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THE THEORY OF MAGNETIC POLE AND THE DIRAC-SCHWINGER QUANTIZATION CONDITION WITHOUT STRING

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THE THEORY OF MAGNETIC POLE
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The motion of nonrelativistic system of a point-like charge and monopole is considered. Lagrange and canonical formalism free from difficulties of the usual Dirac theory with a string is established without an introduction of a potential singular along a string. In particular, from the very beginning the theory is invariant under rotations.

Quantization of the equations of motion was carried out by standard methods. The Dirac condition $\frac{eg}{4\pi} = \frac{1}{2} n$ (n-integer) is obtained as a consequence of the quantization of the system's angular momentum projection on its symmetry axis.

1. Introduction

In 1931 Dirac[1] put forward the suggestion about the existence of a magnetic charge - monopole besides an electric one which would imply an elegant symmetry between the electric and magnetic properties of matter. A remarkable peculiarity of the Dirac theory is the condition

$$\frac{eg}{4\pi} = \frac{1}{2} n \; ; \quad n = 0, \pm 1, \pm 2, \ldots \qquad (1)$$

(e and g - electric and magnetic charges; the system of units $\hbar = c = 1$ is used) which provides an explanation for an empiric fact of the charge quantization.

Unsatisfactory feature of the Dirac theory is, however, the necessity of an artificial introduction of singularity lines for an electromagnetic field (Dirac strings) which do not correspond to any real physical peculiarities of the system. The point is that the equations of motion for the charge in a Coulomb magnetic field of the monopole cannot be represented in a canonical form in an ordinary way. The latter is necessary for the transition to quantum mechanics.

To avoid this difficulty Dirac replaced the monopole by half infinite and infinitely thin solenoid with the end at the point of monopole location. The vector potential of such solenoid with the end at the coordinates origin is equal to

$$\bar{A} = \frac{g}{4\pi r} \frac{[\bar{r}\bar{\nu}]}{r - \bar{r}\bar{\nu}} \qquad (2)$$

($\bar{\nu}$ - a unit vector in the direction of the solenoid) and the magnetic field

$$\bar{H} = \mathrm{rot}\, \bar{A} = \frac{g}{4\pi} \frac{\bar{r}}{r^3} + \bar{H}^f \qquad (3)$$

coincides with the monopole field in all the points except the points arranged on the solenoid itself (string) where $\bar{H}^f$ is not equal to zero and singular.

The classical equation of motion for a charge in the magnetic field (3) coincide with the equation of motion in the infinitely heavy monopole field for all trajectories not intersecting the string. At the same time one can obtain these equations from the standard Lagrangian

$$\mathcal{L} = \frac{m\bar{v}^2}{2} + e\bar{v}\bar{A} \tag{4}$$

and proceed then to Hamiltonian formalism. Here one should, however, impose additional (not following from the action principle) conditions which exclude the trajectories intersecting the string ("Dirac veto"). One should emphasize that the Lagrange function (4) is not invariant under rotations since it includes the fixed external string vector $\bar{\nu}$. The "Dirac veto" restores this invariance.

Further technical complications arise in the Dirac theory when the motion of the finite mass monopole is considered. In this case besides whith the coordinates of the monopole it is necessary to set an infinite number of the coordinates of the solenoid points - the string variables.

Dirac showed[2], however, that if "Dirac veto" is fulfilled non-physical variables of the string are excluded from the equation of motion. Hence, the classical theory of monopole with the string does not contain a real difficulties though seems to be not quite satisfactory from the aesthetic point of view. But the introduction of the string in classical theory is superfluous if one refuses to represent the equations of motion in Lagrange and Hamiltonian forms that is required for quantization only.

The influence of the string is completely revealed in quantum mechanics in which the probability amplitudes are defined by integrals of $\exp\{iS\}$ along all the trajectories, where S is the action corresponding to Lagrange function (4). In this case contributions of the trajectories passing from different sides of the string appear to differ from each other leading to the Bohm-Aharonov[3] effect on the string. Only when condition (1) is fulfilled this effect disappear and the string becomes unobservable so that the invariance of the theory under rotations is restored[4].

