agreement(except at high energy end) is rather good if one notes the crudness of the model, the absence of the free parameters and the strong dependences like $V^N \sim N^N$. In the present calculation we account for predexponent in contrast with /7/, which makes the agreement for small N a little better.

Concluding this point we note, that the nontrivial element of this calculation lies mainly in the assumption (8). The method for the phase space calculation can, of course, be substituted by another one. We prefer this method /7/ because it is simultaneously precise* and physically transparent. It can be used for the derivation of the useful approximations in the particular cases.

Let us discuss the following example, the calculation of the average number of some particle of the kind "i". The probability of the simultaneous N_i and N_π production (according to (11)) divided over that of N_π production only is:

$$W_{N_i} = \frac{1}{N_i!}\left[V n_i(T)\right]^{N_i}\left[\frac{n_\pi(T)}{n_\pi(T_0)}\right]^{N_\pi} \exp\left[E_{tot}\left(T^{-1} - T_0^{-1}\right)\right] \quad (12)$$

where T and T_0 are determined by the equations

$$E_{tot} = N_i\,\chi_i(T) + N_\pi\,\chi_\pi(T) = N_\pi\,\chi_\pi(T_0) \quad (13)$$

When the energy of the particles "i" is much smaller than that of pions, one can expand over $\frac{T-T_0}{T_0}$. The identity

$$\chi_i(T) = \frac{T^2}{n_i(T)}\,\frac{d n_i(T)}{dT} \quad (14)$$

following from the definition (3,5) can be used and the result up to the first order of $\frac{T-T_0}{T_0}$ is the Poissonian distribution

---

* We have checked that (11) gives the true coefficients in $\ln \Gamma_a$ expansion (which are functions of the total energy and the particle masses) for terms $N \ln N$, N, $\ln N$, const. So it is as precise as the famous Sterling formula and can be used even for $N = 3 \div 4$.

$$W_{N_i} = \frac{1}{N_i!}\left[V n_i(T_0)\right]^{N_i} \quad (15)$$

which has a very simple meaning: independent fluctuations, namely the particle "i" productions, in the thermostate - the pions. The average number of particles "i" is

$$\langle N_i \rangle = V \cdot n_i(T_0) \quad (16)$$

This formula is the same as the known macroscopic one, but note that their applicability limits are different. The latter is valid for all particles in the case when the system is so large that $\langle N_i \rangle \gg 1$; and the derived formula (16) for neutral particles only but for any $\langle N_i \rangle$, even very small, implying that the whole system, or $\langle N_\pi \rangle$, is large. The word "neutral" means that it has no other quantum numbers than those of pions, so they can be directly produced by pions. This is so for particles like ρ, ω but not for $K, \bar{p}$, which must be produced in pairs. We return to this case later, in chapter 4.

We finish this chapter with the remark, that the further we go in the description of the system, like fluctuations, the worse is the accuracy of the statistical theory till finally we approach its applicability limits. It is not known when it will happen, some estimates will be given in the chapter 6.

4. The Statistical Theory Applications for the High Energy Collisions.

The secondaries produced in the high energy collisions are distributed unisotropically. Different theoretical approaches explain this fact in different ways. But in any case, there must be also some final stage of the process, which has properties similar to the low energy reactions. In this chapter we try to reveal the characteristics of the process, which are determined by the final stage effects, and to study them with the help of the statistical theory.

It is more convenient to consider the production reaction backward in time for we are interested in the final stage. So we begin from the free secondaries at distance, moving into collision point. Let us divide particles in some groups according to their rapidities. Clear that these groups are also separated in space, the more slow particles closer, the more fast at distance. The volume, occupied by any of these groups shrinks, so at some stage the interaction starts. The condition for it, expressed in the accompanying frame, is again the Landau criterion: the mean free path is comparable with the minimal system dimensions, already discussed above. The minimal dimensions are the transverse ones which are still of the order of m_π^{-1} . So again the condition in question corresponds to the fixed particle density of the order of m_π^3 . Let us also assume the thermodynamical equilibrium at this stage. In contrast with the case of the low energy reactions, this equilibrium is only local since for different particle groups it takes place in different reference frames and even at different moments of time. Of course, in such a system the equilibrium can not be exact due to dissipative phenomena like viscosity, but we still discuss this as a first approximation.

The assumptions made lead to important predictions. First, if the motion of the mentioned groups is longitudinal, the transverse momentum distribution is not affected and so it must be given by the thermal distribution with the temperature of the order of m_π . First this fact has been discussed in /4/, and then in more details by Hagedorn /9/. Let us show how it is connected

with recent experimental data from ISR CERN /14/, plotted at Fig. 3 as $dN/dp_\perp^2$ versus $M_\perp = (p_\perp^2 + m^2)^{1/2}$ for $\pi^-, K^0, K^-, \bar{p}$. The variable $M_\perp$ is useful because the thermal spectrum for all kinds of particles is expressed in it by the single formula

$$\frac{dN}{dp_\perp^2} \propto \int dp_\parallel \left[\exp\frac{(M_\perp^2 + p_\parallel^2)^{1/2}}{T} - 1\right]^{-1} = M_\perp \sum_{N=1}^{\infty} K_1\left(\frac{NM_\perp}{T}\right) \qquad (17)$$

plotted at Fig. 3 by the solid line. One can see, that the dependence (17) really takes place for any particles and with the values of the temperature, very close to each other.

Another less trivial consequence of the hypotesis of the local equilibrium is connected with the calculation of the composition of secondaries /8/. The main idea is that the motion of some volume element does not affect the fluctuation in it and in the thermodynamical sense the whole system in question is equivalent to hadronic plasma at rest in some effective volume V_{eff}, equal to the sum of all volume elements taken at final stage. The calculation of V_{eff} needs the application of the particular theory and in our final results V_{eff} will be excluded.

Since such particles as $K, \bar{p}$ are produced in pairs due to the strangeness and barion number conservation, the formula (16) can not be applied. The probability of such pair production can be taken from (15). The average number of such particles can be easily computed from (15) with the result

$$\langle N_i \rangle = \frac{2 n_i(T_0) V_{eff}\, I_1(2n_i(T_0)V_{eff})}{2I_0(2n_i(T_0)V_{eff}) - 1} \qquad (18)$$

Here I_0, I_1 are Bessel functions, the additional factor 2 corresponds to the number of the isospin states, say $\bar{p}$ can be produced together with n or p . In the case of the large system $V_{eff} \to \infty$ $\langle N_i \rangle = n_i(T_0) V_{eff}$, as it should be, while in the opposite limit $V_{eff} \to 0$ $\langle N_i \rangle = 2 n_i^2(T_0) V_{eff}^2$ /9b,6/. For pions the former case is assumed, and this allows the determination of the V_{eff} : $V_{eff} = \langle N_\pi \rangle / n_\pi(T_0)$. Thus the formula (18) can be understood as the universal connection between the number of K and $\bar{p}$ with the number of pions $\langle N_\pi \rangle$. The corresponding curves for different values of the temperature are plotted at Fig.4 together with data available /15/. The character of the dependence is indeed reproduced rather well and the better agreement is found for temperature value close to those obtained from the study of the $p_\perp$ distribution.

