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V.V.Flambaum

# $K_L \to 2\mu$ AND $K \to \pi \nu \bar{\nu}$ DECAYS IN THE WEINBERG MODEL

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# $K_L \to 2\mu$ AND $K \to \pi \nu \bar{\nu}$ DECAYS IN THE WEINBERG MODEL

V.V.Flambaum

Institute of Nuclear Physics
Novosibirsk 90, USSR

By the method of the $\xi$-formalism the rates of $K_L \to \mu^+\mu^-$ and $K \to \pi \nu \bar{\nu}$ processes are calculated in the Weinberg model based on SU(4) hadron symmetry.

The rates of the rare decay modes of the kaons in the gauge theory of weak and electromagnetic interections have been calculated in the papers /I–4/. However, the results of the papers /I/ and /2/, in which $\Gamma(K_L \to \mu^+\mu^-)$ in the Weinberg model /5/ was calculated, disagree. One of the purpose of this work is the independent calculation of the probability of this process. Our result confirms that of the work /I/. IN the second part of our work the rates of the processes $K \to \pi \nu \bar{\nu}$ and $K \to \pi e \bar{e}$ are found and the results obtained also differ from the expressions presented in /2/. The calculation is performed by the method of the generalized $\xi$-formalism /6/ in the Weinberg model under the assumption $m_p \ll m_{p'} \ll m_W$, where $m_W$, $m_p$ and $m_{p'}$ are the masses of W-bozon, P-quark and P-quark. Use of the $\xi$-formalism provides us with the supplementary verification of the result – the final expression should not contain an arbitrary parameter $\xi$.

I. As it was pointed in /4/, the amplitude of the annihilation of n- and $\lambda$-quarks into $\mu^+\mu^-$ (Fig I and 2) can serve as the effective Lagrangian for the decay $K_L \to 2\mu$. The contribution to this amplitude of the diagram with intermediate W-bozon (Fig I) is equal to

$$M_W = -\frac{g^4 m_{p'}^2}{32\pi^2 m_W^4}\left(2\ln\frac{m_W^2}{m_{p'}^2} + 1 - \frac{7\xi - 1}{2(\xi-1)}\ln\xi\right)\cdot$$
$$\cdot \sin\theta\cdot\cos\theta\ \bar{n}\gamma_\mu(1+\gamma_5)\lambda\ \bar{\mu}\gamma_\mu(1+\gamma_5)\mu \qquad \text{(I)}$$

In our notaition $\frac{G}{\sqrt{2}} = \frac{g^2}{2m_W^2}$, $\theta$ is Cabibbo angle, and W-bozon propogator is

$$-\frac{i}{q^2 - m_W^2}\left[g_{\mu\nu} - \frac{q_\mu q_\nu\left(1-\frac{1}{\xi}\right)}{q^2 - m_W^2/\xi}\right]$$

Now calculate $z\lambda n$-vertex assuming for simplicity that $g' = 0$ (this simplification was proposed by A.I.Vainshtein, who performed also the corresponding calculation in the Feynman gauge). Four-dimensional divergence of Z-bozon current is equel to

$$\partial_\mu J_\mu^z = i m_{p'} g \bar{P}'\gamma_5 P' + 2g\xi i\left[W_\nu^-\partial_\nu\partial_\mu W_\mu^+ - W_\nu^+\partial_\nu\partial_\mu W_\mu^-\right] \qquad (2)$$

Here we omit the terms which contain the neutral scalar bozons and Feynman-DeWitt-Faddeev-Popov ghost particles,because they do not contribute to Zn$\lambda$ -vertex to lowest order of the perturbation theory.Using (2) to obtain Ward identity,we find that Zn$\lambda$ -vertex is

$$E^z_\mu(g'=0) = -\frac{1}{16\pi^2}\frac{m^2_{\rho'}g^3}{m^2_w}\sin\theta\cos\theta\,\gamma_\mu(1+\gamma_5)\cdot$$

$$\cdot\left[-2\ln\frac{m^2_w}{m^2_{\rho'}} + 3 + \frac{7\xi-1}{2(\xi-1)}\ln\xi\right] \tag{3}$$

In this notation the contribution of the diagram with intermediate Z-bozon (Fig 2) is of the form

$$M_z = \bar{n}\,E_\mu(g')\lambda\,\frac{\sqrt{g^2+g'^2}}{2m_z^2}\,\bar{\mu}\gamma_\mu\gamma_5\mu \tag{4}$$

Here we retain only axial muon current because only it contributes to the rate of the decay $K_L\to\mu^+\mu^-$.Adding (4) to (I) with $g'\neq o$,one obtains the expression for the annihilation amplitude which does not depend on $\xi$ .On the other hand, comparing (3),(4) and (I) we see that the parameter $\xi$ is cansellled out for $g'\neq o$ if

$$E^z_\mu(g') = E^z_\mu(g'=0)\,\frac{\sqrt{g^2+g'^2}}{g} = E^z_\mu(g'=0)\,\frac{m_z}{m_w} \tag{3a}$$

Formulas(3),(3a) and (4) in the Feynman gauge ( $\xi$ =I) were obtained also by direct summation of the diagrams which contribute to $E^z_\mu$.