Thus, condition (1) being the most attractive aspect of the monopole theory seems to be tightly conneted whith the usage of the string for introduction of which one does not see any real physical grounds and which appear only at the intermediate stage of the consideration. This circumstance created numerous attempts to modify the Dirac theory in order to neutralize in any way the appearence of the string. However, up to now no variants of the monopole theory have been suggested which would not include the string or other anomalies casting doubt on logic and consistency of the theory.[12]

The aim of the present article is to establish Lagrange and canonical formalisms, not containing such anomalies, in nonrelativistic theory of the interecting point-like charge and monopole. In this connection me should note that all basic features and difficulties of the Dirac theory already arise in this case.

In Sec.2 the classical equations of motion of the system in question are analized. Then the Lagrange function is introduced leading to correct equations of motion for the all trajectories without any exceptions. This appears possible due to the choice of the collective variables similar to those which are used, for instance, when the motion of a symmetric top is described[5]. The most important peculiarity of this function is the absence of external parameters like the string vector $\bar{\nu}$ owing to which the invariance of the theory under rotations is obvious from the very beginning. The standard means allow us to proceed to canonical formalism without any complications. In this case for the Hamilton function and other dynamic variables the expressions obtained correspond to those which in the theory with the string are obtained by using vector potential

$$\bar{A} = \frac{g}{4\pi r}\,\frac{1}{2}\left(\frac{[\bar{r}\bar{\nu}]}{r-\bar{r}\bar{\nu}} - \frac{[\bar{r}\bar{\nu}]}{r+\bar{r}\bar{\nu}}\right) \tag{5}$$

which was considered by Schwinger[6] rather than the Dirac potential. The connection of this circumstance with the requirement of the invariance under rotations is revealed.

In Sec.3 the transition to quantum theory is performed. Condition (1) arises as a consequance of quantization of the angular momentum projection on the symmetry axis of system.

In Conclusion 4 comparison of our and Goldhaber's[7] approaches is presented.

2.Classical Theory

1. In nonrelativistic limit when, in particular, the radiative field may be neglected, the motion of the system consisting of a point-like charge and monopole is described by the equations

$$m_e \frac{d\bar{v}_e}{dt} = \frac{eg}{4\pi} \frac{1}{r^2} [\bar{v}\,\bar{n}]$$

$$m_g \frac{d\bar{v}_g}{dt} = -\frac{eg}{4\pi} \frac{1}{r^2} [\bar{v}\,\bar{n}] \qquad (6)$$

where $\bar{r} = \bar{r}_e - \bar{r}_g$, $\bar{n} = \frac{\bar{r}}{r}$, $\bar{v} = \bar{v}_e - \bar{v}_g$.

It is seen from these equations that the total momentum $m_e\bar{v}_e + m_g\bar{v}_g$ of the system is consered and the centre of inertia moves freely. For the relative motion from eq.(6) we obtain

$$\mu \frac{d\bar{v}}{dt} = \frac{eg}{4\pi} \frac{1}{r^2} [\bar{v}\,\bar{n}] = -\frac{eg}{4\pi} \frac{\bar{L}}{\mu r^3} \qquad (7)$$

(μ - the reduced mass, $\bar{L} = [\bar{r}\,\bar{p}]$).

Both kinetic energy of the relative motion and $|\bar{L}|$ are easily seen to conserve. However the direction of the vector $\bar{L}$ changes in the process of motion. The latter is connected with the fact that although the sum of the internal forces is equal to zero the total moment of these forces does not vanish. Such a situation is not quite ordinary in Newton's mechanics, where a interaction forces are directed along a straight line connecting the interacting particles, but it, nevertheless, can be described within a framework of analytical mechanics.