5. Phenomena at large transverse momentum.

In the discussion above we have used the assumption that the secondaries are closed up in the system before the final stage and everything is determined by the equilibrium final stage thermodynamics. Certanly, there must be also some leakage before the final stage, resulting in the nonequilibrium effects /6,8/. The better place to find them is that one, where the contribution of the equilibrium effects is small, for example at large $p_\perp$.

In the discussion below we use the following special assumption: at intermediate stages of the process, preceeding the final one, there is also thermodynamical equilibrium with temperature higher than T_{fin} . This is so in the hydrodynamical theory of Landau /4/, but this assumption still is less strong than that lying in the basis of this theory. We are going to show that this assumption can explain some phenomena recently observed at high $p_\perp$, at least at semi- qualitative level.

Let the normalized probability of the particle leakage at temperature T is $W(T)$. The essential point is that it is taken to be independent of the particle kind. The transverse momentum distribution can be written as

$$\frac{dN_i}{dp_\perp^2} \approx \frac{\exp\left(\frac{m_i}{T_{fin}}\right)}{2T_{fin}(m_i+T_{fin})} \int_{T_{fin}}^{\infty} dT\, W(T)\, N_i(T) \exp\left(-\frac{M_\perp}{T}\right) \qquad (19)$$

The first factor stands for the normalization, $N_i(T)$ is the particle number at temperature T , the exponent is the formula (17) for $M_\perp \gg T$. The function $W(T)$ is assumed to have a peak at T_{fin} in order the discussed above properties at small $p_\perp$ be valid. But as far as $M_\perp$ becomes larger, the interplay of $W(T)$ and $\exp\left(\frac{M_\perp}{T}\right)$ moves the peak of the integrand to some T_{eff} higher than T_{fin} . This must result in the gradual decrease in the slope parameter of the $\frac{dN}{dp_\perp^2}$ with $M_\perp$, which is indeed seen in data /14/.

The more interesting data are concerned with the composition of the secondaries at high $p_\perp$. Let us begin our discussion of this point from some "zero order approximation", in which the $N_i(T)$ dependence on T is neglected compared to that of other factors in the integrand. In this approximation the particle composition is $M_\perp$ independent. And indeed, the particle ratios change with $M_\perp$ much less rapidly than their absolute yield. As the next approximation let us take $N_i(T_{eff})$, where T_{eff} is the saddle point of the integral(19). Now the composition changes with $M_\perp$ because it changes T_{eff} . It is interesting, that due to $N_i(T)$ dependence on T , the T_{eff} is also a little bit different for different particles, increasing slightly with particle mass. This effect is indeed seen in the discussed above data for small $p_\perp$ range.

Now we proceed to the more detailed discussion of data. Unfortunately, it is not easy to make our model really quantitative, first because $T_{eff} = 0.3 \div 0.5$ Gev. at $M_\perp = 4 \div 5$ Gev. and many resonances must be taken into account, and the second, the composition is very sensitive to the value of T_{eff} in the large $M_\perp$ range. So the accuracy of our discussion is within the factor of two.

Since T_{eff} increases with $M_\perp$, the increase in heavy particle production is predicted. As we have discussed in the chapter 2, the most of such particles are resonances, which then decay into stable particles, pions and kaons. The estimations show that $\frac{N_{K^-}}{N_{\pi^-}}$ ratio remains on the level of 0.2 for any $M_\perp$, in agreement with data* The behaviour of $\frac{N_{\bar p}}{N_{\pi^-}}$ is more interesting, with the increase of the T_{eff} it first increases, and for $T_{eff} > 0.3 \div 0.4$ Gev decreases, because the contribution of the resonance decays into number of pions increases even more rapidly with T_{eff} than $N_{\bar p}$. This behaviour is also in agreement with data, although the absolute value of this ratio is found to be a factor $1.5 \div 2$ smaller than the data. In the estimate we use resonances from the Particle Data Group Tables, although such criterion for an account of resonanaces is rather artificial. The quantitative method for the calculation of the thermodynamics of the hadron plasma in such conditions is still unknown.

---

* Note that in data /14/ the particle ratios at fixed $p_\perp$ are presented. This causes the dip of this ratios at small $p_\perp$, which (according to the discussion in the chapter 4) is absent in $M_\perp$ variable. Of course, at high $p_\perp$ the difference between $p_\perp$ and $M_\perp$ disappears.

6. The Principal questions of the statistical theory.

In spite of its rather long history, the statistical theory still remains at the initial stage of its development. The principal questions, concerning the proof of its basic assumptions and understanding of the applicability limits, are still open. Of course, this is not occasional, since even in the case of classical systems, for which the dynamics is known, the similar problems are not also completely solved. The better argument in favour of the statistical theory now is its agreement with data discussed above. In this chapter some discussion of the principal questions will be made, or more strictly, of the way in which they can be posed. The strong analogy with the statistical theory of the nuclear reactions runs through all this discussion.

Let us remind, that the statistical description always deals with some ensemble of the systems, in which it determines the distribution of different quantities. Thus the proof of such theory must include the determination of the ensemble, corresponding to the conditions of the experiment, and the proof that the distribution is indeed that given by the theory, say canonical or microcanonical ones.

There is important difference between the application of the statistical theory to some subsystem of larger system and to the whole system. In the former case the statistical ensemble corresponds to all states of the rest of the system. Such problems are known as Brownian motion in classical case and the determination of the density matrix in quantum one. In the case of the production reactions this corresponds to inclusive reactions. It is interesting, that the canonical distribution for subsystem can be valid even if the whole system behaviour is not statistical. The classical example of this kind is a particle motion in the system of coupled oscillators. The situation in particle production may also be of this kind, with the statistical behaviour due to the averaging over many unobserved variables.

