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IN THE REACTION  $e^+e^- \rightarrow \pi^+\pi^-$

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The decay  $f \rightarrow 2\pi^+2\pi^-$  proceeding via  $\rho^0\rho^0$ -intermediate state is considered. Using the value of the coupling constant  $g_{f\rho\rho}$  thus obtained the charge asymmetry of  $\pi$ -mesons in the reaction  $e^+e^- \rightarrow \pi^+\pi^-$  is estimated. At the total energy  $2E=1.3$  Gev the asymmetry may be as large as 8.1%.

The possibility of investigating with  $e^+e^-$  colliding beams the resonances with positive charge parity produced via two-photon intermediate state has been first noted in /1/. The problem of the  $\pi$ -meson charge asymmetry in the reaction  $e^+e^- \rightarrow \pi^+\pi^-$  at the energy near the  $f$ -meson mass was considered in detail in /2/ ( see also /3,4/ ). To estimate the value of the charge asymmetry the knowledge of the coupling constant of the  $f$ -meson with  $\rho$ -mesons  $g_{f\rho\rho}$  is needed. Values of  $g_{f\rho\rho}$  used in /2-4/ differ notably from each other. In this work for calculation of  $g_{f\rho\rho}$  we use the assumption consistent with the experimental data /5,6/ that  $f$ -meson decays into  $2\pi^+2\pi^-$  via  $\rho^0\rho^0$ -intermediate state. Comparing the experimental width  $\Gamma_{f \rightarrow 2\pi^+2\pi^-}$  with the decay probability calculated according to the assumption above one can find  $g_{f\rho\rho}$ .

The vertex  $f \rightarrow \rho^0\rho^0$  is written in the form

$$T_{f \rightarrow \rho\rho} = - \frac{g_{f\rho\rho}}{m_f} t_{\mu\nu} (F_{\mu\lambda}^{(1)} F_{\lambda\nu}^{(2)} + F_{\mu\lambda}^{(2)} F_{\lambda\nu}^{(1)}) \quad (1)$$

where  $t_{\mu\nu}$  - symmetrical tensor describing polarisation of the  $f$ -meson with the momentum  $\kappa = \kappa_1 + \kappa_2$  ( $\kappa_\mu t_{\mu\nu} = 0$ ,  $t_{\mu\mu} = 0$ );  $F_{\mu\nu}^{(i)} = \kappa_{i\mu} e_\nu^{(i)} - \kappa_{i\nu} e_\mu^{(i)}$  ( $i=1,2$ );  $\kappa_1, \kappa_2$  - momenta of  $\rho$ -mesons;  $e^{(1)}, e^{(2)}$  - polarisations of  $\rho$ -mesons;  $m_f$  -  $f$ -meson mass.

Two diagrams of Fig.1 corresponding to decays of  $\rho$ -mesons into  $\pi_1^+\pi_2^-, \pi_3^+\pi_4^-$  and  $\pi_1^+\pi_4^-, \pi_3^+\pi_2^-$  should be taken into account.