To achieve this let us rewrite the equations of motion in a form more convenient for the futher. Two purely kinematic relations can be obtained directly from the equality

$$\bar{v} = \frac{d\bar{r}}{dt} = \bar{n}\dot{r} + r\frac{d\bar{n}}{dt} \qquad (8)$$

Those relations are:

$$\frac{d\bar{n}}{dt} = \frac{1}{\mu r^2} [\bar{L}\,\bar{n}] \qquad (9)$$

$$\frac{d\bar{v}}{dt} = \bar{n}\left(\ddot{r} - \frac{\bar{L}^2}{\mu^2 r^3}\right) + \frac{1}{\mu r}\left[\frac{d\bar{L}}{dt}\,\bar{n}\right] \qquad (10)$$

Substituting (10) in (7) we obtain equations

$$\mu\ddot{r} = \frac{\bar{L}^2}{\mu r^3} \qquad (11)$$

$$\frac{d\bar{L}}{dt} = \frac{eg}{4\pi} \frac{1}{\mu r^2} [\bar{L}\,\bar{n}] \qquad (12)$$

If one introduces the vector[7,8]

$$\bar{J} = \bar{L} - \frac{eg}{4\pi}\bar{n} \qquad (13)$$

eq. (12) with the account of (9) may be represented in equivalent form

$$\frac{d\bar{J}}{dt} = 0 \qquad (14)$$

Now we'll derive eqs. (11) and (14) by means of the variation principle. The motion of the system under consideration consists in changing of the distance r between the particles and rotating it as a whole under the action of a pair of internal forces. We'll denote the angular velocity of this rotation by Ω . Then

$$\bar{v} = \bar{n}\dot{r} + r[\bar{\Omega}\bar{n}] \tag{15}$$

and eqs.(11),(14) may be obtained from the Lagrangian

$$\mathcal{L} = \frac{\mu\dot{r}^2}{2} + \frac{\mu r^2[\bar{\Omega}\bar{n}]^2}{2} - \frac{eg}{4\pi}\bar{n}\bar{\Omega} \tag{16}$$

In fact the radial equation corresponding to (16) has a form

$$\mu\ddot{r} = \mu r[\bar{\Omega}\bar{n}]^2 \tag{17}$$

But due to (15)

$$\bar{L} = \mu r^2[\bar{n}[\bar{\Omega}\bar{n}]] \tag{18}$$

as a result of which (17) coincides with (11). Then

$$\frac{\partial\mathcal{L}}{\partial\bar{\Omega}} = \bar{L} - \frac{eg}{4\pi}\bar{n} = \bar{J} \tag{19}$$

The derivative (19) with respect to time is determined by the behaviour of Lagrange function (16) under rotation of the system through the infinitely small angle $\delta\bar{\omega}$ /5/

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\bar{\Omega}} = \frac{\partial\mathcal{L}}{\partial\bar{\omega}} \tag{20}$$

But the Lagrangian (16) is a scalar, which is expressed by vectors $\bar{n}$ and $\bar{\Omega}$ only characterizing the system itself, and does not contain in contrast to (4) any external fixed parameters. Therefore it is invariant under rotations and (20) reduces to (14). The vector $\bar{J}$ in accordance with the mentioned here should be interpreted as a angular momentum of the system.

Thus, the Lagrange function (16) satisfies all the requirements following from homogeneity and isotropy of the space while describing a closed system.

Now proceed to the Hamiltonian formalism. For this purpose one should set explicitly generalized coordinates and velocities of the system. To achieve this we introduce the reference frame K', strictly connected with the system of the charge and monopole, at the same time with the inertial centre-of-mass frame K. This frame rotates with respect to K with the angular velocity $\bar{\Omega}$ and the orientation of its axes with respect to K axes is given by three Euler angles α, β, γ /5/. It is natural to choose the z' axis of the rotating frame along the system's symmetry axis - a line connecting the charge and monopole. In this case the angles α and β coincide with the angles φ and θ of a spherical system of coordinates. At the same time, as it will be seen below, the angle γ disappears from the classical equations of motion, so that the three variables r, α, β are essential generalized coordinates in accordance with a true number of degrees of freedom for the classical system.