The more difficult questions arise in the discussion of the statistical description of the system as whole. Such situation in production reactions corresponds to exclusive studies when all secondaries are detected. Even the formulation of what is the statistical ensemble in this case encounter difficulties. In the classical case it is made for the stochastic systems, that is those with unstable trajectories. The ensemble corresponds to external perturbations, which in contrast to the case of Brownian motion, can be arbitrary small. This approach has revealed the statistical nature of even rather simple systems like the nonlinear oscillator (see e.g. /17/). In the works /18,6/ the same idea is proposed to hadronic and nuclear reactions. The assumption is made that the reaction amplitude is exponentially unstable to external perturbations. We do not use this very strong assumption, for we see no evidence that it is the case. The recent observation of the Ericson fluctuations /21/ is also against this assumption.

The property of the classical systems we are going to explore is the selfmixing /16/. It means that in the system evolution the states, initially occupying small region in the phase space, become distributed in very complicated and irregular way. The coarse grained distribution, averaged over this irregularities, approaches constant, that is, becomes microcanonical. This does not contradict to Liouwille theorem, the volume of the true distribution is conserved while that of the coarse grained one increases. We remind this well known fact in order to stress its strong analogy with the discussion below.

The analog of the selfmixing in our case is the strong and irregular dependence of the transition amplitude, the S-matrix, on its variables. If the precision of the experiments is not high enough, the coarse grained distribution is found, which may be described by the statistical theory. The principal difference with the true stochastic systems lies in the fact, that with the accuracy increase one can reach the region of the dynamical behaviour, while in the case of unstable system this is impossible to do. Note, that according to estimates below, this difference is not of practical importance, since the nessesary accuracy in the multiple production seems to be out of reach.

The importance of the amplitude irregularity can be also understood in other way. Note, that the derivative of the amplitude over energy is connected with the characteristic duration of the process. This is known, for example, for elastic scattering in which $\tau = \frac{d\delta(E)}{dE}$, where $\delta(E)$ is the scattering phase. Thus the irregularity of the amplitude means long duration of the process, which may lead to its mixing up to equilibrium.

The general reason for irregularity of the amplitude is that the system, created in the collision, has many ways for the decay into secondaries. For example, different intermediate resonances of some groups of particles can be formed. The interference of these possibilities causes irregularities with the characteristic parameter - the width of the resonances. The experiments reveal only the most prominent resonances, while all other together form the homogeneous in average background in phase space.

This situation has caused the misunderstanding of the statistical theory, namely the statement that this theory assumes the amplitude to be constant. Such a situation really takes place at the threshold of the reaction, when the secondaries have large wavelength and are produced in one quantum state. In this case the secondaries are completely coherent (see the chapter 8) which is not so apart from the threshold. This situation has nothing to do with what is usually understood by the statistical behaviour. The constancy of the amplitude leads to the unphysical result, that the N particle production increases with energy as N particle phase space. The origin of this misunderstanding is the confusion of the amplitude with the coarse grained distribution. The imaginary contradiction with unitarity we speak about, is the direct analog /6/ of the classical paradox: contrast of the Liouwille theorem to the entropy grouth. Another misunderstanding is caused with the confusion of the phase space of the system during the process, say at final stage, with that at infinity, just the product of d^3p for independent particles. It results in wrong opinion that the existence of resonances contradicts to statistical assumptions and also hinders the studies of the interference effects, caused by quantum statistics.

7. The Ericson fluctuations.

In this chapter we illustrate our general discussion above by the consideration of the cases in which the amplitude irregularities of the amplitude are known under the name of Ericson fluctuations. We begin with the reactions like πp and $K^- p$ scattering, in which many resonances are known, then proceed to exotic channels like pp and K^+p and finally to some estimates for the many-body reactions.

Let us write the reaction amplitude in some partial wave as a sum of the resonance contributions

$$A^J_{ab} = \sum_c \frac{\gamma^*_{ac}\gamma_{cb}}{E-E_c+i\Gamma_c/2} \qquad (20)$$

Here γ_{ac} are partial and Γ_c the total decay widths. Two cases are possible. If the average spacing of the resonances $\mathcal{D} \gg \Gamma$, where Γ is the typical width, they do not interfere significantly. But in the opposite limit $\mathcal{D} \ll \Gamma$, as Ericson has noted /19/, the interference causes strong fluctuations of the cross section. Indeed, let γ_{ac} be random variables, then the amplitude (20) is also random, probably with gaussian distribution. The cross section $\sigma \propto |A|^2$ is then distributed according to the Releigh low:

$$dW(\sigma) \propto \exp(-\sigma/\sigma_0)\, d\sigma \qquad (21)$$

Note that the fluctuations are large and the more probable cross-section value is zero.

The essential difference with this case is that of the elastic scattering in which $a=b$ and $\gamma^*_{ac}\gamma_{cb} = |\gamma_{ac}|^2$ in (20) is positive for all terms. Let γ_{ac} be random, then σ becomes distributed according to χ^2 distribution with $2N$ degrees of freedom, where N is the number of overlapping resonances.

$$N \sim \Gamma/\mathcal{D} \qquad (22)$$

and the fluctuating part of the cross section σ^F is small: $\sigma^F \sim \sigma/N$. These estimates, presented in /22,23/ are in agreement with data /21/ if one uses the exponentially rising resonance spectrum*.

It is interesting to see how the transition to statistical behaviour settles from the low energy end. When resonances are sepa-

* It was taken from the statistical bootstrap model /9/. But in fact, only the constancy of the temperature is needed, so Pomeranchuck approach fits also well. It can be easily seen, $\rho(E) \propto \exp S(E)$ and since $dS = \frac{dE}{T}$ and T_{fin} =const, then $\rho(E) \propto \exp(\frac{E}{T_{fin}})$.

rate, the partial amplitude make circmles in the Argand diagram. When they overlap the motion becomes more complicated and finally reminds the random walk, which can be described statistically. This remineds the motion of the nonlinear oscillator in the phase space, where the criterium for the statistical behaviour is the same - the overlapping of the resonances /17/. As discussed above, in the elastic scattering the amplitude of this random walk is very small compared to the nonfluctuating average amplitude.