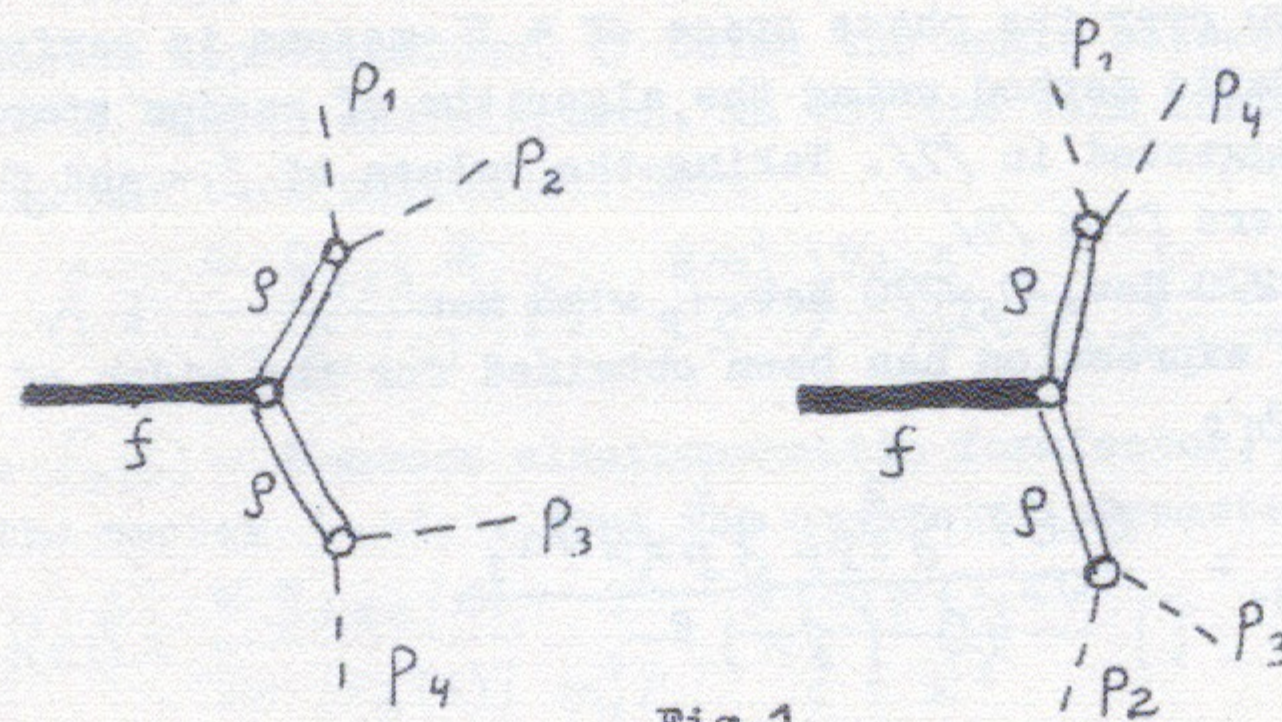


Fig.1.

After simple calculations the amplitude of the decay  $f \rightarrow 2\pi^+ 2\pi^-$  takes the form

$$T_{f \rightarrow 4\pi} = - \frac{g_{fpp} g_{p\pi\pi}^2}{m_f} \epsilon_{\mu\nu} (R_{\mu\nu}^{(1)} + R_{\nu\mu}^{(1)} + R_{\mu\nu}^{(2)} + R_{\nu\mu}^{(2)}) \quad (2)$$

where

$$R_{\mu\nu}^{(1)} = \frac{4[(P_2 P_4) P_{1\mu} P_{3\nu} - (P_2 P_3) P_{1\mu} P_{4\nu} + (P_1 P_3) P_{2\mu} P_{4\nu} - (P_1 P_4) P_{2\mu} P_{3\nu}]}{[m_p^2 - (P_1 + P_2)^2 - i m_p \Gamma_p] [m_p^2 - (P_3 + P_4)^2 - i m_p \Gamma_p]} \quad (3)$$

Here  $P_i$  - momentum of the  $i$ -th  $\pi$ -meson,  $R_{\mu\nu}^{(2)}$  is obtained from  $R_{\mu\nu}^{(1)}$  substituting  $P_2 \leftrightarrow P_4$ .

Averaging over  $f$ -meson polarisations the square of the amplitude is obtained:

$$|T_{f \rightarrow 4\pi}|^2 = \frac{g_{fpp}^2 g_{p\pi\pi}^4}{5 m_f^2} (S_{\alpha\beta} S_{\alpha\beta}^* - \frac{1}{3} S_{\alpha\alpha} S_{\beta\beta}^*) \quad (4)$$

where

$$S_{\alpha\beta} = R_{\alpha\beta}^{(1)} + R_{\beta\alpha}^{(1)} + R_{\alpha\beta}^{(2)} + R_{\beta\alpha}^{(2)} \quad (\alpha, \beta = 1, 2, 3).$$

Integration over the phase space of 4  $\pi$ -mesons is performed by the Monte-Carlo method using the algorithm of random stars generation suggested in /7/. Taking the values of  $f$ - and  $p$ -meson parameters from /8/

$$m_f = 1270 \text{ Mev}, m_p = 770 \text{ Mev}, \Gamma_p = 146 \text{ Mev}$$

the following expression has been obtained for the width of the decay  $f \rightarrow 2\pi^+ 2\pi^-$ :

$$\Gamma_{f \rightarrow 2\pi^+ 2\pi^-} = \frac{0.67 g_{fpp}^2 g_{p\pi\pi}^4 m_f}{40 (2\pi)^8} \quad (5)$$

Averaging of  $\Gamma_{f \rightarrow 2\pi^+ 2\pi^-}$  from /8/ and recent experimental data /6,9/ gives

$$\Gamma_{f \rightarrow 2\pi^+ 2\pi^-} = (5.5 \pm 1.0) \text{ Mev} \quad (6)$$

From (5) and (6)  $g_{fpp}$  is found:

$$g_{fpp} = 22.4 \pm 2.5 \quad (7)$$

It is interesting to compare  $g_{fpp}$  with the coupling constant of the  $f$ -meson with  $\pi$ -mesons  $g_{f\pi\pi}$ . Using the expression

$$\Gamma_{f \rightarrow 2\pi} = \frac{g_{f\pi\pi}^2 m_f}{320 \pi} \left(1 - \frac{4m_\pi^2}{m_f^2}\right)^{5/2}$$

and the experimental value from /8/

$$\Gamma_{f \rightarrow 2\pi} = (134 \pm 9) \text{ Mev}$$

$g_{f\pi\pi}$  is calculated:  $g_{f\pi\pi} = 10.9 \pm 0.4$ . Thus

$$g_{fpp} / g_{f\pi\pi} = 2.1 \pm 0.2$$

implying that  $f$ -meson coupling with  $p$ -mesons is notably stronger than that with  $\pi$ -mesons.