In the paper/2/where the calculation are performed in the Feynman gauge,Zn$\lambda$ -vertex was also found by using the Ward identity, but the form of this identity differs from identity obtained in our work. One can obtain Zn$\lambda$ -vertex presented in /2/ using formula (2),but without the second term on the right hand side.But the contribution of this term is essential.E.g.,using $\xi$ -formalism we can see that without this term the final resalt depends on $\xi$ that is certainly wrong.

Adding up the contribution of the diagram I and 2,we obtain the amplitude of the process $\lambda\bar{n}\to\mu^+\mu^-$



$$M = -\frac{G^2 m^2_{\rho'}\cos\theta\sin\theta}{4\pi^2}\,\bar{n}\gamma_\mu(1+\gamma_5)\lambda\,\bar{\mu}\gamma_\mu\gamma_5\mu \tag{5}$$

It coincides with the result of the article /I/. Thus,the ratio of the rates of $K_L\to\mu^+\mu^-$ and $K^+\to\mu^+\nu$ decays is equel to /T/

$$\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)} = \frac{G^2 m^4_{\rho'}\cot^2\theta}{2\pi^4} \tag{6}$$

It is necessary to point that in the formula (6) the contribution of the graph with two intermediate photons(see /7/) is not taken into account .Therefore,one can use (6) only for obtaining upper limit on the mass of P'-quark (see /I/).

2. If we know Zn$\lambda$ -vertex it is easy to estimate the probability of the decay $K\to\pi\nu\bar{\nu}$. In the quark model this process is described by the graphs I,2,3.The sum of the diagram I and 2 does not depend on the parameter $\xi$ and is of the form

$$M = \frac{G^2 m^2_{\rho'}}{8\pi^2}\left(3\ln\frac{m^2_w}{m^2_{\rho'}} - 1\right)\bar{n}\gamma_\mu(1+\gamma_5)\lambda\,\bar{\nu}\gamma_\mu(1+\gamma_5)\nu \tag{7}$$

Due to the difference between the Zn$\lambda$ -vertices which we discuss above,the formula (7) does not coincide with the corresponding formula in /2/,but in this case the difference consists only in another non-logarithmic term in the bracket.

Now estimate the contribution of the diagram 3 . The characteristic parameter which defines value of this graph, is the ratio $\frac{m^2}{m^2_w}$ where m is of order of the mass of uncharmed quark or the mass of K-or $\pi$ -mesons.In the diagrams I and 2 this parameter is $\frac{m^2_{\rho'}}{m^2_w} \gg \frac{m^2}{m^2_w}$ .Thus, the relative contribution of the diagram 3 is of the order $\left(\frac{m}{m_{\rho'}}\right)^2$ and we can neglect it with the taken accuracy.

Using (7) and taking into account the relations

$$\langle\pi^0|\bar{\lambda}\gamma_\mu(1+\gamma_5)n|K^0\rangle = -\langle\pi^0|\bar{\lambda}\gamma_\mu(1+\gamma_5)p|K^+\rangle,$$

$$\langle\pi^+|\bar{\lambda}\gamma_\mu(1+\gamma_5)n|K^+\rangle = \langle\pi^-|\bar{\lambda}\gamma_\mu(1+\gamma_5)p|K^0\rangle$$

and $$2\Gamma(K^+\to\pi^0\bar{e}\nu)=\Gamma(K^0\to\pi^-\bar{e}\nu)$$

we find

$$\frac{\Gamma(K_S\to\pi^0\nu\bar{\nu})}{\Gamma(K_L\to\pi e\nu)}=\frac{\Gamma(K^+\to\pi^+\nu\bar{\nu})}{2\Gamma(K^+\to\pi^0\bar{e}\nu)}=\left[\frac{G m^2_{\rho'}}{4\pi^2}\left(3\ln\frac{m^2_W}{m^2_{\rho'}}-1\right)\right]^2 \qquad (8)$$

where $$\Gamma(K\to\pi\nu\bar{\nu})=\Gamma(K\to\pi\nu_e\bar{\nu}_e)+\Gamma(K\to\pi\nu_\mu\bar{\nu}_\mu),$$
$$\Gamma(K_L\to\pi e\nu)=\Gamma(K_L\to\pi^+e\bar{\nu})+\Gamma(K_L\to\pi^-\bar{e}\nu)$$