In the exotic channels, like pp, there are no resonances. The manybody reactions in such channels are, of course, still irregular for it is caused by the resonances of subsystems. And indeed, the statistical models are as good for πp reactions as for pp ones. There must be some trace of it in the elastic scattering also, in a form of the threshold singularities, say those due to reactions $pp \to N_1^* N_2^*$, where $N_{1,2}^*$ is some barion resonance. Note, that such singularity although weak, still gives the infinite derivative of the amplitude over energy and so corresponds to the long living state. The influence of such singularity decreases slowly from the threshold, so at any energy a lot of them "overlap". Let us estimate their number roughly as

$$N = \int \rho(m_1)\,\rho(m_2)\,\theta(E - m_1 - m_2)\,dm_1\,dm_2 \qquad (23)$$

In this case the fluctuating part of the cross section $\sigma^F \propto$ $\propto \exp\left(-\frac{\sqrt{s}}{T_{fin}}\right)$ and so it is not surprising that the search for them at s *34 Gev^2 gave the negative result/20/.

In the case of the multibody reactions the fluctuations can be large, but here another difficulty arises: the number of the variables is too large. Any averaging decreases the fluctuations, so inclusive data are smooth. The complete exclusive analysis with

---

* This is also true for reactions with close connections between in and out states, like $\pi^- p \to \pi^0 n$.

the necessary accuracy needs too high statistics. Let the typical fluctuation parameter Γ be of the order of 100 Mev. and the typical energy $E_0 \sim 1$ Gev. Then the total number of fluctuations in the reaction 2 to N particles is something like $(E_0/\Gamma)^{\frac{(N-1)(N+2)}{2}} \sim 10^{\frac{N^2}{2}}$. So the multibody reactions can hardly be used for the Ericson fluctuations observation and thus the statistical description is inevitable. As for the two body reactions, especially at large angles, the fluctuations can be studied and important information about the phenomenon, like the lifetime, can be obtained.

8. Are there produced pions coherent ?

Our first aim in this discussion is to attract the attention of the experimentalists to the study of the interference effects which can give valuable information about the phenomenon. Another one is to clarify the role of the quantum statistics effects, which are little discussed and are the matter of quite controversial arguments.

The term "coherency" is used in the same sense as, say, in optics for photons emitted by some source. It means that there is some special relation between the phases of quanta, so one can expect the interference effects. Unfortunately the wavelengh of secondaries in the hadronic production is too small to make the usual interference experiments possible. But there exists the so called second order interference, or the intensity interference, first studied in optics by Brown and Twiss and then applied to the measurements of the star radii /24/. This effect means the following: if one observes two quanta by two detectors, he can see their interference if they overlap in space and time. The last condition, or coherency condition, is better expressed in momentum and energy variables, for it is what is measured in experiments. The condition is:

$$|\vec{p}_1-\vec{p}_2| R \lesssim 1 \qquad |\omega_1-\omega_2|\cdot\tau \lesssim 1 \qquad (24)$$

where $p_{1,2}$ ($\omega_{1,2}$) are particle momenta (energies), R is the dimension of the source and τ is the emmission duration. The derivation of this condition can be found in any textbook.

As far as (24) is valid, the interference effect must be seen in the two particle spectrum, the peak for bosons and the dip for fermions. It is the width of the peak which is of interest. The ideas presented were developed in works /25-27/. The most detailed theory, connecting the two particle spectrum with the space-time properties of the source is developed in /27/.

Thus the question in the title of this chapter can be put as follows; what is the connection between the reaction range R and its duration τ and the wavelength λ ? According to our discussion of the low energy case (1) $R \sim N_\pi^{1/3}/m_\pi$ while the typical $\lambda = 1/(2\div3m_\pi)$. So the R and τ are several times larger than λ and so pions are mainly incoherent. In the high energy case this is true with even better precision unless the short range effects are considered.

The fundamental importance of this statement can be seen in different ways. First it is consistent with our general discussion of the amplitude irregularities because the number of the oscillations in any variable is something like kR or $\omega\tau$. As is estimated in the previous chapter, it is rised into large power - the number of variables- so there is no need to have $kR, \omega\tau$ really large in order to statistical description holds. Second, if pions are incoherent one may pass to classical description, say to use Boltsmann statistics, as we did in chapter 3.

The opposite point of view, that the pions are completely coherent, can also be found in the literature. It is the confusion of the microcanonical distribution with the constant reaction

amplitude, discuseed above. Note, that in this case there are no correlations of the Goldhaber type, which is not so experimentally. The complete coherency is also assumed in the Heisenberg theory /1/ based on the classical wave approach. In this respect this theory is the opposite case to the hydrodynamical model which implies the case of incoherency. Let us mention to this end the attempt /28/ to connect the Bose statistics effects with the multiplicity distribution, in particular with the positive value of the $f_2 \equiv \langle n(n-1)\rangle - \langle n\rangle^2$. Although this effect contributes into right direction, the degree of the coherency of pions seems to be too low to explaine the data. On the other hand, the multiplicity distribution can be completely explained by the leading particle effects /5/, and the positive value of f_2 by the admixture of the diffractive processes /29/.

We finish this chapter with the following comment. The lifetime τ of the system, created in the collision, is of greate interest. It can be measured as a width of the interference peak in variable $|\omega_1-\omega_2|$. There are arguments /27/ that this effect is more prominent than angular correlations /25/. This is so because the random motion of secondaries makes the lifetime larger than the system dimensions. The estimations in the Pomeranchuck model in/27/ is $\tau \sim N_\pi^{2/3}/m_\pi$. In the statistical bootstrap model /9c/ the successive evaporation of pions makes it even larger $\tau \propto N_\pi$. The only available data on τ, that on Ericson fluctuations /21/, give large lifetime indeed, τ = 1/30 Mev= 6.6 fermi! The interference studies of this quantity seems to be possible with the available statistics at bubble chambers.

Acknowledgements.

The author is much indebted to valuable discussions with E.L.Feinberg, M.I.Podgoretsky, B.V.Chirikov.

References.