Using known /5/ components of the angular velocity $\bar{\Omega}$ along the axes of the moving reference frame we obtain

$$\mathcal{L} = \frac{\mu\dot{r}^2}{2} + \frac{\mu r^2}{2}\left(\dot{\beta}^2 + \sin^2\beta\,\dot{\alpha}^2\right) - \frac{eg}{4\pi}\left(\dot{\alpha}\cos\beta + \dot{\gamma}\right) \tag{21}$$

from which we find canonical impulses

$$P_r = \frac{\partial\mathcal{L}}{\partial\dot{r}} = \mu\dot{r}, \quad P_\beta = \frac{\partial\mathcal{L}}{\partial\dot{\beta}} = \mu r^2\dot{\beta}$$

$$P_\alpha = \frac{\partial\mathcal{L}}{\partial\dot{\alpha}} = \mu r^2\sin^2\beta\,\dot{\alpha} - \frac{eg}{4\pi}\cos\beta, \quad P_\gamma = \frac{\partial\mathcal{L}}{\partial\dot{\gamma}} = -\frac{eg}{4\pi} \tag{22}$$

The equations of motion which are derived by variation of the action corresponding to the Lagrange function (21) over r, α, and β are equivalent, as can be easily seen, to eqs. (11) and (12). While the variation over γ provides a trivial relationship $dP_\gamma/dt = 0$ only. However, the presence of the canonical impulse P_γ will play an important role in quantization.

In an ordinary way the Hamiltonian function is obtained from eqs. (21) and (22)

$$H = \frac{P_r^2}{2\mu} + \frac{P_\beta^2}{2\mu r^2} + \frac{(P_\alpha - P_\gamma\cos\beta)^2}{2\mu r^2\sin^2\beta} \qquad (23)$$

Components of the vector $\bar{J}$ along the axes of the fixed K and moving K'reference frames are also of interest. In the frame K

$$J_1 = -\cos\alpha\, ctg\beta\, P_\alpha - \sin\alpha\, P_\beta + \frac{\cos\alpha}{\sin\beta} P_\gamma$$
$$J_2 = -\sin\alpha\, ctg\beta\, P_\alpha + \cos\alpha\, P_\beta + \frac{\sin\alpha}{\sin\beta} P_\gamma \qquad (24)$$
$$J_3 = P_\alpha$$

from which Poisson brackets for the angular momentum components follow

$$\{J_i, J_j\} = -\varepsilon_{ijk} J_k, \quad \{J_i, J^2\} = 0 \qquad (25)$$

In the frame K'

$$J'_1 = -\frac{\cos\gamma}{\sin\beta} P_\alpha + \sin\gamma\, P_\beta + \cos\gamma\, ctg\beta\, P_\gamma$$
$$J'_2 = \frac{\sin\gamma}{\sin\beta} P_\alpha + \cos\gamma\, P_\beta - \sin\gamma\, ctg\beta\, P_\gamma \qquad (26)$$
$$J'_3 = P_\gamma$$

and

$$\{J'_i, J'_j\} = \varepsilon_{ijk} J'_k, \quad \{J'_i, \bar{J}^2\} = 0 \qquad (27)$$

As it is seen from (26) the canonical impulse P_γ has the meaning of the projection of the angular momentum $\bar{J}$ along the system's symmetry axis

$$P_\gamma = \bar{n}\bar{J} \qquad (28)$$

Taking into account (24) and (28) the Hamiltonian function may be represented in the form

$$H = \frac{P_r^2}{2\mu} + \frac{\bar{J}^2 - (\bar{n}\bar{J})^2}{2\mu r^2} \qquad (29)$$

Finally, write down the components of the kinetic momentum of relative motion $\bar{P} = \mu\bar{v}$ along the axes of the frame K

$$P_1 = P_1^{(0)} + \sin\alpha\, \frac{1}{r}\, ctg\beta\, P_\gamma$$
$$P_2 = P_2^{(0)} - \cos\alpha\, \frac{1}{r}\, ctg\beta\, P_\gamma \qquad (30)$$
$$P_3 = P_3^{(0)}$$

where $\bar{P}^{(0)}$ is the momentum at $y = 0$ ($P_\gamma = 0$).