The corresponding expression presented in /2/ is four times larger than (8), perhaps due to computational error in /2/. In our opinion the branching ratio of the $K\to\pi e\bar{e}$ process should be also four times smaller than that given in /2/. Namely, it is

$$\frac{\Gamma(K^+\to\pi^+e\bar{e})}{\Gamma(K^+\to\pi^0\bar{e}\nu)}=\frac{2\Gamma(K_S\to\pi^0e\bar{e})}{\Gamma(K_L\to\pi e\nu)}=\left(\frac{\alpha}{3\pi}Q\ln\frac{m^2_{\rho'}}{m^2_h}\cdot\cot\theta\right)^2 \qquad (9)$$

where $m_h$ is the typical uncharmed hadron mass, Q is the charge of P-quark. This result does not contradict to the experimental upper bound $\Gamma(K^+\to\pi^+e\bar{e})/\Gamma(K^+\to all)\lesssim 0.4\cdot 10^{-6}$ /8/, if $\ln\frac{m^2_{\rho'}}{m^2_h}\lesssim 6$, that is $m_{\rho'}\lesssim 20m_h$.

Some words about the assumptions under which our results are valid. The rates of the processes $K_L\to\mu^+\mu^-$ and $K\to\pi\nu\bar{\nu}$ are obtained under the assumption that we can neglect the strong interection in the region of the virtual momenta q $m_{\rho'}\lesssim q\lesssim m_W$. As to the process $K\to\pi e\bar{e}$, here the small momenta ($m_\rho\lesssim q\lesssim m_{\rho'}$) give the main contribution to its rate. Besides, we neglect the contribution of the diagram of the type 3 with $\gamma$-quantum substituted for Z-bozon (this diagram does not depend on the P'-quark and, hence, does not contain the large factor $\ln\frac{m^2_{\rho'}}{m^2_h}$). Therefore, formula (9) can be true if $\ln\frac{m^2_{\rho'}}{m^2_h}\gg 1$.

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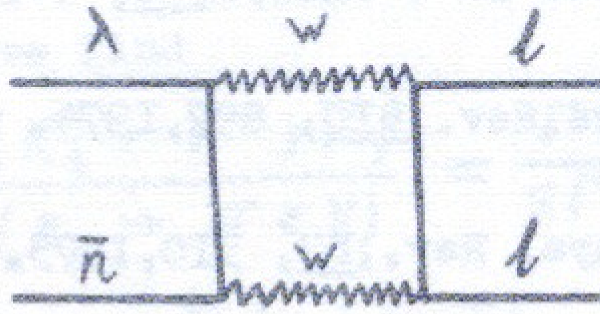


Fig 1.

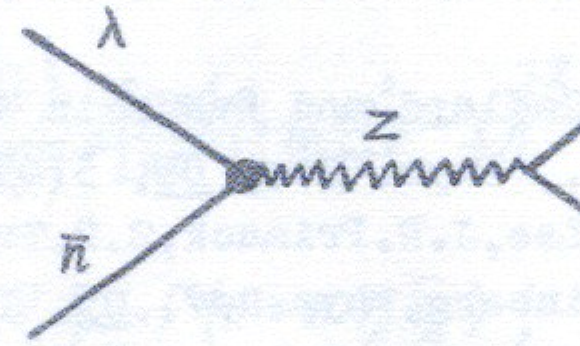


Fig 2.

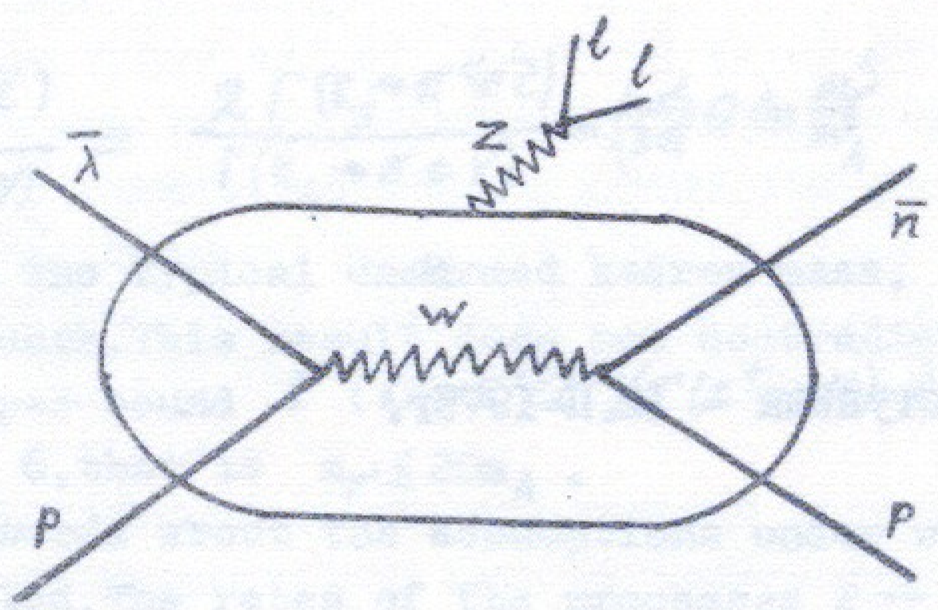


Fig 3.

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