1. W.Heisenberg. Zs. Phys. 101 (1936) 533, 113 (1939) 61, 164 (1949) 65.
2. E.Fermi. Progr. Theor. Phys. 5 (1950) 570.
3. I.Ya. Pomeranchuck. Doklady Akademii Nauk 78 (1951) 889.
4. L.D.Landau. Isvestia Akademii Nauk, ser. fis. 17 (1953) 51.
S.Z.Belenky, L.D.Landau. Uspekhi Fis. Nauk. 56 (1955)309.
5. E.V.Shuryak. "High Energy Multiple Production of Hadrons and Landau Hydrodynamical Theory". Proceedings of the V Simposium on Many Particle Hadrodynamics, Leipzig, 1974; and submitted to Nuclear Physics.
6. I.V.Sisakyan, E.L.Feinberg, D.S.Chernavsky. Trudy FIAN (Lebedev Inst. Proceedings) 16 (1971) 50.
E.L.Feinberg. Uspekhi Fis. Nauk. 104 (1971) 539.
E.L.Feinberg. Physics Reports 5c (1972)237.
7. E.V.Shuryak. Phys. Lett. 42B (1972) 357.
8. E.V.Shuryak. Yadernaya Fisika (Soviet Journal of Nuclear Physics) 20 (1974) 549.
9. R.Hagedorn. Nuovo Cimento 35 (1965) 216.
R.Hagedorn, J.Ranft. Nuovo Cimento Suppl. 6 (1968) 169.
S.Frautchi. Phys. Rev. 3D (1971) 2821.
10. T.Erwin et al. Phys. Rev. Lett. 27 (1971) 1534.
11. T.F.Hoang, D.Rhims, W.A.Cooper. Phys. Rev. Lett. 27 (1971) 168.
12. S.Z.Belenky, V.M.Maximenko, A,N.Nikishov, I.L.Rosental. Uspekhi Fis. Nauk. 62 (1956) N2, page 1.
13. E.Beth, G.E.Uhlenbeck. Physica. 3 (1936) 729.
14. British- Scandinavian colloboration. Phys. Lett. 44B (1973) 521, 527; and report to International Conference on High Energy Physics, London, 1974.

15. H.Antinucci et al. Nuovo Cimento Lett. 6 (1973) 121.
16. N.S.Krylov. Raboty po obosnovaniyu statisticheskoy fisiki. (Works on the Statistical Physics Foundation) Moskow, 1950.
17. G.M.Zaslavsky, B.V.Chirikov. Uspekhi Fis. Nauk. 105 (1971) 3.
18. N.M.Pukhov, D.S.Chernavsky. Teor. Mat. Fisika. 7 (1971) 219.
19. T.Ericson, R.Mayer-Kuckuck. Ann. Rev. Nucl. Sci. 16 (1966) 183.
20. J.Allaby et al. Phys. Lett. 23 (1966) 389.
21. F.H.Schmidt et al. Phys. Lett 45B (1973) 157.
22. S.Frautchi. Nuovo Cimento 12A (1972) 133.
23. P.J.Carlson . Phys. Lett. 45B (1973) 161.
24. R.H. Brown, R.Q.Twiss. Phyl. Mag. 45 (1954) 633, Proc. Roy. Soc. A242 (1957) 300, A243 (1957) 291.
25. G.Goldhaber, S.Goldhaber, W.Lee, A.Pais. Phys. Rev. 120(1960) 300.
26. G.I.Kopylov, M.I.Podgoretsky. Yadernaya Fisika. 14 (1972) 1081, 15 (1972) 392.
G.I.Kopylov. Phys. Lett. 50B (1974) 472.
27. E.V.Shuryak. Phys. Lett. 44B (1973) 387, Yadernaya Fisika 18 (1973) 1302.
28. F.Cooper, E.Schonberg. Phys. Rev. 8D (1974) 334.
W.I.Knox. Phys. Rev. 10D (1974) 65.
29. H. Harari, E.Rabinovichi. Phys. Lett. 43B (1973) 49.
K.Fialkowski, H.I.Miettinen. Phys. Lett. 43B (1973) 61.
L.Van Hove. Phys. Lett. 43B (1973) 65.