Proceed now to estimation of the  $\pi$ -meson charge asymmetry in the reaction  $e^+ e^- \rightarrow \pi^+ \pi^-$ . Using the results of /2/ the differential cross-section of this reaction can be written as

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} (1 + 2 \text{Re} \delta \cos \theta) \quad (8)$$

where  $d\sigma_0/d\Omega$  is the cross-section of the reaction in the one-photon approximation,  $\theta$  being the angle between the momenta of  $\pi^-$ -meson and electron. In (8) the term containing  $\delta^2$  squared has been neglected. Here

$$\delta = \frac{\alpha g_{f\pi\pi} x}{4\pi F_\pi(s)} \left(1 - \frac{4m_\pi^2}{s}\right)^{1/2} \frac{s}{m_f^2} \frac{s}{s - m_f^2 + i m_f \Gamma_f} \quad (9)$$

where  $F_\pi(s)$  -  $\pi$ -meson electromagnetic formfactor,  $x$  determines the vertex  $f \rightarrow e^+ e^-$ . Thus the asymmetry parameter

$$2 \text{Re} \delta = \frac{\alpha g_{f\pi\pi} |x|}{2\pi |F_\pi(s)| m_f^3 \Gamma_f} \frac{s^2}{1+x^2} \frac{x \cos \varphi + \sin \varphi}{\left(1 - \frac{4m_\pi^2}{s}\right)^{1/2}} \quad (10)$$

where

$$x = (s - m_f^2) / m_f \Gamma_f, \quad \varphi = \arg x - \arg F_\pi = \varphi_x - \varphi_\pi$$

To estimate  $|x|$  its lower limit obtained from the unitarity condition in /2/ is taken:

$$|x| \geq \text{Im } x = \frac{1}{6} g_{f\gamma\gamma} \quad (11)$$

where  $g_{f\gamma\gamma}$  - the coupling constant of the  $f$ -meson with  $\gamma$ -quanta. Vector dominance model gives

$$g_{f\gamma\gamma} = g_{f\rho\rho} \cdot 4\pi g_\rho^{-2} \quad (12)$$

where the value of  $g_\rho$  determining the  $\rho$ - $\gamma$  transition is known from  $e^+e^-$  colliding beam experiments /10/. In (12) we have neglected the  $\omega$ -meson contribution since  $4\pi g_\omega^{-2}$  is small as well as  $f$ - $\phi$  interaction. Using the value of  $g_{f\rho\rho}$  from (7)

$$g_{f\gamma\gamma} = 8.8 \pm 1.3 \quad (13)$$

Then (11) gives

$$|x|_{\min} = 1.5 \pm 0.2 \quad (14)$$

$e^+e^-$  colliding beam experiments indicate /11/ that in the energy region of interest the formfactor  $|F_\pi(s)|$  is approximately constant:

$$|F_\pi(s)| \approx 1.8 \pm 0.2$$

Using this value and  $|x|$  from (14)  $2\text{Re } \delta$  can be calculated. The values of the asymmetry parameter  $2\text{Re } \delta$  thus obtained are shown in Fig.2 for  $\varphi = 0$  and  $\varphi = 1.31$ . If  $\varphi$  were equal to 1.31 the asymmetry parameter should be maximal and equal to  $(8.1 \pm 1.4)\%$  at  $2E = 1.3$  GeV. Note that it is a lower limit of the asymmetry parameter because of (11). For comparison the non-resonant contribution to the asymmetry parameter due to soft virtual photons is 7% at  $\theta = 45^\circ$  and  $\Delta E/E \sim 1\%/12/$ .

The value of  $g_{f\gamma\gamma}$  found can be used to calculate the decay width of  $f$ -meson into  $2\gamma$ :