Now compare obtained expressions with those arising from an ordinary theory with the string. It is easily seen that they practically coincide with those that follow from Lagrange function (4) with a singular Schwinger potential with the string directed along the Z axis of the K frame. The term $e\bar{v}\bar{A}$ in (4) equal in our designations to

$$e\bar{v}\bar{A} = -\frac{eg}{4\pi}(\dot{\alpha}\cos\beta + \dot{\alpha}) \qquad (31)$$

in case (2) and

$$e\bar{v}\bar{A} = -\frac{eg}{4\pi}\dot{\alpha}\cos\beta \qquad (32)$$

in case (5) should be compared with the term

$$-\frac{eg}{4\pi}\bar{n}\bar{\Omega} = -\frac{eg}{4\pi}(\dot{\alpha}\cos\beta + \dot{\gamma}) \qquad (33)$$

in (21). All three expressions differ only by terms, which are the total derivatives of angular coordinates with respect to the time and lead therefore to equal almost for all trajeqtories equations of motion. (As, for example, the α coordinate is not a simple function of the point(while going round the Z axis it changes by 2π), the addition of the term $\dot{\alpha}$ to the Lagrangian changes, strictly speaking, the equations of motion. That is this addition causes the appearance of a magnetic field which is not equal to zero along z axis only.) Nevertheless, the canonical impulses and the expressions H, $\vec{J}$, $\vec{p}$ appear to be, generally speaking, different. The latter is essential for transition to quantum mechanics.

It is important to note that neither (31) nor (32) have by themselves invariant forms under rotations and can be represented in contrast to (33) as a scalar products without introducing fixed external vectors. However, (32) leads to the difference only in the impulse P_γ, while (31) distors all the expressions discussed. As a result the quantum equations obtained in the theory with the Schwinger potential also appear to be closer to the true ones than those obtained by using the Dirac potential.

3. Quantum Theory

To proceed to the quantum theory the classical function of Hamilton should be substituted, in accordance with a common rules, by the operator

$$\hat{H} = \frac{\hat{P}_r^2}{2\mu} + \frac{\hat{\vec{J}}^2 - (\vec{n}\hat{\vec{J}})^2}{2\mu r^2} \qquad (34)$$

$$\hat{P}_r^2 = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) \qquad (35)$$

$$\hat{\vec{J}}^2 = -\left\{\frac{1}{\sin\beta}\frac{\partial}{\partial\beta}\left(\sin\beta\frac{\partial}{\partial\beta}\right) + \frac{1}{\sin^2\beta}\left(\frac{\partial^2}{\partial\alpha^2} - 2\cos\beta\frac{\partial^2}{\partial\alpha\partial\gamma} + \frac{\partial^2}{\partial\gamma^2}\right)\right\} \qquad (36)$$

$$\vec{n}\hat{\vec{J}} = -i\frac{\partial}{\partial\gamma} \qquad (37)$$

In a similar way, instead of (24) we have operators

$$\hat{J}_\pm = \hat{J}_1 \pm i\hat{J}_2 = i e^{\pm i\alpha}\left(\mathrm{ctg}\,\beta\frac{\partial}{\partial\alpha} \mp i\frac{\partial}{\partial\beta} - \frac{1}{\sin\beta}\frac{\partial}{\partial\gamma}\right) \qquad (38)$$

$$J_3 = -i\frac{\partial}{\partial\alpha} \qquad (38)$$

satisfying the commutation relations

$$[\hat{J}_i, \hat{J}_j] = i\varepsilon_{ijk}\hat{J}_k, \quad [\hat{J}_i, \hat{\vec{J}}^2] = 0 \qquad (39)$$

Besides, it is obvious that

$$[\hat{J}_i, \vec{n}\hat{\vec{J}}] = [\hat{\vec{J}}^2, \vec{n}\hat{\vec{J}}] = 0 \qquad (40)$$

Hence, the angular part of the relative motion wave function is an eigenfunction of the operators $\hat{\vec{J}}^2$, $\hat{J}_3$ and $\vec{n}\hat{\vec{J}}$. From the theory of representations of the rotation group it is known [9] that eigenvalues of these operators are equal respectively to j(j+1); m,m'=-j, -j+1,..., j,where j is a integer or half-integer number and the eigenfunction coincide with the matrix elements of the operator of finite rotations

$$D^j_{m'm}(\alpha,\beta,\gamma) = e^{im'\gamma}\, d^j_{m'm}(\beta)\, e^{im\alpha} \qquad (41)$$

Now taking into account the fact, that the eigenvalues of the operator $\hat{p}_\gamma = \vec{n}\hat{\vec{J}}$ coincide with $-\frac{eg}{4\pi}$, we come to condition (1). Thus, in the monopole theory there is a remarkable relation between the space and charge properties of the system.