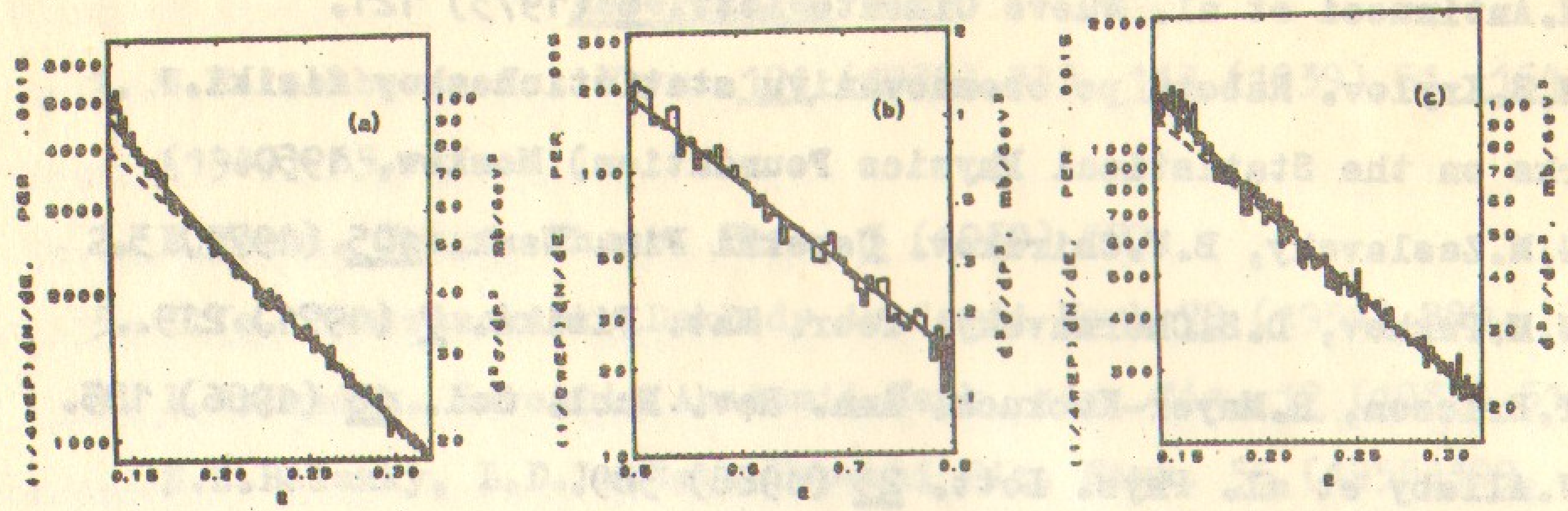
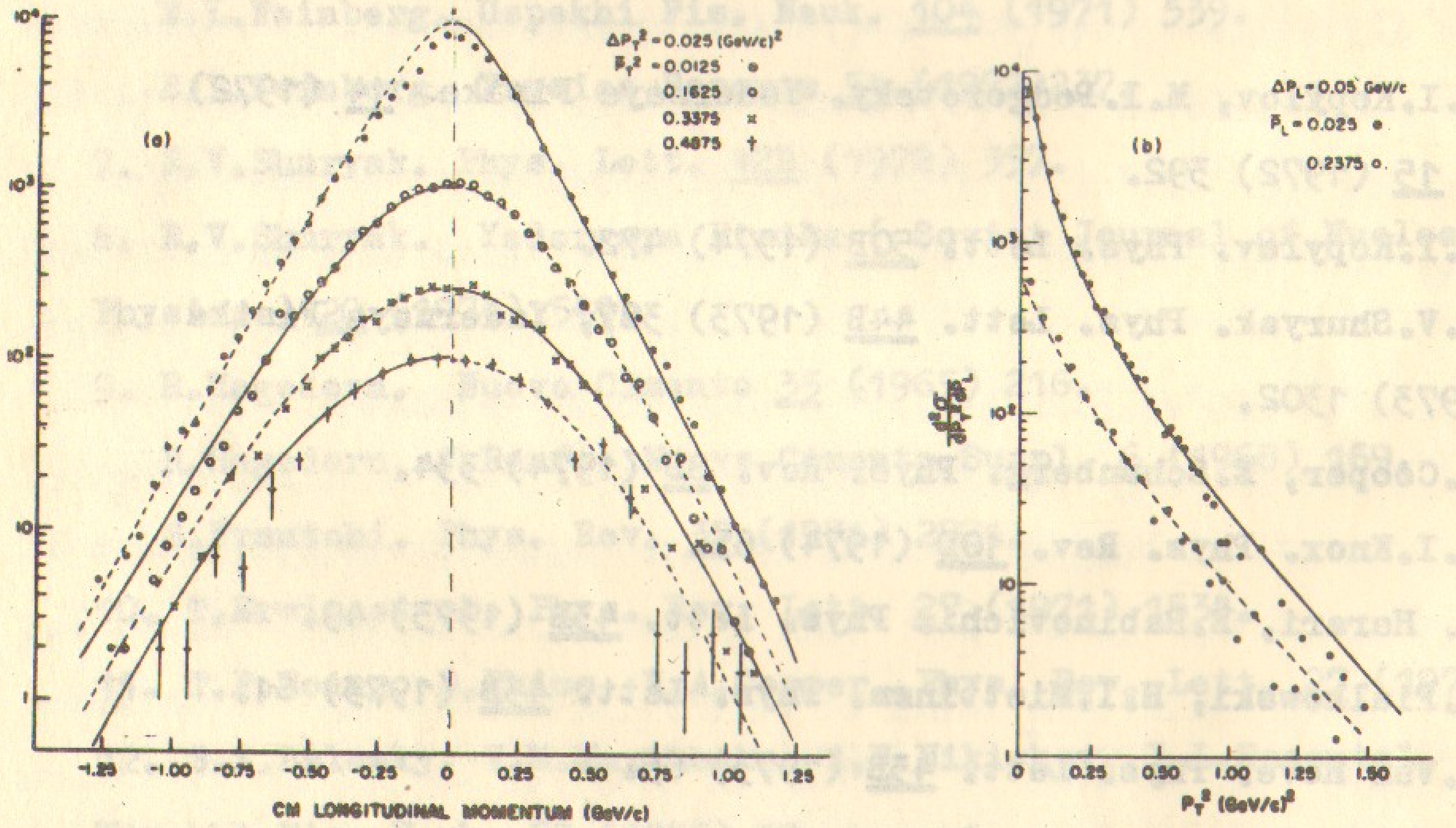
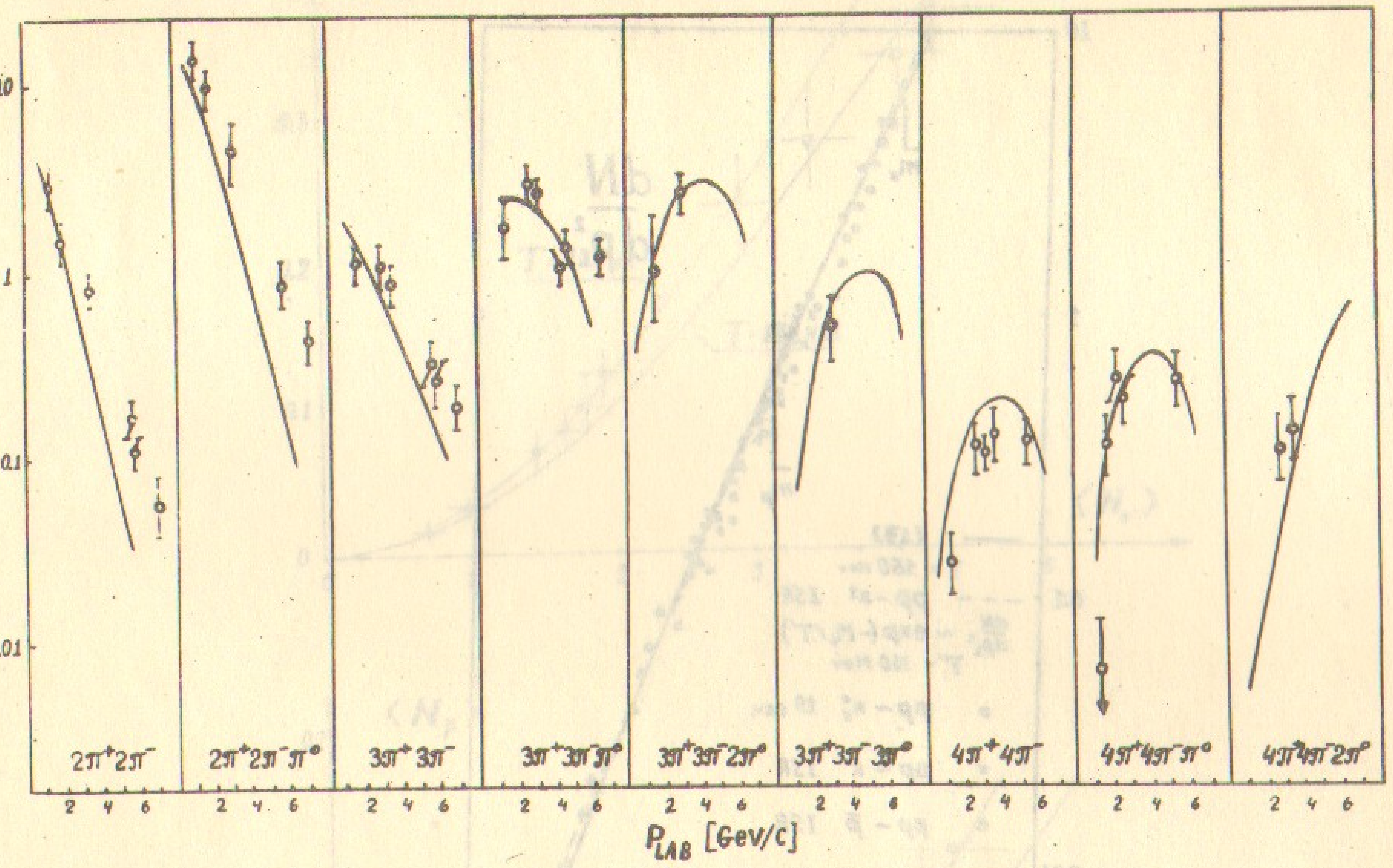