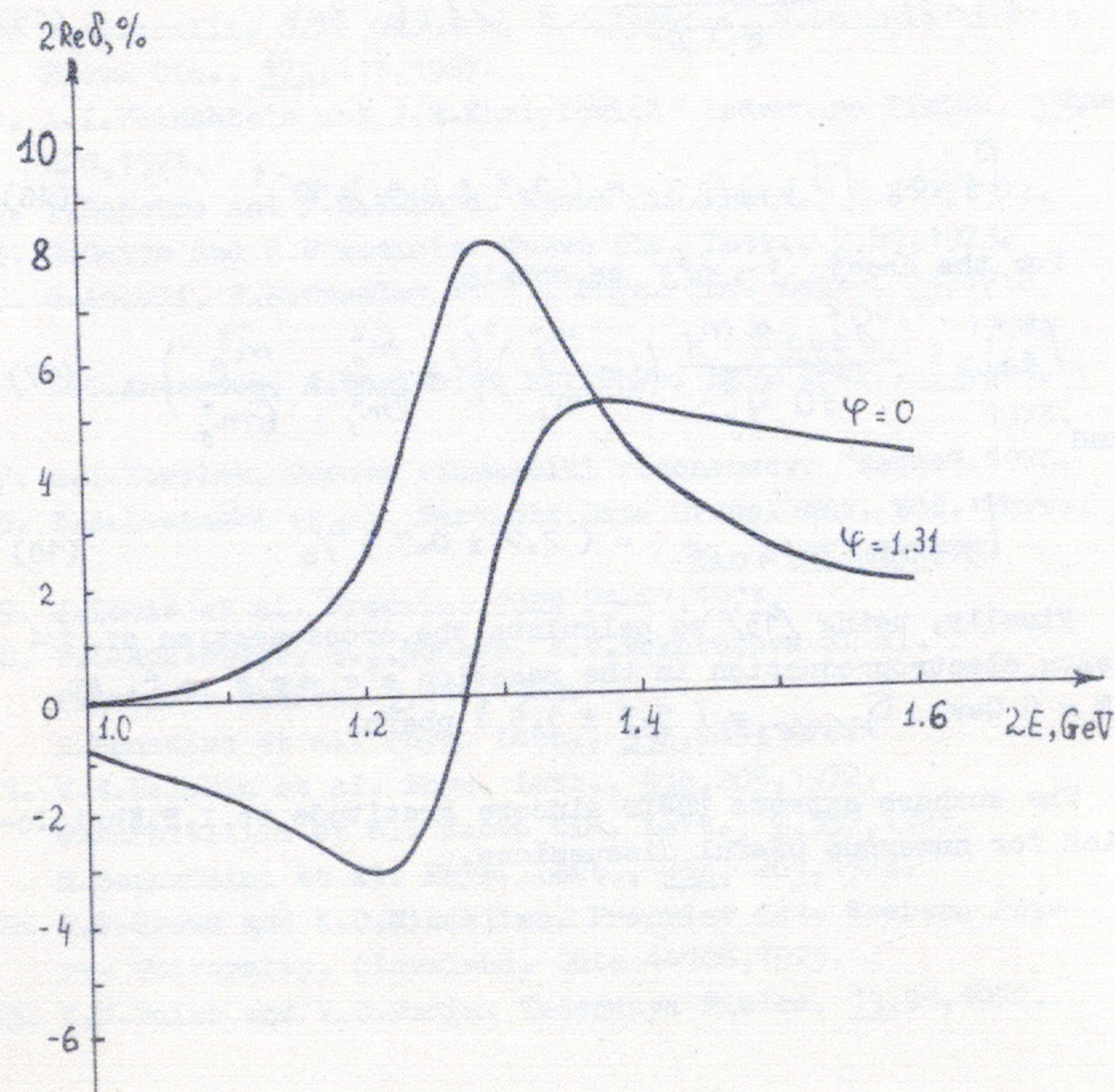


Fig.2. Energy dependence of the asymmetry parameter  $2\text{Re } \delta$ .

$$\Gamma_{f \rightarrow 2\gamma} = \frac{\alpha^2 g_{f\gamma\gamma}^2 m_f}{80\pi} = (21 \pm 6) \text{ keV} \quad (15)$$

and

$$\Gamma_{f \rightarrow 2\gamma} / \Gamma_{f \rightarrow \text{all}} = (1.3 \pm 0.4) \times 10^{-4} \quad (16)$$

For the decay  $f \rightarrow \rho^0 \gamma$  we obtain

$$\Gamma_{f \rightarrow \rho\gamma} = \frac{g_{f\rho\rho}^2 \alpha m_f}{10 g_\rho^2} \left(1 - \frac{m_\rho^2}{m_f^2}\right)^3 \left(1 + \frac{m_\rho^2}{2m_f^2} + \frac{m_\rho^4}{6m_f^4}\right) \quad (17)$$

and

$$\Gamma_{f \rightarrow \rho\gamma} / \Gamma_{f \rightarrow \text{all}} = (2.7 \pm 0.7) \% \quad (18)$$

Finally, using /13/ we calculate the cross-section of  $f$ -meson electroproduction in the reaction  $e^+e^- \rightarrow e^+e^- + f$ . At  $2E = 6 \text{ GeV}$   $\sigma_{e^+e^- \rightarrow e^+e^- f} = (5.3 \pm 1.5) \text{ nbarn}$ .

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