As it follows from previous discussions, the wave function of a relative motion depends on a new variable γ - the angle of the rotation around the axis of the system symmetry. We emphasize once more that only by virtue of introduction this variable it is possible to establish the theory in an obviously invariant under rotations form. Nevertheless when $-\frac{eg}{4\pi} = m'$ is fixed the dependence on γ comes into the phase of the wave function only and disappears from all matrix elements.

Two classes of charge - monopole pairs exist depending on whether j is integer or half-integer. One of them corresponds to the integer j·s and is described by the simple functions α and γ. The parameter $\frac{eg}{4\pi}$ may be a integer number only in this case. The other class corresponding to the half-integer j's is described by two-valued functions and $\frac{eg}{4\pi}$ is a half-integer number. The appearance of two-valued functions seems surprise in considering the system consisting of spinless particles, however within the framework of nonrelativistic quantum mechanics there are perhaps no arguments excluding this possibility. At the same time in the relativistic quantum field theory, as Schwinger believe/8/, only integer $\frac{eg}{4\pi}$ are allowed.

Finally, the operators

$$\hat{J}'_\pm = \hat{J}'_1 \pm i\hat{J}'_2 = i e^{\mp i\gamma}\left(\frac{1}{\sin\beta}\frac{\partial}{\partial\alpha} \mp i\frac{\partial}{\partial\beta} - \mathrm{ctg}\,\beta\frac{\partial}{\partial\gamma}\right) \quad (42)$$

change the eigenvalue of the operator $\hat{P}_J = \bar{n}\hat{\bar{J}}$ by one unit and thus connect the wave functions of the system of the given class with various values $\frac{eg}{4\pi}$.

4. Conclusion

The work of Goldhaber/7/, where, in particular, the role of the conserving vector $\bar{J}$ is emphasized, was of great importance for the author while considering the problems of this article. At the same time there is an important difference between Goldhaber's and our approaches which is worth of a more detailed consideration.

Having introduced the vector (see (13))

$$\bar{S} = -\frac{eg}{4\pi}\bar{n}, \quad (43)$$

the author of the paper/7/ considers it (in direct inconsistence with its real properties) as a angular momentum with independent degrees of freedom and assumes, in particular, that

$$\{S_i, S_j\} = -\varepsilon_{ijk} S_k \quad (44)$$

The system of charge and monopole is considered to be essentially unclosed. In this consideration the angular momentum $\bar{S}$ is

ascribed to the (static!) electromagnetic field. As Wilson/13/ noted, the quantization of this angular momentum in units of a half Planck constant leads to condition (1). Goldhaber's arguments are close to those of Wilson in this point. It is clear, however, that the assumption about the character of quantization of the additional angular momentum $\bar{S}$ is arbitrary since its properties in fact remain unclear. Thus the basic relations (44) in the theory of angular momentum do not follow from definition (43) but simply contradict to it.

From our point of view the consideration of $\bar{S}$ as an independent angular momentum is incorrect. There exists only the angular momentum $\bar{J}$ of a closed system, consisting of a charge and monopole. To make this statement clearer compare our system with the system of two electric charges. The electrostatic energy of this system may be represented in two forms: either as the energy of an electric field $\bar{E}_1 + \bar{E}_2$ distributed in space with density $\frac{1}{4\pi}\bar{E}_1\bar{E}_2$ or as the potential energy of the charges interection. In the latter case it is expressed through the variables (coordinates) of the particles only, and the static field appears to be completely excluded from the consideration. Therefore the nonrelativistic two charge system is closed.

There is a similar situation in case of the system of a charge and monopole. The angular momentum of this system becomes equal $\bar{J} = [\bar{r}\bar{p}] - \frac{eg}{4\pi}\bar{n}$ after exluding static fields analogous to the energy of two charges system that gains the additional term $e_1 e_2 / r$. Then, we again deal with the closed system of particles.

In both cases only the regarding of the radiation field necessary for the transition to relativistic consideration requires the introduction of a new degrees of freedom for the photon spin description.

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