Fig. 1. a) The distributions of π^+, K^o, π^- in CM energy in K^+p collisions at 12 Gev./10/. The solid line is Bose distribution with T= 142 Mev., the dashed one is simple exponent.



b) The π^+ distribution in $p\bar{p}$ annihilation at 2.32 Gev. /11/. The lines are Bose distribution with T=130 Mev.



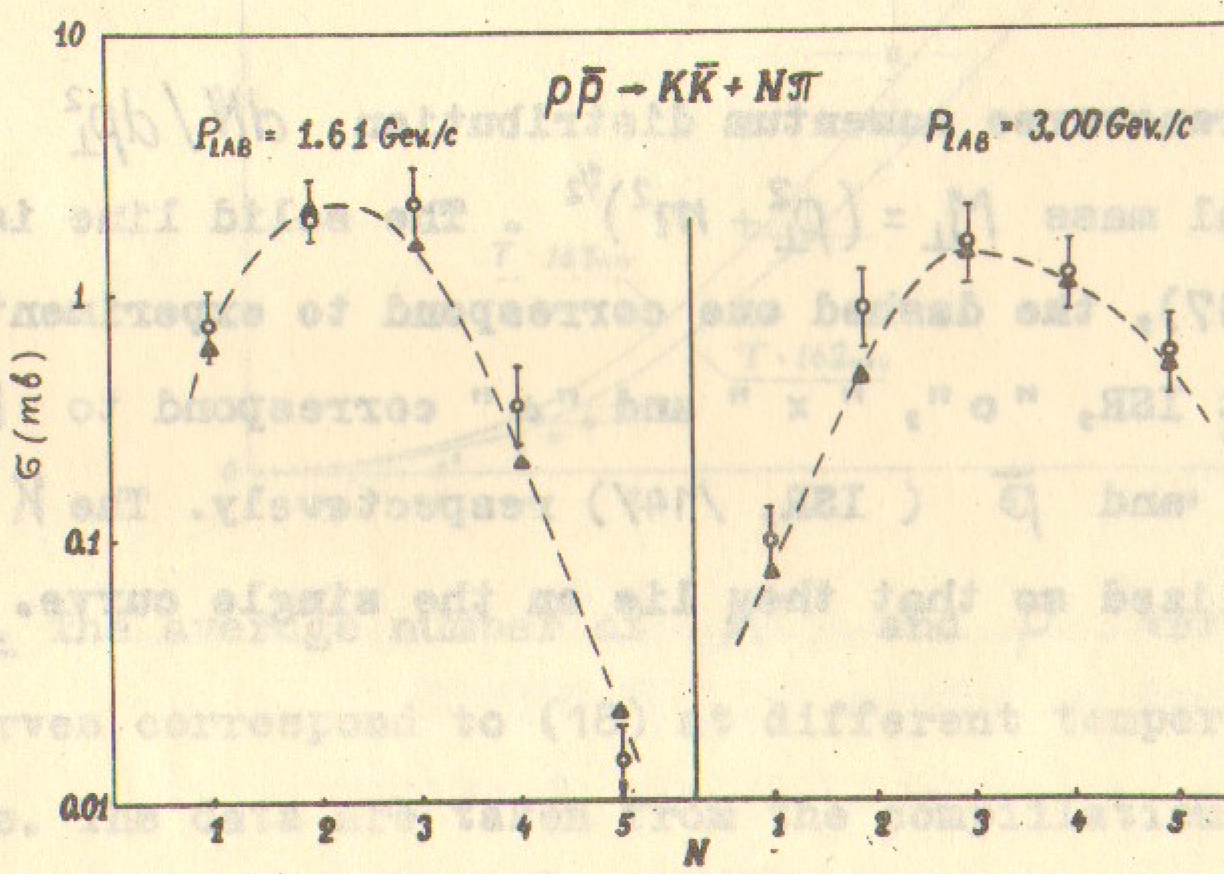


Fig. 2. The production of particular channels in $p\bar{p}$ annihilation versus P_{LAB}. The curves correspond to the calculations described in the text, normalised to total annihilation cross section. The data are taken from the compillation CERN HERA 70-3.

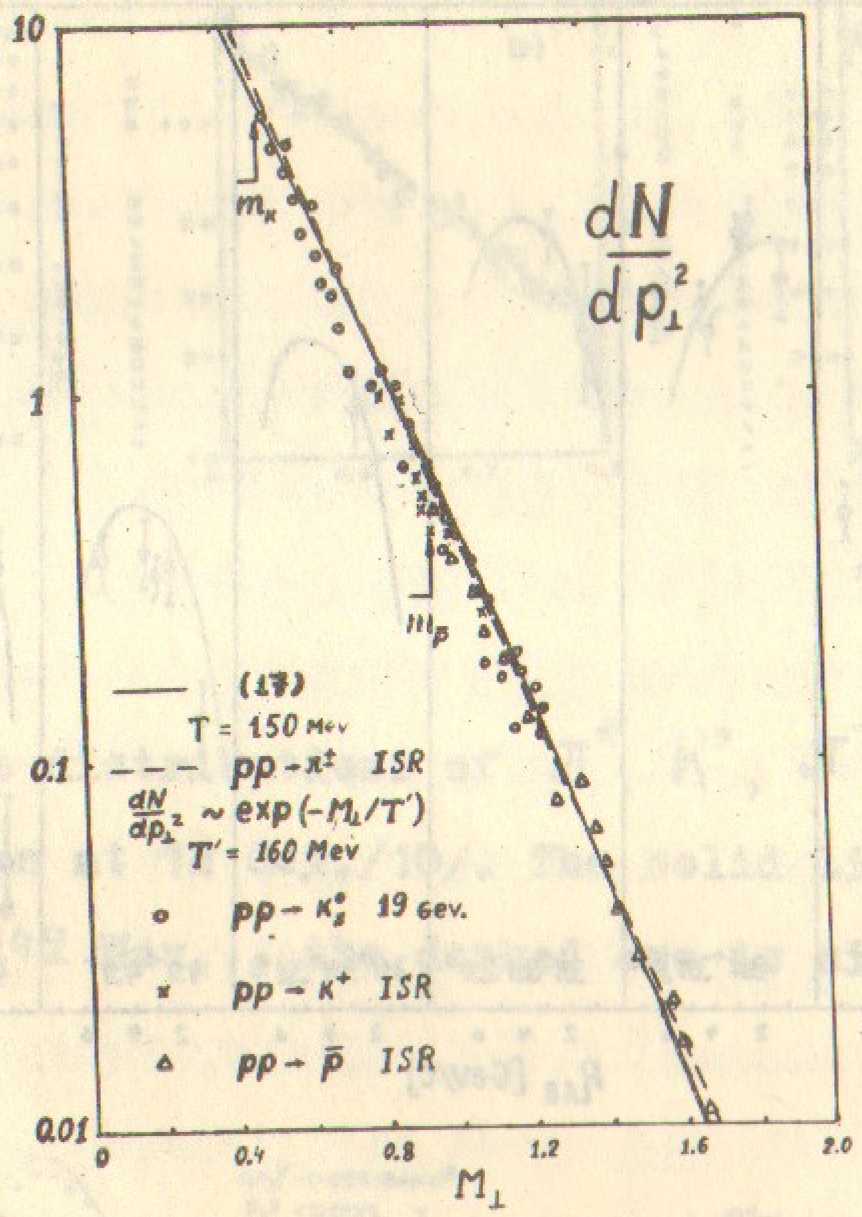


Fig. 3. The transverse momentum distribution $dN/dp_\perp^2$ versus the longitudinal mass $M_\perp = (p_\perp^2 + m^2)^{1/2}$. The solid line is the distribution (17), the dashed one correspond to experimental distribution at ISR, " o ", " x " and " Δ " correspond to K^0_S (at 19 Gev.), K^- and $\bar p$ (ISR, /14/) respectevely. The K and $\bar p$ data are normalized so that they lie on the single curve.

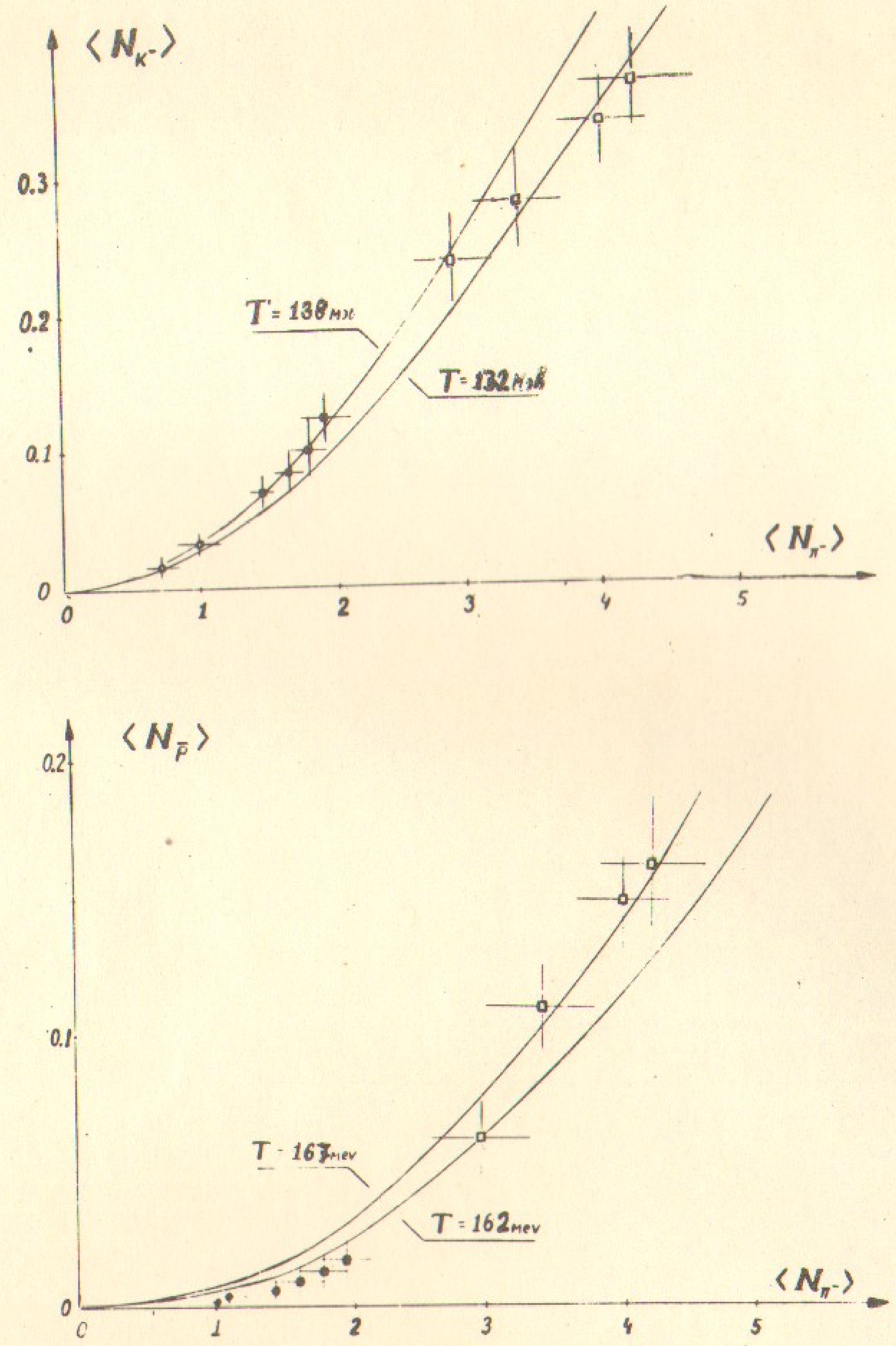


Fig. 4. The average number of K^- and $\bar p$ versus that of pions. The curves correspond to (18) at different temperatures, shown at figures. The data are taken from the compilation /15/.

Ответственный за выпуск Г.А.СПИРИДОНОВ

Подписано к печати 28.XI-1974г. МН 08597

Усл. 2,0 печ. л., тираж 200 экз. Бесплатно

Заказ № I08

Отпечатано на ротапринте в ИЯФ СО АН СССР